

ISSN 2318-2377



TEXTO PARA DISCUSSÃO Nº 647

A PROCESS OF DEMAND DISCOVERY FROM A SMITHIAN PERSPECTIVE

**Michele Bee
Juan Pablo Gama**

Setembro de 2022

Universidade Federal de Minas Gerais

Sandra Regina Goulart Almeida (Reitora)
Alessandro Fernandes Moreira (Vice-Reitor)

Faculdade de Ciências Econômicas

Hugo Eduardo Araujo da Gama Cerqueira (Diretor)
Kely César Martins de Paiva (Vice-Diretora)

Centro de Desenvolvimento e Planejamento Regional (Cedeplar)

Frederico Gonzaga Jayme Jr (Diretor)
Gustavo de Britto Rocha (Vice-Diretor)

Laura Rodríguez Wong (Coordenadora do Programa de Pós-graduação em Demografia)

Rafael Saulo Marques Ribeiro (Coordenador do Programa de Pós-graduação em Economia)

Ana Paula de Andrade Verona (Chefe do Departamento de Demografia)

Pedro Vasconcelos Maia do Amaral (Chefe do Departamento de Ciências Econômicas)

Editores da série de Textos para Discussão

Aline Souza Magalhães (Economia)
Adriana de Miranda-Ribeiro (Demografia)

Secretaria Geral do Cedeplar

Maristela Dória (Secretária-Geral)
Simone Basques Sette dos Reis (Editoração)

<http://www.cedeplar.ufmg.br>

Textos para Discussão

A série de Textos para Discussão divulga resultados preliminares de estudos desenvolvidos no âmbito do Cedeplar, com o objetivo de compartilhar ideias e obter comentários e críticas da comunidade científica antes de seu envio para publicação final. Os Textos para Discussão do Cedeplar começaram a ser publicados em 1974 e têm se destacado pela diversidade de temas e áreas de pesquisa.

Ficha catalográfica

B414p	Bee, Michele -. 2022	A process of demand discovery from a smithian perspective / Michele Bee, Juan Pablo Gama. - Belo Horizonte: UFMG / CEDEPLAR, 2022. 20 p. : il. - (Texto para discussão, 647) Inclui bibliografia. ISSN 2318-2377 1. Concorrência. 2. Preços. 3. Mercado . I. Gama, Juan Pablo. II. Universidade Federal de Minas Gerais. Centro de Desenvolvimento e Planejamento Regional. III. Título. IV. Série. CDD: 330
-------	-------------------------	---

Elaborado por Rosilene Santos CRB-6/2527
Biblioteca da FACE/UFMG. – RSS/109/2022

As opiniões contidas nesta publicação são de exclusiva responsabilidade do(s) autor(es), não exprimindo necessariamente o ponto de vista do Centro de Desenvolvimento e Planejamento Regional (Cedeplar), da Faculdade de Ciências Econômicas ou da Universidade Federal de Minas Gerais. É permitida a reprodução parcial deste texto e dos dados nele contidos, desde que citada a fonte. Reproduções do texto completo ou para fins comerciais são expressamente proibidas.

Opinions expressed in this paper are those of the author(s) and do not necessarily reflect views of the publishers. The reproduction of parts of this paper of or data therein is allowed if properly cited. Commercial and full text reproductions are strictly forbidden.

**UNIVERSIDADE FEDERAL DE MINAS GERAIS
FACULDADE DE CIÊNCIAS ECONÔMICAS
CENTRO DE DESENVOLVIMENTO E PLANEJAMENTO REGIONAL**

A PROCESS OF DEMAND DISCOVERY FROM A SMITHIAN PERSPECTIVE

Michele Bee

Department of Economics, Federal University of Minas Gerais Antônio Carlos Avenue, 6627 Belo Horizonte - MG - Brazil.
Zip 31270-901. Email: michelebee@cedeplar.ufmg.br

Juan Pablo Gama

Department of Economics, Federal University of Minas Gerais Antônio Carlos Avenue, 6627 Belo Horizonte - MG - Brazil.
Zip 31270-901. Email: jpgamat@cedeplar.ufmg.br

**CEDEPLAR/FACE/UFMG
BELO HORIZONTE
2022**

SUMÁRIO

1. INTRODUCTION	6
2. MARKETPLACE WITH INSTANT COMPETITION AMONG CONSUMERS AND PRODUCERS	7
2.1. Market place model with a deterministic natural demand.....	7
2.2. The marketplace game.....	10
2.3. Results	10
3. MARKETPLACE WITH INSTANT COMPETITION AMONG CONSUMERS AND NOT AMONG PRODUCERS	13
3.1. Market place model with a deterministic natural demand.....	13
3.2. The one-day marketplace game.....	13
3.3. The marketplace game.....	15
3.4. Results	15
4. CONCLUSIONS	18
REFERENCES	19

ABSTRACT

We propose a theoretical representation of how agents estimate demand through a market learning process. To this end, we model a traditional marketplace by interpreting Smith's theory of the convergence of the market price to the natural price, understood as the reserve price. In this model, natural price is obtained through a bargaining process between consumers and producers. We use a repeated multi-period game with producers deciding their offers and both, consumers and producers, in an imperfect type of competition. Producers estimate the demand at the reserve price thanks to information provided by competition as rivalry between consumers and between producers. But the stronger this competition is, the slower the discovery process.

RESUMO

Propomos uma representação teórica de como os agentes estimam a demanda por meio de um processo de aprendizado de mercado. Para esse fim, modelamos um mercado tradicional, interpretando a teoria de Smith sobre a convergência do preço de mercado no preço natural, entendido como o preço da reserva. Neste modelo, o preço natural é obtido através de um processo de negociação entre consumidores e produtores. Utilizamos um jogo repetido de vários períodos com os produtores decidindo suas ofertas e ambos, consumidores e produtores, em um tipo de competição imperfeita. Os produtores estimam a demanda pelo preço reserva devido às informações fornecidas pela concorrência como rivalidade entre os consumidores e entre os produtores. Mas quanto mais forte essa competição for, mais lento será o processo de descoberta.

Keywords: Demand discovery, natural price, rivalry, imperfect competition.

JEL Classification: B12, C72, C73, D43.

1. INTRODUCTION

In this paper we describe the process that leads producers to discover consumer demand at the reserve price. We develop an inter-temporal model based on a repeated game representing a marketplace. Agents do not act as in a perfect competition model. Consumers compete on prices in the case of excess of demand. Producers compete on prices in the case of excess of supply. Producers are concerned with the proper evaluation of goods that adequately remunerates costs. This evaluation is done through a bargaining process, in which the reserve price is established as the ‘natural price’ in Adam Smith’s theory. Proposing an interpretation of his theory, we show that competition may play a different role in price formation from that assumed in neoclassical models. Competition as rivalry among consumers as well as among producers move the market price away from the natural price. The convergence of the market price towards the natural price is due to the fact that producers are concerned with the market price being as close as possible to the natural price, tending to satisfy the demand of all consumers willing to pay the natural price. Therefore, the stronger the competition as rivalry, the slower the convergence towards the natural price.

Whether one can present Smith’s price formation argument as in the neoclassical model is still debated. Garegnani (1983) argues that in Smith there is no curve of demand, but only a point in the Cartesian axes that represents the effective demand, i.e. the demand of consumers ready to pay at the natural price (see also Tsoulfidis 2010). Thus, he does not consider in Smith a process of demand discovery. On the other hand, Blaug (1997) presents Smith’s theory in terms of a standard supply and demand diagram, while stating that too often scholars overlook that Smith describes the ‘process’ leading to natural price and not the ‘final state’ of the timeless perfect competition. He sees in this process competition as rivalry among producers as well as among consumers (as also Stigler 1957). But differently from the model proposed here, he states that rivalry drives the market price towards, and not away from, natural prices. Smith (2012) proposes an explanation of this process through simulation of agent-based models, in which producers adapt their strategies based on their sales history. Differently from the model proposed here, this model is not strategic and assumes profit maximization of producers instead of their willingness to get as close as possible to natural price. More recently, Inoua and Smith (2020) define a large market model based on Adam Smith’s theory of the market design using abundance or scarcity of the good to characterize the equilibrium price and its relationships with the natural price and the monopoly price, and Inoua and Smith (2021a) develop an information theory of market price formation using multilateral haggling and bargaining (see also Inoua and Smith 2021b). In economic theory, the price formation is usually associated with Auction models in which buyers and sellers have private information about how they value the negotiated good or asset. Grossman (1976) and Grossman and Stiglitz (1976) had showed that when agents are price-takers there is no equilibrium since, in any of them, agents do not have incentives to negotiate the information needed by the other agents. Jackson (1991) showed that if agents are not price takers, negotiations of information occur, and equilibrium is ensured. Several authors including Sznajd-Weron and Weron (2002) and Smith (2012) use agent-based models to analyze the price formation. However, in most of the existing literature, producers are maximizing their profits while consumers look for the lowest possible prices.

Our model is concerned about the dynamics of the price formation in the absence of information on consumer demand and its properties in the presence of over-demand or over-supply. Moreover, our

model shows the existence of competition among consumers in the former and competition among producers in the latter. Therefore, agents are far from being price-takers, and the convergence to the natural price is not a consequence of the competition of firms as free-entry and free-exit. Instead, it is due to the haggling and bargaining process that defines the natural price and to the producers' strategies to bring the market price back to the natural price in response to imbalances in the market that instead push it away.

Strong competition as rivalry gives information to the other agents involved at the marketplace since the change on the market price help them to compute the demand or supply elasticity. However, in our model, it slows down the convergence to the natural price since it is more difficult to the agent to estimate properly the natural price. Competition as rivalry informs producers of an imbalance in the market. But the stronger this information is, the less information producers have on how to reduce this imbalance. This counter-intuitive result goes in a similar direction as Kaya and Liu (2015) in which the observability leads to a reduce the "Coasian effect" in a sequential bargaining model of price formation.

2. MARKETPLACE WITH INSTANT COMPETITION AMONG CONSUMERS AND PRODUCERS

Let us represent the negotiations in a traditional marketplace as a dynamic model with a finite number of producers and consumers. The goal of producers is not the maximization of profit, as in the neoclassical model, but the proper evaluation of their good by consumers, that is, the natural price. By gaining the right appreciation from consumers, producers find confirmation of their self-esteem, which for Smith is perhaps the strongest of human motives for acting (see Smith 1759 VI.i.3, see Bee 2021, on exchange and other's approval in Smith see also Sugden 2002). Producers do not know the market demand, that is, the amount of goods demanded by the consumers at each market price. Thus, producers do not know even the effective demand, that is, the amount of goods demanded by the consumers at the natural price. However, they use the information that they observed in previous days to choose their production strategy so as to get as close as possible to the natural price. Each day, after producers decide the amount of good that is produced, they go to the marketplace and sell all their products in a price competition where all produced goods are perfect substitutes. Note that goods are perishable, that is, all the production that is not sold cannot be offered to the consumers in the future. Therefore, the model is a repeated game in which each subgame represents one day at the marketplace.

2.1. Market place model with a deterministic natural demand

Each day can be represented as a three-period game. In the first period, which is before the marketplace is open, producers decide their production strategy in function of the information they have. In the second period the haggling and bargaining process takes place in which the natural price is defined. In the third period the competition among consumers or producers takes place in the case of excess of demand or supply and the market price is defined.

At the beginning of the day t , producers define their amount of good produced, that is, y_t^j .

If their supplies are such that no more no less is demanded than it was originally produced, the market price “naturally comes to be either exactly, or as nearly as can be judged of, the same with the natural price” (see Smith 1776 I.vii.11). When there is “perfect liberty” the natural price is the “lowest price”, that is, the reserve price (Smith 1776 I.vii.6). Note that, at the beginning of the day, producers do not know precisely the reserve price of the good since part of the information needed to do so is not known, that is, they only know that the natural price is in the following interval $[p_{f_j}^N, \bar{p}_{f_j}^N]$. This interval comes from an expectation based on the ordinary rates of wages, rent and profit, according to which producers must remunerate their production costs (Smith 1776 I.vii.1-5). The estimation of these rates is based on an assessment of ‘the general circumstances of the society’ and ‘the particular nature of each employment’ (see Smith 1776 I.vii.1; see also Garegnani 1983, Rashid 1992, Roncaglia 2006, Aspromourgos 2009, Naldi 2013, Menudo 2013). But, as Smith says, “it is not easy to find any accurate measure” to assess production costs (Smith 1776 I.v.4). However, producers can go to the market and, through the bargaining process, use the information available to producers and consumers. Producers transparently share their information with consumers because they want a proper evaluation of their good (see Bee 2021; on the transparent language of bargaining in Smith see Brown 1994). Through information sharing that takes place in “the higgling and bargaining of the market”, the assessment of production costs and thus the reserve price is established “according to that sort of rough equality which, though not exact, is sufficient for carrying on the business of common life” (Smith 1776 I.v.4).

Therefore, at the end of the bargaining process, the exchangers agree about the value of the natural price p^N

$$p^N \in [p_{f_1}^N, \bar{p}_{f_1}^N] \cap [p_{f_2}^N, \bar{p}_{f_2}^N].$$

Note that in $t = 1$, the bargaining process is done. Since the information needed to obtain the natural price is known after the date $t = 1$, it is not needed this type of bargaining process at every $t \geq 2$ unless the natural price changes which is not the case since we are modeling a market with constant costs. In the case of non-constant costs, the bargaining process must be restarted at the beginning of each day.

In the case of an enormous oversupply or overdemand, producers will try to sell at the beginning of the day at an intermediate price at the interval $[p_f^N, \bar{p}_f^N]$, but competition will start so early that the necessary bargaining to establish the natural price will be prevented. The more the excess is reduced in the following days (by adjusting the quantities brought to market), the later the competition will be triggered, giving room at the beginning of the day for the bargaining process.

If the supplies $\{y_t^j\}_{j=1}^2$ are such that more is demanded than it was originally produced $D(p^N) \geq \sum_j y_t^j$, the prices rise from p^N to $p_t^j \geq p^N$ in which the demand is equal to the supply $\sum_j D^j(p_t^1, p_t^2) = \sum_j y_t^j$. This happens because in this case not all the consumers who are willing to pay the natural price can be satisfied. Some of them, therefore, might be willing to pay more than the natural price, thus giving rise to competition among consumers to grab the few goods. The competition is more or less strong depending on the consumers’ purchasing capacity and their desire or need not to return

from the market empty-handed (see Smith 1776 I.vii.9). Note that in this model the competition starts immediately because of the importance of this desire or need.

On the other hand, if the supplies $\{y_t^j\}_{j=1}^2$ are such that less is demanded than it was originally produced $D(p^N) \leq \sum_j y_t^j$, the prices falls from p^N to $p_t^j \leq p^N$ in which the demand is equal to the supply $\sum_j D^j(p_t^1, p_t^2) = \sum_j y_t^j$. This happens because in this case producers noticed that they will not sell all at the end of the day at the natural price. Some of them, therefore, might be willing to sell less than the natural price, thus giving rise to competition among producers to grab the few consumers. The competition is more or less strong depending on the size of the oversupply and the desire or need of the producers not to return from the market with unsold goods (see Smith 1776 I.vii.10). Note that in this model the competition starts immediately because of the importance of this desire or need.

Note that in this case, consumer behaves as price takers in the presence of excess of supply, but they compete on prices in the presence of excess of demand at the natural price as it was mentioned above. Since we are in an imperfect type of competition with perfectly substitute goods, we have that the demand of the producer j is given by

$$D^j(y^1, y^2, p^1, p^2) = \begin{cases} 0 & \text{if } p^j > p^{3-j}, \\ D(p^j) & \text{if } p^j < p^{3-j}, \\ D(p^j) & \text{if } p^j = p^{3-j}, \end{cases}$$

where p^1 and p^2 are the price of the producer 1 and 2 respectively.

When producers decide their strategy, they are concerned that consumers evaluate properly the good, that is, the market price is as close as possible to the natural price p^N . Then,

$$\pi^j(y_t^1, y_t^2, p_{t,1}^2, p_{t,2}^2) = -(p^N - p_t^j)^2.$$

The aim of producers of selling at the natural price is different from maximizing profit as in the neoclassical theory. Several authors used economic theory framework to analyze the price formation as Adam Smith did in chapter VII of Book I of Smith (1776), but always assuming the profit maximization of producers. Our model tries to analyze another possible interpretation of Adam Smith's explanation of price formation in Smith (1776).

If producers know the demand function, the Nash equilibrium for this game is a production allocation (y^1, y^2) such that $y^j = D^j(y^1, y^2, p^N, p^N) = D(p^N)/2$, that is, all producers chooses their "natural production", the production plan that induces the natural price as the market price. This happens already at the beginning of the day only in the neoclassical model. In our model, instead, it is a matter of discovering an unknown demand, based on the demand in previous days. Each producer observes only the realization D^j at the prices during the day and at the end of day $\{p_t^j\}_{j=1}^2$. However, producers can estimate the demand function based on the demand in previous days.

2.2. The marketplace game

It is a repeated game in which a one-day marketplace game takes place every day as described above. In $t = 1$, the producers do not have any information about the consumers. Then, each producer chooses a production strategy $y_1^j \in \mathbb{R}^+$.

In $t = 2$, the producers will use the information that they obtained from $t = 1$ about the effective demand to choose the new production strategy, y_2^j . In $t > 2$, the producers will decide y_t^j based on the market price and production strategy history. We will now analyze the properties of the sequence of the producer strategy $\{y_t^j\}_t$ and the sequence of the market prices $\{p_t^j\}_t$.

2.3. Results

If $p_1^j < p^N$ in $t = 1$, that is in the case of excess of supply, the producers only have information about one point of the demand at a price below the natural price. Then, the only information that they can obtain from it is that the production level needed to reach the natural price is below of the one chose by them. Therefore, $y_2^j < y_1^j$ for all $j = 1, 2$, then $\bar{p}_2^j < \bar{p}_1^j$ for $j = 1, 2$. If the aggregate production level is still lower than $D(p^N)$, the producers will decrease their production level in $t = 3$ similarly as from day $t = 1$ to day $t = 2$.

Analogously, if $\bar{p}_1^j > p^N$ in $t = 1$, that is in the case of excess of demand, the producers only have information about one point of the demand at a price above the natural price. Then, the only information that they can obtain from it is that the production level needed to reach the natural price is above of the one chose by them. Therefore, $y_2^j > y_1^j$ for all $j = 1, 2$, then $\bar{p}_2^j < \bar{p}_1^j$ for $j = 1, 2$. If the aggregate production level is still higher than $D(p^N)$, the producers will increase their production level in $t = 3$ similarly as from day $t = 1$ to day $t = 2$. Therefore, we have the following result.

Proposition 1. *For any initial production strategy $y_1^j \in \mathbb{R}^+$, any production strategy that is sequentially optimal for the producers, $\{y_t^j\}_t$, are such that the market price induced p_t^j converges to the natural price p^N . Moreover,*

1. *if there is a day t such that the market price is lower than the natural price, $p_t^j < p^N$, the market price satisfies that $p_{t+k}^j > p_t^j$ for all $k \in \mathbb{N}$, and*
2. *if there is a day t such that the market price is larger than the natural price, $p_t^j > p^N$, the market price satisfies that $p_{t+k}^j < p_t^j$ for all $k \in \mathbb{N}$.*

Proof. Let us see the first case. Since producers know that by reducing quantity there will be less oversupply and thus less competition will be triggered among them and therefore the market price will be lower, which means that the demand function $D(\cdot)$ is a decreasing function and the market prices are such that $p_t^j < p^N$, they know that the natural aggregated quantity must be less than the quantity

produced at date t . Therefore, it is optimal for the producers to reduce their production plans onward, making that $p_{t'}^j > p_t^j$ for all $t' > t$. The proof of the second part is analogous. ■

When the price is higher than the natural price due to upward competition among consumers, producers realize that they are in a supply deficit. However, they do not know by how much they need to increase supply to meet effective demand. The only thing they know is that in the future they do not have to offer less and that by offering more the market price will be lower (because there will be less competition among consumers) and thus closer to the natural price. They thus bound from day one a Region of demand uncertainty “R” (see the top left of Figure 1 and Figure 3). Similarly, regarding oversupply, they bound “R” (see Figure 2). Later, they can reduce “R” by increasing (Figure 1) or decreasing (Figure 2) quantities slightly, or they can go back and forth between excess demand or excess supply (in case, for example, they are less prudent, see Figure 3). In any case “R” is reduced, and producers will reach the effective demand.

FIGURE 1
Representation of the discovering process (gray region) of a more prudent producer who started with excess of demand

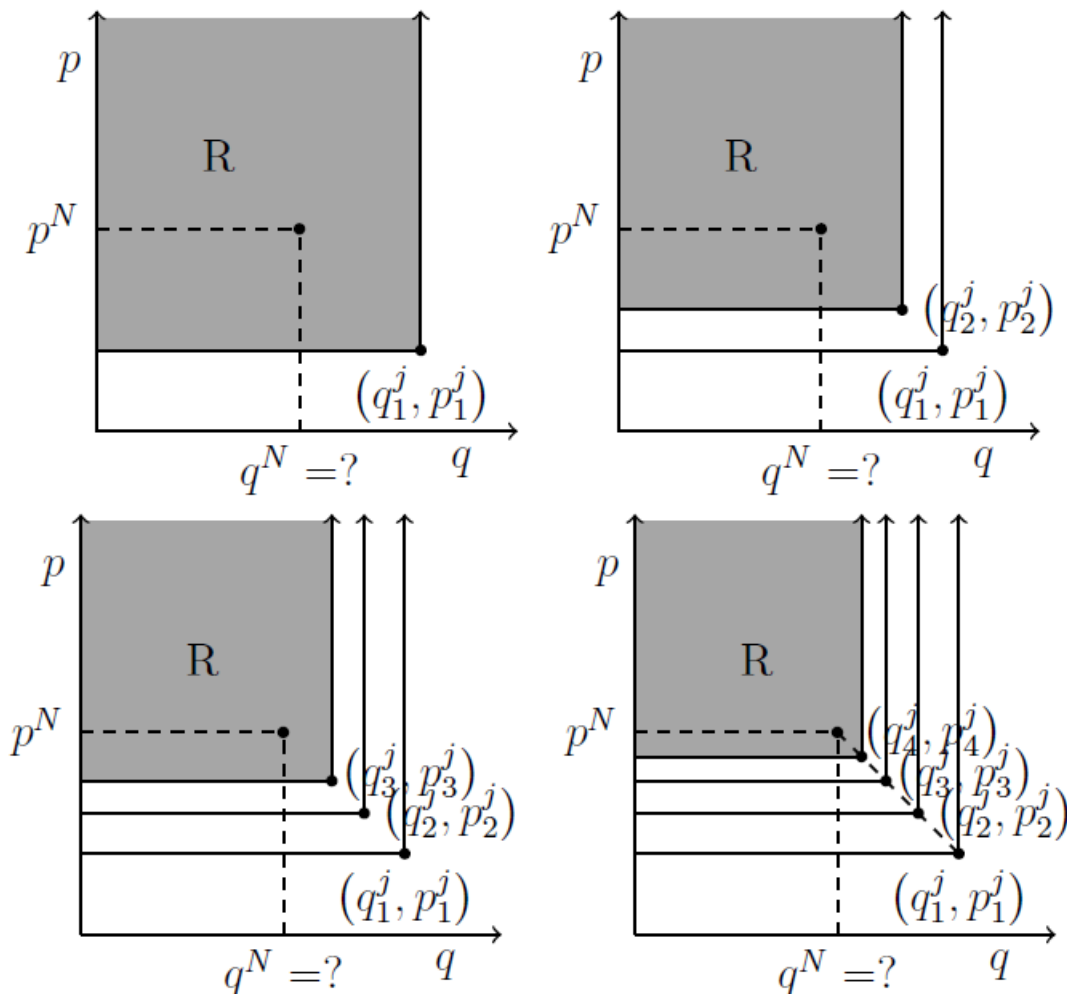


FIGURE 2
Representation of the discovering process (gray region) of a more prudent producer who started with excess of supply

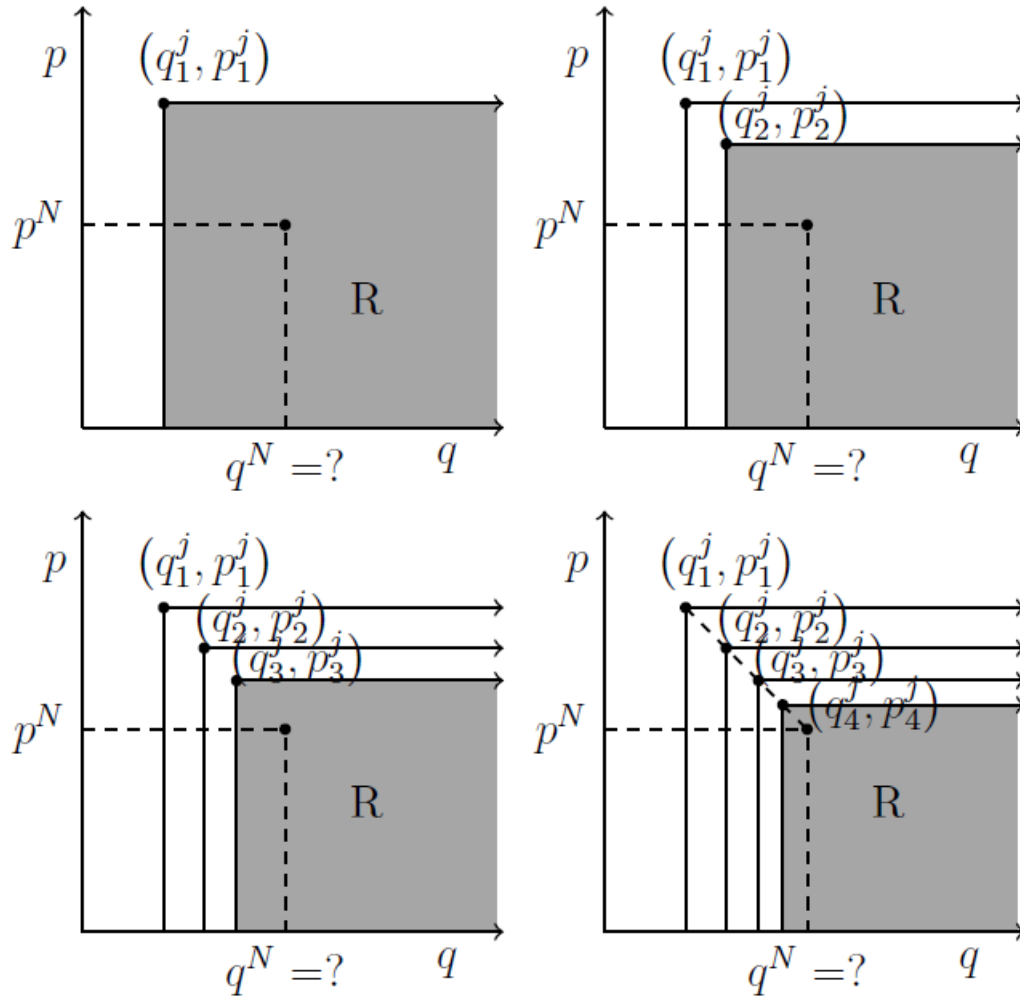
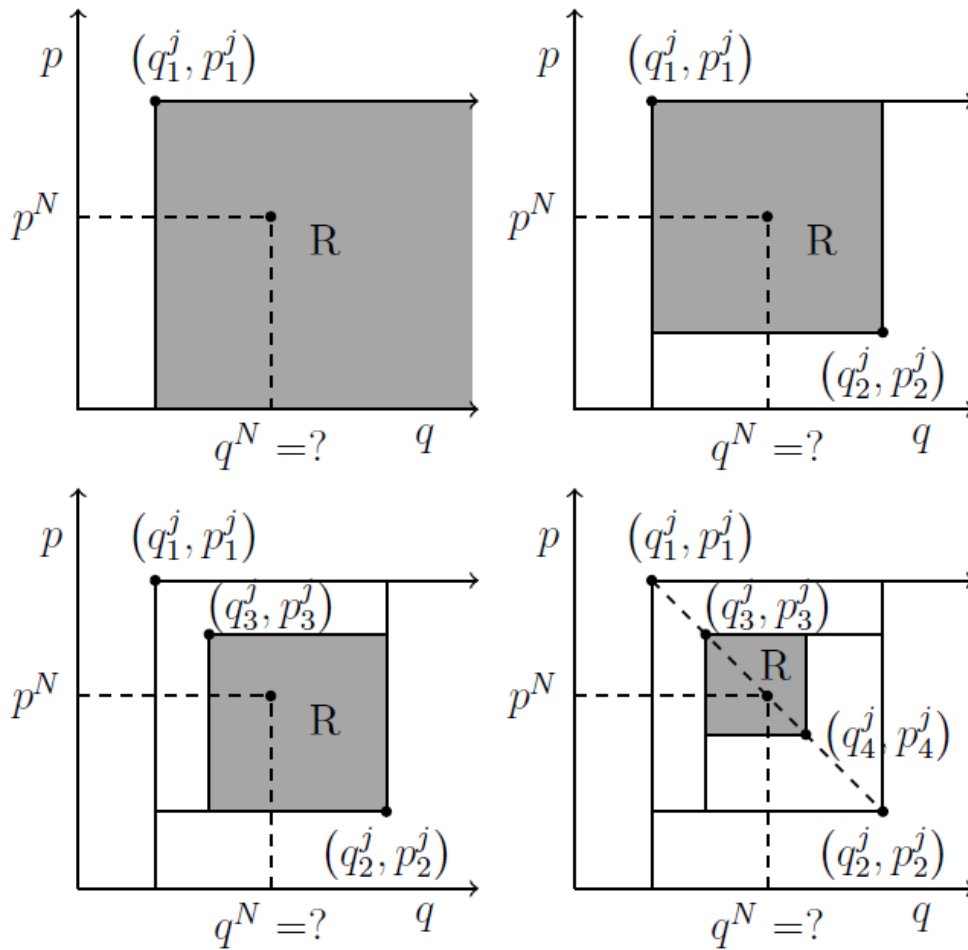


FIGURE 3
Representation of the discovering process (gray region) of a less prudent producer



3. MARKETPLACE WITH INSTANT COMPETITION AMONG CONSUMERS AND NOT AMONG PRODUCERS

3.1. Market place model with a deterministic natural demand

Let us represent the negotiations in a traditional marketplace as in the previous model, but in this case, producers are not competing instantly in the presence of an excess of supply. This happens because in this case it is “less important to them to get rid immediately of the commodity” (see Smith 1776 I.vii.10).

3.2. The one-day marketplace game

Each day can be represented as a four-period game. The first two periods are the same as in the previous model. In the third period producers sell at the natural price because competition among

producers does not start immediately. In the fourth period takes place the competition among producers in the case of excess of supply.

At the beginning of the day t , producers define the amount of good produced by the producer j , y_t^j . Analogously to the previous model, producers do not know precisely the reserve price of the good, then the natural price is obtained as before, that is,

$$p^N \in [p_{f_1}^N, \bar{p}_{f_1}^N] \cap [p_{f_2}^N, \bar{p}_{f_2}^N]$$

which only occurs in $t = 1$.

However, if the aggregate supply is higher than the aggregate demand at the initial prices, producers will sell at the end of the day what it was not bought $\{y_t^j - D^j(p^N, p^N)\}_{j=1}^2$ at price $p_{t,2}^j$ as close as possible to the natural price p^N to the consumers still interested in buying the good, that is,

$$\sum_{j=1}^2 (D^j(p_{t,2}^1, p_{t,2}^2) - D^j(p^N, p^N)) = \sum_{j=1}^2 (y_t^j - D^j(p^N, p^N)).$$

Therefore, even in the existence of excess of supply or demand at the natural price, producers will sell all the production that was chosen at the beginning of the day.

Note that the competition among producers occurs at the end of the day. Since producers want to sell at a price that is as close as possible to the natural price, they will start selling down as late in the day as possible (taking into account, anyway, their desire to sell at the end of the day all goods brought to market). If the daily distribution is not known by the producers, they will start selling at a lower price before. In Remark 1, we discuss more this case and its implications.

The demand of the producer j in this case is given by

$$D^j(y^1, y^2, p^1, p^2) = \begin{cases} 0 & \text{if } p^j > p^{3-j}, \\ D(p^j) & \text{if } p^j < p^{3-j}, \text{ where } p^1 \text{ and } p^2 \text{ are the price of the producer 1} \\ D(p^j) & \text{if } p^j = p^{3-j}, \end{cases}$$

and 2 respectively.

When producers decide their strategy as follows

$$\pi^j(y_t^1, y_t^2, p_{t,1}^2, p_{t,2}^2) = (p^N - \bar{p}_t^j)^2$$

where \bar{p}_t^j is the average price for the producer j along the day t , that is,

$$\bar{p}_t^j = f(x) = \begin{cases} p_{t,2}^j, & \text{if } D^j(y_t^1, y_t^2, p^N, p^N) \geq y_t^j \\ \frac{D^j(p^N, p^N)p^N + y_t^j - D^j(p^N, p^N)p_{t,2}^j}{y_t^j}, & \text{if } D^j(y_t^1, y_t^2, p^N, p^N) < y_t^j \end{cases}$$

If producers know the demand function, the Nash equilibrium for this game is a production allocation (y^1, y^2) such that $y^j = D^j(y^1, y^2, p^N, p^N) = D(p^N)/2$, that is, all producers chooses their “natural production”, the production plan that induces the natural price as the market price.

Note that producers do not know the demand function D . Each producer observes only the realization D^j at the prices during the day and at the end of day $\{p_{t,k}^j\}_{j,k=1}^2$. However, producers can estimate the demand based on the demand in previous days.

3.3. The marketplace game

It is a repeated game in which a one-day marketplace game takes place every day as described above. In $t = 1$, the producers do not have any information about the consumers. Then, each producer chooses a production strategy $y_1^j \in \mathbb{R}^+$.

In $t = 2$, the producers will use the information that they obtained from $t = 1$ about the effective demand to choose the new production strategy, y_2^j . In $t > 2$, the producers will decide y_t^j based on the market price and production strategy history. We will now analyze the properties of the sequence of the producer strategy $\{y_t^j\}_t$ and the sequence of the market prices $\{p_t^j\}_t$.

3.4. Results

If $\bar{p}_1^j \leq p^N$, the aggregate production was such that at the end of the day part of supply in the first period was not bought by the consumers at the natural price. Then, they discovered exactly the amount of the good that the consumers will need in future days to the market price will be equal to the natural price which is equal to $D(p^N)$.

If $\bar{p}_1^j > p^N$ in $t = 1$, the producers only have information about one point of the demand at a price above the natural price. Then, the only information that they can obtain from it is that the production level needed to reach the natural price is above of the one chosen by them. Therefore, $y_2^j > y_1^j$ for all $j = 1, 2$, then $\bar{p}_2^j < \bar{p}_1^j$ for $j = 1, 2$. If the aggregate production level is still higher than $D(p^N)$, the producers will increase their production level in $t = 3$ similarly as from day $t = 1$ to day $t = 2$. Therefore, we have the following result.

Theorem 2. For any initial production strategy $y_1^j \in \mathbb{R}^+$, any production strategy that is sequentially optimal for the producers, $\{y_t^j\}_t$, are such that the market price induced \bar{p}_t^j converges to

the natural price p^N . Moreover, if there is t such that $\bar{p}_t^j \leq p^N$, the market price satisfies that $\bar{p}_{t+1}^j = p^N$.

Proof. If the market prices $\bar{p}_t^j = p^N$, it means that the aggregated production plans are equal to the consumers demand at the natural price. Then, the producers do not have incentives to deviate from this production plan onward implying that $\bar{p}_{t'}^j \leq p^N$ for all $t' > t$.

If the market prices $\bar{p}_t^j < p^N$, it means that the aggregated production plans are such that it was necessary to sell part of the production at a lower price at the end of the day. Therefore, producers could observe how much it was bought by the consumers at the natural price during the day, that is $D(p^N)$, implying that the following days their optimal production strategy are $D^1(y^1, y^2, p^N, p^N)$ and $D^2(y^1, y^2, p^N, p^N)$ because this production plans implement the natural price as the market price, which concludes the proof. ■

To obtain this result, producers must know the daily distribution of potential consumers since they can estimate after some number of hours if he will be able to sell everything that it was brought to the marketplace. If this information is not known, the latter part of the result is not true.

Remark 1. If producers do not know the daily distribution of potential consumers, the last part of Theorem 2 is not valid since they cannot estimate properly if there will be enough consumers to buy all that brought to the marketplace. However, each producer will estimate that distribution after certain time. Therefore, the convergence result is still valid similarly as in Theorem 1.

If producers know the daily distribution of potential consumers at the natural price, they can estimate the effectual demand once the natural price has been established through bargaining at the beginning of the day (i.e., before competition is triggered). The less they know about this distribution, the less they can estimate the effectual demand (even if they have already discovered the natural price). The faster they can discover this distribution, the faster they can estimate the effectual demand. The longer they manage to sell at the natural price, the quicker they know the daily distribution of potential consumers at the natural price. The later competition is triggered, the more time they are able to sell at the natural price. The less competition there is among producers or consumers, the later it starts and the sooner producers can discover the effective demand. The lower the competition, the faster the discovery process.

The two extreme cases are: 1. the longest process - the competition is triggered at the beginning of the day, before establishing the natural price through bargaining, and the producers have to discover the natural price and the daily distribution of potential consumers at the natural price; 2. the shorter process - the producers discover the natural price and the daily distribution of potential consumers at the natural price before competition is triggered. If competition never occurs, this means that producers have brought to the market exactly what they need to meet effective demand. Therefore they will end up with enough information about the natural price, the quantity demanded, and its daily distribution at that price.

Remark 2. If the producers start with an auction, all the bargaining process does not occur, and then they are unable to estimate the natural price. Moreover, even if the natural price is known, they can

not know the daily distribution of the demand at the natural price since they only know demand at prices different to the natural price. Therefore, in the best case, this case is similar to the longest process described above. However, this similarity can occur only at the early beginning of the process described by Adam Smith, since in that process a stronger competition occurs only when there is a large excess of demand (and this excess is reduced day after day). Therefore, everyday auctions at the marketplace implies a slower convergence of the market price to the natural price describes in Remark 1 (case 1) due to the lack of information about the distribution of potential buyers at the natural price.

Remark 3. Note that if consumers can only realize that there is not enough good for all of them at the natural price, p^N , then they will not decide to compete among them immediately. They will wait until the available good in the marketplace reaches some threshold after in a reasonable number of hours. In other words, consumer will compete in this case similarly to producers in the excess of supply case. Therefore, the convergence to the natural price is also true.

Remark 4. Our results are also true for any fixed number of producers with unbounded possible production strategies. Moreover, if free-entry and free-exit of producers occur, the convergence results are also true even if the set of possible strategies are bounded. Competition as freedom of entry only ensures that there are no consumers ready to pay at the natural price who remain unsatisfied, and thus that the market price remains at a higher point than the natural price because of their rivalry. Additionally, competition as freedom of exit only ensures that producers can relocate their investments, and thus apply their quantity reduction strategy aimed at selling at the natural price. Their strategy reduces their rivalry and thus increases the market price towards the natural price.

But when producers arrive in the proximity of the natural price, it is possible that – given the freedom of entry and exit – new entries or exits will be sufficient for the market price to oscillate between values below and above the natural price. This means that we would be faced with a kind of market price orbit around the natural price. Thus, the third graph should be viewed as a zoom. In the case of excess demand, producers produce more or new producers enter the market because everyone knows that there are consumers willing to buy at the natural price who are still not satisfied. However, in the short run, new producers may not have enough information and therefore produce more than necessary. In such a case, the market price might fall too far and instead of converging toward the natural price we would move away from it, finding the market in an oversupply situation. New producers then might decide to exit the market, just as they entered it, causing the market to return to a situation of excess demand. The same is true in the opposite case. But in the long run the information obtained by all agents and the producers desire to have a market price as close as possible to the natural price make that the distance between the aggregate production plan and the quantity demanded by the consumers at the natural price goes to zero. Therefore, in the long run the market price is also converging to the natural price. This means that competition, as free entry and free exit, while not preventing the convergence process, also slows it down.

4. CONCLUSIONS

We have shown an intertemporal model describing the process that leads producers to discover the quantity that meets the effective demand. This process, based on supply and demand, only serves to bring the market price back to the natural price in case of imbalances, not to establish the latter. The natural price is the price that results from bargaining. Unlike the neoclassical model, here we do not have supply and demand curves known by the agents and an equilibrium price to be discovered from them. Instead, we have a natural price established in advance by bargaining and to which producers tend in a process of discovering the market and the actual demand. The supply curve of the neoclassical model is not relevant here, as producers adjust their strategy according to the natural price and the discovered market demand. Since the natural price coincides with the reserve price, the eventual supply curve of the neoclassical model can be found in the process of natural price discovery, regardless of whether costs are constant or not. In contrast to the neoclassical model, the convergence of the market price towards the natural price is not given by competition in the sense of freedom of entry or exit, i.e. hypothetical competition between producers who sell at a lower price than one another or who eliminate the other by selling at a lower cost. In the model proposed here, what moves the market price away from and towards the natural price are two other factors. What moves it away is competition as rivalry, while what moves it closer is the strategy of producers who tend to sell as much as possible close to the natural price, satisfying the entire effective demand. Competition, in the sense of freedom of entry or exit, ensures that a sufficient number of producers enter or leave the market in such a way that the quantity can in any case increase or decrease in order to satisfy the entire effective demand. Thus, competition as freedom of entry or exit does not affect price movements. It does only ensure that the market price continues to move on the basis of producers' strategies in the opposite direction to its movement generated by rivalry. In the neoclassical model, competition is just seen as what leads the market price to the natural price. In the interpretation of Smith's theory proposed here, competition as rivalry warns producers of excess demand or supply, and thus of the imbalance between the market price and the natural price, prompting them to adjust quantities to best meet effective demand. At the same time, however, competition as rivalry slows down the process of bringing the market price into line with the natural price, because it slows down the process of discovering effective demand. The process of discovering of the effective demand is faster or slower depending on the speed at which competition between producers and consumers begins. The weaker the rivalry, the faster the convergence towards the natural price. In the proximity of the natural price, it is possible that also competition as free exit or entry slows down, in the short run, the convergence process.

REFERENCES

- Aspromourgos, T. (2009). *Adam Smith: a Moral Philosopher and His Political Economy*. Taylor & Francis.
- Bee, M. (2021). The pleasure of exchange: Adam Smith's third kind of self-love. *Journal of the History of Economic Thought* 43 (1), 118–140.
- Blaug, M. (1997). *Economic Theory in Retrospect*. Cambridge university press.
- Brown, V. (1994). Higgling: The language of markets in economic discourse. *History of Political Economy* 26 (1), 66–93.
- Garegnani, P. (1983). The classical theory of wages and the role of demand schedules in the determination of relative prices. *The American Economic Review* 73 (2), 309–313.
- Grossman, S. (1976). On the efficiency of competitive stock markets where trades have diverse information. *The Journal of finance* 31 (2), 573–585.
- Grossman, S. J. and J. E. Stiglitz (1976). Information and competitive price systems. *The American Economic Review* 66 (2), 246–253.
- Inoua, S.M. & Smith, V.L. (2020). Adam Smith's theory of value: A reappraisal of classical price discovery. *ESI Working Paper 20-10*.
- Inoua, S. M. and V. L. Smith (2021a). Classical theory of competitive market price formation. *ESI Working Paper 21-09*.
- Inoua, S. and Smith, V.L. (2021b). Classical versus neoclassical equilibrium discovery processes in market supply and demand theory. *History of Political Economy*, 54(1), 37-73.
- Jackson, M. O. (1991). Equilibrium, price formation, and the value of private information. *The Review of Financial Studies* 4 (1), 1–16.
- Kaya, A. and Q. Liu (2015). Transparency and price formation. *Theoretical Economics* 10 (2), 341–383.
- Menudo, J. M. (2013). Market stability in Adam Smith: competitive process and institutions. *Journal of Economic Issues* 47 (3), 719–744.
- Naldi, N. (2013). Adam Smith on value and prices. *The Oxford handbook of Adam Smith*, 290–306.
- Rashid, S. (1992). Adam Smith and the Market Mechanism. *History of Political Economy* 24 (1), 129–152.
- Roncaglia, A. (2006). *The Wealth of Ideas: A History of Economic Thought*. Cambridge University Press.
- Smith, A. (1759). 1976. *The Theory of Moral Sentiments*, ed. DD Raphael and AL Macfie. Oxford: Clarendon Press.
- Smith, A. (1776). 1976. *An Inquiry into the Nature and Causes of the Wealth of Nations*. The Glasgow Edition of the Works and Correspondence of Adam Smith.

- Smith, N. (2012). *Complexity, competition and growth: Key ideas from Adam Smith, modeled using agent-based simulation*. Lap Lambert Academic Publishing.
- Stigler, G. J. (1957). Perfect competition, historically contemplated. *Journal of political economy* 65 (1), 1–17.
- Sugden, R. (2002). Beyond sympathy and empathy: Adam Smith's concept of fellow-feeling. *Economics & Philosophy* 18 (1), 63–87.
- Sznajd-Weron, K. and R. Weron (2002). A simple model of price formation. *International Journal of Modern Physics C* 13 (01), 115–123.
- Tsoufidis, L. (2010). *Competing schools of economic thought*. Springer Science & Business Media.