RegGae: Macroprudential Policy and Stress Testing with DSGEs

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Abstract

We set up a framework for macroprudential policy analysis and for macroprudential stress testing with DSGEs, dubbed *RegGae*. Financial crises are built into DSGEs as regime switches, such as occasionally binding collateral constraints, to introduce non-linearities in otherwise linear solutions. The transition probabilities governing the switch are endogenous. This allows DSGE models to quantify the benefits from macroprudential policy as the expected costs of financial crises averted. The endogeneity captures the idea that the probability of financial crises depends on the state of the economy whereas its timing cannot be forecasted. The framework unifies DSGE modeling with early warning methods. Because financial markets do not price-in crises risks until late, *RegGae* assumes that agents depart from rational expectations by acting as if they knew the future sequence of regimes with certainty. The regime-wise linear solution method is easier than the alternative and its accuracy is the same — close to the exact, non-linear solution. We illustrate the framework with a toy New-Keynesian DSGE augmented and macroprudential policy. We conduct macroprudential stress tests with generalized impulse responses and compute GDP-at-risk ("GaR") and other "at-risk" metrics. The illustration showcases the functionalities of the framework for forecasting probability distributions under the risk of financial crises. *RegGae* paves the way for the optimal calibration of macroprudential tools and optimal coordination with monetary policy.

Keywords: DSGE, regime-switching, occasionally binding constraint, endogenous probability, financial crises, macroprudential policy, optimal policy, macroprudential stress tests, growth-at-risk (JEL codes: C68, G01, G28).

1 Introduction

We set up a framework for macroprudential policy analysis and for macroprudential stress testing with regime-switching DSGE models (RS-DSGEs). The framework is dubbed *RegGae: Regime-Switching General Equilibrium with Endogenous Probabilities and Deterministic Expectations*. It can be thought of as a *framework*, rather than a specific model, as its high generality allows it to be applied to most existing DSGE models being used. This article presents the framework, its solution, and provides a "toy" example. The contribution is both methodological and substantive. On the methodological side, the article characterizes closed-form solutions for RS-DSGEs with deterministic expectations. On the substantive side, it applies the framework to analyze macroprudential policy and financial stability. These ideas will be developed in this article.

The overhaul in banking regulation that followed the 2008-09 global financial crisis created, or provided renewed interest in, policy tools that reduce the probability of the financial crises or limit their severity. These policies were labeled "macroprudential" due to their nature that incentivizes prudent risk-taking by financial institutions that behave in ways that produce system-level, or macro, effects. An example of such tools is Basel III's countercyclical capital buffer. The buffer level is to be managed by regulators along with the credit cycle under a macroprudential policy *rule*. But doubts remain on how to calibrate these rules: what level should be chosen by regulatory authorities at each moment? What are the optimal parameters of the reaction function of macroprudential policy? How should we explain policy trade-offs for policy-makers so that they are accountable to society? Can macroprudential stress testing helping to elaborate a narrative to communicate decisions to the public? These questions can be answered with DSGE models augmented for this purpose.

At its most basic level, policy analysis is weighting the costs *and* benefits of each option for policy-makers to pick the best. The tools for this exercise should provide both sides of the coin. DSGE models of the constant-parameter family can quantify the *costs* of policy tightening — macroprudential policy, monetary or other — in terms of output forgone. This is commonly done with impulse response functions displaying the time trajectory of output and other welfare-relevant variables following a policy "shock"

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or "innovation" (a tightening). The cost is the difference between the trajectories of the welfare relevant variables with and without the policy action. However, the *benefits* of macroprudential policy with DSGEs cannot be quantified with impulse responses. Macroprudential policy is geared towards ensuring the smooth, uninterrupted functioning of the financial system. It does so by strengthening the system's resilience, thereby lowering the likelihood of the realization of a destabilizing event sufficiently severe so as to tip the system into a financial crises. The exogenous shocks introduced in these DSGEs do not represent financial crises: financial crises are not "just" extraordinarily large shocks, periods of greater volatility. Instead, financial crises are much graver events, occurrences "outside the curve" in both nature and magnitude, with discontinuities in the fundamental parameters of operation of the economy. In the absence of financial crises in DSGEs, macroprudential tools appear simply as an alternatives for *monetary* policy, not for financial stability policy. This is because macroprudential policy operates through many of the same channels of monetary policy and have effects on aggregate demand, the output gap, and prices. Constant-parameter DSGEs allow for measuring the demand-management impact of macroprudential tools but not their financial stability benefits. The absence of financial crises in typical DSGEs implies that there is no financial stability metric against which to perform cost-benefit assessment of policy.

To augment DSGEs for macroprudential policy, we introduce regime-switches to build financial crises inside DSGEs. A financial crisis can be seen as a regime-switch, a change, a self-confirming change, in the system's parameters (such as market liquidity) when over-leveraged institutions face credibility problems and when stabilizing forces no longer dominate (Borio and Zhu, 2012). The switch often occurs when agents realize they had been making mistakes. Under these circumstances, a new equilibrium emerges in which financial intermediation is severely disrupted and a credit crunch ensues. The regime switches considered in this article correspond, but are not limited, to the switches caused by the occasionally binding constraint ("OccBin" or "OBCC", if the constraint is a collateral constraint) method to embed financial crises into DSGE (Guerrieri and Iacoviello, 2015; Bianchi and Mendoza, 2018; Laséen et al. 2017; IMF, 2017). Because of this correspondence and because regime switches in financial crises are typically seen as resulting from binding constraints, this article will use the terms "regime switch" and "constraint binding" interchangeably, unless the regime change in question is not strictly a binding constraint — it can be, for example, a low technology regime in a growth DSGE model. A regime change is a reduced form representation of a fully microfounded model of triggers of financial crises. The occbin models in Laséen and al. (2017) and Bianchi and Mendoza (2018) microfound to some extent what happens in a typical financial crisis — fire sales, asset price drops, liquidity hoarding — and endogenize its probability. They do not microfound why the regime switches, the crisis trigger itself. *RegGae* goes along the same lines and proposes a solution for any DSGE with reduced-form representations of financial crises. The disrupted-intermediation regime when the constraint binds — that is, when the rules (parameters and equations) governing the system change — is consistent with a drastically lower *equilibrium* output and prices that can be modeled through regime switches (Woodford, 2012).

In order to conduct macroprudential stress tests and policy analysis, we augment RS-DSGEs by introducing also endogenous probabilities of regime changes. This is because shocks in RS-DSGEs are typically assumed to be exogenous, that is, their distributions are time-invariant, independent from the system's state. However, financial crises are shocks whose probability and magnitude vary with time and depend on the state of the system. In fact, "[t]he [2008] shock was endogenous..." (Stiglitz, 2018). For example, credit growth affects the probability of financial crises. To augment DSGE and thereby overcome the "exogeneity" inadequacy, we propose that the transition probabilities governing the regime-switch be endogenous (state-contingent). It is a generalization of the endogenous mechanism of binding collateral constraint of Bianchi and Mendoza (2017) and Laséen et al. (2016). This modeling choice would be consistent with what have come to be common knowledge in the financial stability field: while the timing of crises is unpredictable, the buildup of the vulnerabilities that make crises more likely can be observed.

We introduce an additional component that distinguishes *RegGae* from other RS-DSGE methods: the *deterministic* expectations. We provide the solution for a general RS-DSGE under the assumption that agents expect a *certain* future path of regimes: agents believe they know the future regime sequence and form expectations accordingly. This assumption is used in OccBin models of Guerrieri and Iacoviello (2015) — GI, henceforth —, although not explicitly stated. While information may be available that financial vulnerabilities are building up and a crisis is more likely, agents may have other incentives to behave as if a financial crisis would never happen (Ajello et al., 2015; Aikman et al., 2018; Gennaioli et al., 2015). Indeed, bankers may incur moral hazard, compensation may be based on benchmarking, behavior may be overoptimistic, limited liability may cause risk underpricing, until the music stops. Indeed, such market failures justify the adoption of prudential regulation standards such as Basel III. For simplicity, we assume that even the policy-makers, who implement the policy rules, have deterministic expectations. It is a departure from fully rational expectations but it still implies that agents are forward-looking and act rationally, consistently with (the deterministic and possibly wrong) expectations. We claim that this assumption can be appropriate to study financial crises based on empirical evidence. Nevertheless, we admit that the validity of this assumption is ultimately an empirical question and depends on the specifics of the DSGE being used and of the expectation formation protocol assumed by the analyst.

RegGae's adopts a solution strategy of "regime-wise linearization": a first order perturbation around each regime-specific steady-state, equivalent to GI's solution. The difference between GI and *RegGae* is one of context, application and method, not of solution. GI is concerned with a constraint — such as $i_t = \max\{0, \text{Taylor Rule}\}$, which is the zero lower bound — that bound today and look for the policy function with a "guess-and-verify" algorithm: guess how long the constraint would bind, solve the model backwards starting from the last constrained period (guessed), and verify that the constraint would bind exactly the guessed duration (or try again).

RegGae provides a closed form solution for all time-varying coefficients, including for the variance-covariance matrices, which is absent in GI. This is key for applying shocks during the constraint binding, which is necessary for macroprudential stress testing but impossible to be derived with GI's method. *RegGae* provides the solution for each duration expected by agents that the analyst may wish to assume. *RegGae* is concerned with switches in parameter values when financial crises erupt. It looks for the policy function that depends on what agents expect to happen next, and solve the model backwards from the starting period of the "s-infinite" regime (the regime expected to last forever). The solution in GI also assumes deterministic expectations: agents do not weight future outcomes with their probability and the "volatility paradox" also holds. While GI has already established the high-quality of the regime-wise linearization, we also compare *RegGae's* solution for a specific example with the exact non-linear solution and found that they are remarkably close (Appendix D). *RegGae's* theorem formalizes sufficiency conditions for uniqueness, a "Long-Run Taylor Principle for Macroprudential Policy" of sorts. Our contribution is to shed new light on GI's OccBin, to bring up a different and easier way to use it, applied to macroprudential policy analysis and macroprudential stress testing.

RegGae is to be used with a new heuristics for analyzing the DSGE's results and it paves the way for macroprudential stress testing and for deriving optimal macroprudential policy and coordination with monetary policy. Instead of focusing on impulse response functions as the typical DSGE exercise, *RegGae* is to be used with *generalized* impulse response functions for macroprudential stress testing and for deriving probability distributions of future variable paths (Koop, Pesaran and Potter, 1996). It is a tool for macroprudential stress testing thanks to the fact that it allows for identifying the financial stability risks involved in each policy choice or shock, including "GDP-at-risk" (GaR) and other "value-at-risk" (VaR) metrics (IMF, 2017; Anderson et al., 2018).

RegGae provides an avenue for further developments needed in DSGE modeling (*Ox-*

ford Review of Economic Policy, January 2018):

1. **Non-linearities:** *RegGae* provides a convenient way to introduce non-linearities in DSGEs while maintaining the linearized, usual structure of existing DSGEs. It does so to the extent that a regime-switch is a highly non-linear event. While each regime is linear or linearized, the switch (or the expectation of a switch) introduces a non-linearity conveniently in an otherwise fully linearized system.

2. **Deterministic expectations:** *RegGae* introduces an expectation formation structure that departs from full rationality while maintaining the principle that agents care about the future in the spirit called for by Blanchard (2018). *RegGae* assumes that expectations are imperfectly rational in the sense that regime expectations are *deterministic*: in each period, agents assign a probability of zero or 1 to each future regime. Agents believe they know the future regimes and act accordingly. This puts DSGE modeling closer to the literature on behavioral economics and to the empirically observed behavior during bubbles, euphoria, and "irrational exhuberance" (Ajello et al., 2015; Aikman et al., 2018; Gennaioli et al., 2015). *RegGae* contains, as a special case, some elements of the heuristics of Woodford (2018): agents may be assumed to care about only a finite number of periods ahead.

3. **Endogeneity of shocks:** Pre-crisis DSGEs did not model properly the buildup of vulnerabilities to shocks, which are endogenous (Woodford, 2012; Stiglitz, 2018). *RegGae* addresses this concern frontally by assuming endogenous probabilities of financial crises.

In order to showcase and exemplify *RegGae*, we postulate a simple "toy" DSGE under *RegGae*. It is the standard "new-keynesian" (with nominal frictions) model augmented (arbitrarily, without explicit microfoundations) with credit and macroprudential policy equations. We simulate the model, perform an event-history analysis useful for the elaboration of narratives, and conduct a macroprudential stress test by computing the generalized impulse responses and probability distributions of future outcomes (growth-at-risk). This exercise shows the way forward for optimization and identification of the optimal policy mix. The optimization is now possible based on the assumption that the social planner takes the expectational biases of agents while it makes full use of the public information about the probability of financial crises.

In the next section we present how *RegGae* relates to previous literature. In section 3, we introduce *RegGae*, the general framework for RS-DSGEs with endogenous transition probabilities and deterministic expectations. We outline the way forward for optimizing the policy rules. Then, in section 4, we present a toy example applying the framework.

This example will showcase the functionalities of the framework to address financial crises and macroprudential policy.

2 Relation to previous literature

Regime-switching DSGEs are not new but only recently started to make their way into the study of financial stability. A series of theoretical studies have paved the way for the introduction of RS-DSGEs into the financial stability realm. Starting from the seminal work by Hamilton (1989) came the work by Davig and Leeper (2007) and by Farmer, Waggoner and Zha (2009 and 2011), which characterized equilibrium and derived necessary conditions for existence and stability of RS-DSGEs. The idea that financial crises probabilities might be endogenous was put forth by Woodford (2012) and formalized and solved by Maih (2015) and Barthélemy and Marx (2017). Nevertheless, these works apply the technique to study monetary policy regimes where expectations are fully rational (take into account the true probability of future regime switches). Our application and, therefore, assumptions about expectations will differ from this line of work and be closer to GI's OccBin framework. We bypass the debate on system stability concepts given our focus in *deterministic* expectations justified by the specific application of interest.

We build on a stream of work that proposes the use of RS-DSGE models with endogenous probabilities to analyze financial stability and crises. Davig and Leeper (2009) mention that monetary policy changes during financial crisis but does not model financial crises as regime changes explicitly. Woodford (2012) elaborates the idea and formalizes it but does not provide a general solution for infinite periods. Benes et al. (2014a, 2014b) come close to using regime switching to assess financial stability by modeling expectation reversals (deterministic) causing hard crashes. More recently, it was proposed that the study of rational bubbles and financial stability would follow regime-switching models but with fixed probabilities of regime switches (Martin and Ventura, 2018). From the stream of literature of "occasionally binding constraint" used for analyzing the zero lower bound of interest rates emerged some work that applied the same technique for modeling non-linearities and engineering financial crises. He and Krishnamurthy (2014), Laséen et al. (2017), Bianchi and Mendoza (2018) introduce DSGEs with occasionally binding *collateral* constraints: a stochastic (auto-correlated) variable governing the activation. In this line of work, the endogenous probability of regime switches emerge naturally from the model instead of being assumed.

Our paper also contributes to the debate on the welfare benefits of "leaning-against-the-wind" (LAW, i.e., using monetary policy to counter financial stability risks). Svensson (2016 and 2017) argues that LAW does not improve welfare. However, their method is a cross-section comparison of the welfare of different states weighted by probabilities, not a dynamic weighting of regimes' welfares as we do here. As indicated by Laséen et al. (2017), it applies to a surprise episode of LAW, not to a systematic LAW rule. Regarding techniques to measure welfare, *RegGae* is better equipped to assess welfare than other studies that hardwire, into welfare functions, financial stability objectives. This is because our framework takes full account of crises probabilities and the post-crisis welfare developments and normalization of regime. Indeed, articles such as Angelini et al. (2014) and Laureys and Meeks (2018) hardwire in welfare functions volatilities and deviations from "sustainable" levels of a financial stability-relevant variables (respectively). This is because our framework takes full account of the benefits of macroprudential policy by reducing crises probabilities. The framework for welfare assessment in Laséen et al. (2017) is equivalent to ours.

The toy model used to exemplify *RegGae* generalizes in DSGE structure the 2-period games of Ajello et al. (2015) and Aikman et al. (2018) while keeping equivalent assumptions about expectations and endogenous crisis probability.

3 *RegGae*, the framework

RegGae has three elements: a regime-switching DSGE, an endogenous probability function for regimes, and an expectation formation protocol. In this section we introduce one of these elements at a time, starting with the RS-DSGE. We present the solution after the first two elements to highlight that the third element is not needed for the solution.

3.1 The regime-switching DSGE

Consider a generic RS-DSGE where some parameter values may change every period randomly according to the a realization of a state variable, or regime variable, s_t. The RS-DSGE can be represented by:

$$E_t[g(x_{t+1}, x_t, x_{t-1}, \Gamma(s_t), \Upsilon(s_t), \varepsilon_t)] = 0 \tag{1}$$

where:

- $E_t[\cdot] \equiv E[\cdot|\Omega_t]$ is the expectation operator conditional on the information set known at time t;
- $g(\cdot)$ is a system of difference equations in x_t, its lead and lag;
- x_t is the vector of endogenous variables;

- $\Gamma(s_t)$ is a vector of regime-specific parameters of g outside the control of the social planner. It can contain (and be thought of as) both parameters that enter $g(\cdot)$ and actually shift the equations as well as *sunspot* variables that coordinate agents to play a particular equilibrium (in multiple equilibrium settings);
- $\Upsilon(s_t)$ is a vector of (possibly) regime-contingent parameters chosen by the social planner;
- ε_t is a vector of exogenous shocks.

The assumptions about the properties of the distributions of the regime variable s_t and the exogenous shocks ε_t will be introduced as we present the framework.

As examples, regimes may include a "normal" regime, a bubble/credit boom regime, a crisis regime, a zero lower-bound regime, and/or a post-crisis regime, etc. Further, the framework allows for modeling crises of different severities by assuming each crisis severity corresponds to a different regime such as "severe crisis regime" or "mild crisis regime".

Each regime may possess a regime-specific steady-state, $\bar{x}(s_t)$. A regime-specific steady-state is the steady-state that would prevail if there was only that regime. The system can be linearized around each of these regime-specific steady-states, "regime-wise". Then, the generic regime-wise linear RS-DSGE can be rearranged as:

$$A_1(s_t)\begin{bmatrix} x_t^p \\ E_t[x_{t+1}^j] \end{bmatrix} = A_2(s_t)\begin{bmatrix} x_{t-1}^p \\ x_t^j \end{bmatrix} + \bar{G}(s_t) + A_3(s_t)\varepsilon_t \tag{2}$$

where

- x_{t-1}^p is a column vector, a partition of x_{t-1}, of p predetermined variables;
- x_t^j is a column vector, the complementary partition of x_t, of j non-predetermined, forward-looking variables ("jumpers");
- $A_i(s_t)$ are conformable matrices corresponding to the Jacobians of the DSGE[1] and the variance-covariance matrix of shocks. They are functions of $\Gamma(s_t)$ and $\Upsilon(s_t)$.
- The non-homogenous terms $\bar{G}(s_t)$ are regime-specific vectors of constants resulting from the steady state of each regime. Specifically,

$$\bar{G}(s_t) \equiv [A_1(s_t) - A_2(s_t)]\bar{x}(s_t) \tag{3}$$

[1] The Jacobians can be obtained with Dynare for each regime-specific steady state.

Naturally, the regime-specific steady-states $\bar{x}(s_t)$ are also functions of $\Gamma(s_t)$ and $\Upsilon(s_t)$.

We normalize the fundamental shocks vector ε_t to unit variance. Thus, $A_3(s_t)$ is a regime-specific matrix of standard deviations and correlations. Therefore, *RegGae* comprehends the stochastic volatility case.

With this specification, all elements of the DSGE are regime-dependent: the matrices that govern the system's motion direction, $A_1(s_t)$ and $A_2(s_t)$, the system's level, $\bar{G}(s_t)$, and shock variance-covariances, captured via $A_3(s_t)$.

We adopt the "end-of-period" timing convention for state-variables and expectations, in keeping with Dynare: x_t^p is the value at the *end* of period t, *after* all decisions in t are taken and all t-indexed random variables are realized.

We make the following simplifying assumption:

Assumption 1. ***Invertibility:*** *$A_1(s_t)$ are invertible for all s_t.*

This assumption allows using the eigenvector decomposition method for solving the DSGE, instead of using the QZ decomposition suitable for singular lead matrix case. Future work will attempt to extend the regime-switching framework for singular lead matrix case. Given Assumption 1, we can premultiply (2) by $A_1^{-1}(s_t)$ to obtain

$$\begin{bmatrix} x_t^p \\ E_t[x_{t+1}^j] \end{bmatrix} = A_1^{-1}(s_t)A_2(s_t)\begin{bmatrix} x_{t-1}^p \\ x_t^j \end{bmatrix} + A_1^{-1}(s_t)\bar{G}(s_t) + A_1^{-1}(s_t)A_3(s_t)\varepsilon_t \tag{4}$$

Equation (4) can be recast as

$$\begin{bmatrix} x_t^p \\ E_t[x_{t+1}^j] \end{bmatrix} = A(s_t)\begin{bmatrix} x_{t-1}^p \\ x_t^j \end{bmatrix} + B(s_t) + C(s_t)\varepsilon_t \tag{5}$$

where

- $A(s_t) \equiv A_1^{-1}(s_t)A_2(s_t)$
- $B(s_t) \equiv A_1^{-1}(s_t)\bar{G}(s_t)$

- $C(s_t) \equiv A_1^{-1}(s_t)A_3(s_t)$

Equation (5) is the expectational linear system to be solved. To that end, we need to know the expectation formation protocol.

3.2 Deterministic Expectations

Solving equation (5) requires specifying the expectation formation protocol. The model solution depends on which sequence of regimes materialized and what regimes are therefore expected going forward. In constant-parameter DSGEs, this is not an issue as there is only one regime and therefore expectations conform with the true process. But in regime switching DSGEs there is a need to specify whether expectations are rational or something else. Solving the model with rational expectations and endogenous probabilities of regime switch is a technical challenge that only recently has been overcome (Barthélemy and Marx, 2017; Maih, 2015). Notwithstanding these technical advances and because the application of interest here are financial crises, a different route is taken by *RegGae*. We provide a solution for the framework under the assumption that all agents, including the policy authority, do not incorporate in their expectations the true probability that the regime may switch and that a crisis may occur.[2] Agents are assumed to believe that the current regime and their future path is *certain*: they believe they know the current regime and their future sequence and attribute only probabilities 0 or 1 to current and future regimes, never a probability strictly between 0 and 1. This assumption is also imposed in GI with the additional requirement that the expected duration until return to baseline regime depends on the "tightness" of the occasionally binding constraint. How long one expects to be in a regime may depend on the state vector. But conditional on the state vector, expectations are deterministic. Agents cannot properly assess regime duration but they believe they can do it with precision. In a sense, this presents one answer to the question asked in Blanchard (2018): "How can we deviate from rational expectations, while keeping the notion that people and firms care about the future?".

Put formally, for each node in the history tree $(..., s_{t-2}, s_{t-1}, s_t) \equiv \Im_t$, for each history of states $(..., x_{t-2}, x_{t-1}) \equiv X_{t-1}$ and current exogenous shock ε_t, agents form deterministic expectations that current and future regimes will be $\Re(\Im_t, X_{t-1}, \varepsilon_t) = (s_t, s_{t+1}, s_{t+2}, ...)$ where $\Re(\cdot)$ is an ordered set-valued mapping from past and present histories of regimes $\Im_t$ to expected present and future regimes. The framework allows the modeler to assume that expectations deviate from the true current regime (that is, a crisis may erupt but

[2]Note that the social planner is not an agent in the framework. Its role is only to choose only once, at the "beginning", the reaction function to be implemented by the policy authority during the evolution of the system.

agents wouldn't believe they are in a crisis). This flexibility differentiates *RegGae* from OccBin's algorithm in which expectations abide by regime feasibility.

Formally, we specify expectations with the following assumption:

Assumption 2. ***Deterministic Expectations:*** *Let R be the set of regimes, $\mathbb{R}$ be the real line, n be the number of endogenous variables, n_e be the number of exogenous shocks. Let there be a mapping $\Re$ from past and present histories to present and future sequence of regimes*

$$\Re : \prod_{-\infty}^{t} R \times \{\mathbb{R}^n\}_{-\infty}^{t-1} \times \mathbb{R}^{n_e} \to \prod_{t}^{\infty} R$$

For any history up to end of period $t-1$ plus the random variables realized in t, namely, (s_t, ε_t), denoted $(\Im_t, X_{t-1}, \varepsilon_t)$, agents assign probability 1 that the current regime and its future sequence will be $\Re(\Im_t, X_{t-1}, \varepsilon_t)$, that is,

$$E_t[s_t, s_{t+1}, s_{t+2}...|\Im_t, X_{t-1}, \varepsilon_t] \equiv \Re(\Im_t, X_{t-1}, \varepsilon_t)$$

We call $\Re(\Im_t, X_{t-1}, \varepsilon_t)$ an *expectation formation protocol.* One specific case is the assumption in our example of section 4 where agents expect that a crisis will never happen and, if one does happen, it will last a deterministic and bounded number of periods. There is a number of possibilities to explore with this specification, which were not considered in GI. One example is to assume that agents expect that the central bank will comply with the commitment to keep interest rates at zero for a certain number of periods k even after the zero-lower bound constraint ceases to bind. The number of periods k can be set optimally so that inflation rises faster. Then there is a third regime — not considered by GI — where interest rates are zero but the zero-lower bound is slack (the reference regime monetary policy rule would call for a positive interest rate). The expected future regime sequence is revised every period according to the realization of random variables.

RegGae allows sufficient flexibility for the researcher to calibrate the expectation formation protocol for the specific application of interest. Whether expectations are deterministic is ultimately an empirical question. The assumption may be valid for some specific DSGEs capturing some specific features of the real world but not for others. The extent of the difference it makes relative to the fully rational expectations equilibrium may also depend on the specific DSGE being used and on the specific expectation formation protocol. Because the validity of this assumption is DSGE-specific and $\Re$-specific, we do not compare its results with a fully rational regime-switching framework.

We note that, while this assumption may appear strong,[3] a closer look shows that it is justified for the specific objective of modeling financial crises and macroprudential policy. First, there is empirical evidence that suggest it. This assumption would be consistent with findings from the behavioral economics literature about how agents factor in infrequent events such as financial crises. This assumption can represent the stylized fact that individual lenders and borrowers do not account for financial stability risks when deciding whether to lend and borrow (remember the subprime?). After a hiatus without crises, investors fall into believing that "this time is different" and neglect crises risks (Gennaioli et al., 2015). As stated by Stiglitz (2018), "[b]anks engage in contracts with each other that may be individually rational, but result in greater systemic risk...". Borio and Zhu (2012) argue that "even when risks are recognized, it may sometimes be difficult for market participants to withdraw from the fray, as the short-term pain is not seen as offset by future potential gains". Ajello et al. (2015) find evidence in forecasters survey that the subjective crisis probability was close to zero in early 2008.

Perhaps the most compelling empirical observation suggesting that agents behave as if financial crises are a zero probability event is the fact that financial markets are typically calm (low spreads and low volatility) until the crisis eve, when vulnerabilities are already in place ("volatility paradox"). In fact, it is understood that long periods of low volatility and low spreads may increase risk appetite, fuel disregard for risk buildup, and actually increase the vulnerabilities and, thus, the probability of crisis (Borio and Zhu, 2012; Anderson et al., 2018). Central banks and financial stability authorities actually monitor *low* volatilities, not high, as leading indicators of financial crises (Aikman et al., 2017). He and Krishnamurthy (2014) suggest that lack of information about leverage and of repo market transparency can be blamed for the *lack of market anticipation* of the 2007-09 crisis. They state categorically that "*is not possible to construct a model in which spreads are low ex-ante, as in the data, and yet the probability of a crisis is high. [...] our model offers little advance warning of the crisis that followed. That is, without the benefit of hindsight, in both the model and data the probability of the 2007-2009 crisis is low.*" These findings provide empirical support for our assumption that expectations of financial crisis are deterministic (Assumption 2). With this assumption, we provide a framework for constructing such models He and Krishnamurthy (2014) state are impossible to be built.

Second, there are game theoretical foundations for this modeling choice. In any equilibrium of a repeated game with multiple equilibria, players do not expect that the equilibrium path will switch to a different one during the play of an equilibrium (or it would

[3] GI also argues that while this assumption may appear "draconian", it is routinely imposed when solving DSGE models by standard first-order perturbation.

not be an equilibrium). Thus, when an equilibrium shift does happen, it is fully unexpected. Naturally, the regime switch under *RegGae* is a closed-form solution that "black-boxes" a microfounded game governing the equilibrium path. This argument is valid for unexpected regime switches, not for expected regime switches such as the return to normality after crises assumed in the example of section 4 illustrating *RegGae*. For expected switches, the effects of the switch are fully built in current actions.

And third, the assumption does *not* imply that the probability of crises play no role. In fact, if the social planner conditions the reaction functions (to be implemented by the policy authority) on the parameters of the distribution of s_t (which will be the case presented later), then by implementing the macroprudential policy rule, agents will indirectly (via the policy rule) internalize the probability of crises in their actions and expectations, albeit not explicitly.

3.3 Solution of *RegGae*

Equation (5), together with the specification of an expectation formation protocol $\Re(\Im_t, X_{t-1}, \varepsilon_t)$ can be solved by regime-wise linearization under certain conditions yielding a solution in state-space representation in the following format:

$$\begin{bmatrix} x_t^p \\ x_t^j \end{bmatrix} = \begin{bmatrix} M_\iota \\ V_\iota \end{bmatrix} x_{t-1}^p + \begin{bmatrix} Q_\iota \\ Z_\iota \end{bmatrix} + \begin{bmatrix} S_\iota \\ W_\iota \end{bmatrix} \varepsilon_t \tag{6}$$

where the general solution for block-matrices M_ι, Q_ι, S_ι, V_ι, Z_ι and W_ι is presented below and derived in Appendix A. The upper block of matrix equation (6) corresponds to the state-equation while the lower-block corresponds to the signal-equation. While the quality of the regime-wise linear solution has already been established by GI, the quality of the regime-wise linear solution compared to the exact (non-linear) solution is assessed in Appendix C for one specific example. These matrices are grouped into history node "types", denoted ι. A node type is, for example, a normal-regime period, a crisis period, a period where regime is $s_t = s$ but agents believe $s_t = s'$, etc. The set of node types depends on $\Re$.

Thus the solution, if one exists at a specific history node, depends on the expectation formation protocol but not on the actual process governing the regime switch as, by assumption, this does not affect expectations. Let the set of matrices that solve the model at the nodes where a solution exists be denoted $\mathbb{S}$ and the set of period types be denoted $\mathbb{I}(\Re)$:

$$\mathbb{S}(\Re) \equiv \{M_\iota, Q_\iota, S_\iota, V_\iota, Z_\iota, W_\iota | \iota \in \mathbb{I}(\Re)\} \tag{7}$$

An expectation formation protocol may culminate in history nodes which are s-

infinite. In an s-infinite expectations history node at time t, *agents expect regime $s \in R$ to last forever*. It is an identical situation to the well known, constant parameter, single regime case. This definition will be convenient when characterizing the solution (Appendix A).

Definition 1. ***s-infinite histories:*** *History $(\Im_t, X_{t-1}, \varepsilon_t)$ is s-infinite if and only if, for some $s \in R$,*

$$\Re(\Im_t, X_{t-1}, \varepsilon_t) = (s, s, s, ...)$$

In the following propositions we present the regime-wise linear solutions for two types of history nodes.

Proposition 1 (**Solution in s-infinite histories**). *Let $\Im_t$ be an s-infinite history node and let $\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t))$ denote the subset of the solution set at history node $(\Im_t, X_{t-1}, \varepsilon_t)$:*

$$\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t)) \equiv \{M_{s_t,s}, Q_{s_t,s}, S_{s_t,s}, V_s, Z_s, W_s | s_t, s \in \mathbb{I}(\Re(\Im_t, X_{t-1}, \varepsilon_t))\}$$

Let the Blanchard-Kahn conditions hold in regime s. Then, with the notation of Appendix A, the solution $\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t))$ is:

$$M_{s_t,s} = A_{pj}(s_t)V_s + A_{pp}(s_t) \tag{8}$$

$$Q_{s_t,s} = A_{pj}(s_t)Z_s + B_p(s_t) \tag{9}$$

$$S_{s_t,s} = A_{pj}(s_t)W_s + C_p(s_t) \tag{10}$$

$$V_s = -(P_{jj,s})^{-1}P_{jp,s} \tag{11}$$

$$Z_s = -(P_{jj,s})^{-1}[(I_j - \Lambda_{j,s}^{-1})^{-1} - I_j]P_{j,s}B(s) \tag{12}$$

$$W_s = -(P_{jj,s})^{-1}\Lambda_{j,s}^{-1}P_{j,s}C(s) \tag{13}$$

Proof. The proof is Appendix A. □

This solution looks very much like the solution for constant-parameter DSGEs. Indeed, for the special case where $s = s_t$, this solution matches the solution matrices obtained with Dynare (when the actual current regime s_t is the regime s that agents believe that will last forever). But there is one difference: we allow for the flexibility that agents believe that the regime that is expected to last forever s be different from the actual current regime s_t. The modeler is able to assume that even unfeasible trajectories can be expected, such as when an occasionally binding constraint is violated.[4] The next proposition introduces the solution when the regime is expected to change eventually.

[4]GI's OccBin assumes that expectations respect feasibility, which is what the "guess-and-verify" algorithm is all about finding.

Proposition 2 (Solution at finite histories). *Let s and s' be two regimes, not necessarily the same. Let $(\Im_t, X_{t-1}, \varepsilon_t)$ be a history node where $\Re(\Im_t, X_{t-1}, \varepsilon_t) = (s, s', s_{t+2}, ...)$. Let $\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t))$ denote the subset of the solution set at history node $\Im_t$:*

$$\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t)) \equiv \{M_{s_t,s}, Q_{s_t,s}, S_{s_t,s}, V_s, Z_s, W_s | s \in \mathbb{I}(\Re(\Im_t, X_{t-1}, \varepsilon_t))\}$$

Let $E_t[\Re(\Im_{t+1}, X_t, \varepsilon_{t+1})] = (s', s_{t+2}, ...)$ and let there exist an expected unique solution for the next history $(\Im_{t+1}, X_t, \varepsilon_{t+1})$:

$$E_t[\mathbb{S}(\Re(\Im_{t+1}, X_t, \varepsilon_{t+1}))] = \{M_{s'}, Q_{s'}, S_{s'}, V_{s'}, Z_{s'}, W_{s'} | s' \in \mathbb{I}(E_t[\Re(\Im_{t+1}, X_t, \varepsilon_{t+1})])\}$$

Then, with the notation of Appendix A, the solution $\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t))$ is:

$$M_{s_t,s} = A_{pj}(s_t)V_s + A_{pp}(s_t) \tag{14}$$

$$Q_{s_t,s} = A_{pj}(s_t)Z_s + B_p(s_t) \tag{15}$$

$$S_{s_t,s} = A_{pj}(s_t)W_s + C_p(s_t) \tag{16}$$

$$\begin{aligned} V_s = [I_j - (P_{jj,s})^{-1}\Lambda_{j,s}^{-1}(P_{jp,s} + P_{jj,s}V_{s'})A_{pj}(s)]^{-1} \\ [-(P_{jj,s})^{-1}P_{jp,s} + (P_{jj,s})^{-1}\Lambda_{j,s}^{-1}(P_{jp,s} + P_{jj,s}V_{s'})A_{pp}(s)] \end{aligned} \tag{17}$$

$$\begin{aligned} Z_s = [I_j - (P_{jj,s})^{-1}\Lambda_{j,s}^{-1}(P_{jp,s} + P_{jj,s}V_{s'})A_{pj}(s)]^{-1} \\ (P_{jj,s})^{-1}\Lambda_{j,s}^{-1}\{(P_{jp,s} + P_{jj,s}V_{s'})B_p(s) + P_{jj,s}Z_{s'} - P_{j,s}B(s) + P_{jj,t}W_{s'}E_t[\varepsilon_{t+1}]\} \end{aligned} \tag{18}$$

$$\begin{aligned} W_s = [I_j - (P_{jj,s})^{-1}\Lambda_{j,s}^{-1}(P_{jp,s} + P_{jj,s}V_{s'})A_{pj}(s)]^{-1} \\ (P_{jj,s})^{-1}\Lambda_{j,s}^{-1}[(P_{jp,s} + P_{jj,s}V_{s'})C_p(s) - P_{j,s}C(s)] \end{aligned} \tag{19}$$

Proof. The proof is Appendix A. □

Proposition 2 shows that the solution today depends on the expected solution tomorrow. It implies that existence and uniqueness today is ensured by expected existence and uniqueness tomorrow. With that we can state the theorem of *RegGae:*

Theorem 1 (*RegGae*: Global Existence and Uniqueness). *Let $\Re(\cdot)$ be such that, in all attainable regime history nodes $(\Im_t, X_{t-1}, \varepsilon_t)$, agents expect the system to culminate in s-infinite histories in a globally bounded number of periods. Let R_∞ be the subset of the regime set R of all regimes s for which s-infinite histories are expected somewhere in* *the attainable history tree. Let also the Blanchard-Kahn conditions hold for all regimes in R_∞. Then a unique solution $\mathbb{S}(\Re)$ exists everywhere in the attainable history tree.*

Proof. Let $(\Im_t, X_{t-1}, \varepsilon_t)$ be an attainable history node. Then, by assumption, $\Re(\Im_t, X_{t-1}, \varepsilon_t)$ is expected to culminate in s-infinite history for some regime $s \in R_\infty$ from time $t+n+1$ onwards, that is, $\Re(\Im_t, X_{t-1}, \varepsilon_t) = (s_t^e, s_{t+1}^e, s_{t+2}^e, ..., s_{t+n}^e, s, s, s...)$ where superscript e denotes the specific regime expected. Let $\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t))$ denote the subset of the solution set at history node $(\Im_t, X_{t-1}, \varepsilon_t)$. Let's show that a unique $\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t))$ exists:
$s \in R_\infty \Rightarrow$ the Blanchard-Kahn conditions hold for s
$\Rightarrow \exists$ unique $E_t[\mathbb{S}(\Re(\Im_{t+n+k}, X_{t+n+k-1}, \varepsilon_{t+n+k}))]$ for $k \geq 1$ (by Proposition 1)
$\Rightarrow \exists$ unique $E_t[\mathbb{S}(\Re(\Im_{t+n}, X_{t+n-1}, \varepsilon_{t+n}))]$ (by Proposition 2)
$\Rightarrow \exists$ unique $E_t[\mathbb{S}(\Re(\Im_{t+n-1}, X_{t+n-2}, \varepsilon_{t+n-1}))]$ (by Proposition 2)
... (by backward induction)
$\Rightarrow \exists$ unique $E_t[\mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t))] = \mathbb{S}(\Re(\Im_t, X_{t-1}, \varepsilon_t))$ (by Proposition 2) □

The theorem follows from the fact that, at any s-infinite history node, the Blanchard-Kahn (BK) conditions ensure expected existence and uniqueness at that history node (Blanchard and Kahn, 1980). Intuitively, if expectations at all history nodes culminate in s-infinite histories after a finite, bounded number of periods, then the unique solution at that s-infinite history node can be rolled back to any starting point.

The main idea of Theorem 1 is also stated (informally) in GI. Theorem 1 simply states that the BK conditions are needed only for those regimes for which there are s-infinite histories. Regimes that are expected to last a bounded number of periods do not have to satisfy BK. But agents must expect that the history of regimes will eventually reach s-infinity at some node down the regime history tree. It is the deterministic expectation parallel of the result of Davig and Leeper (2007) that enlarged the determinacy set beyond the set where all regimes satisfy BK conditions, a "Long-Run Taylor Principle for Macroprudential Policy" of sorts. Note that Theorem 1 is a sufficiency result for global determinacy. Other configurations of expectations — such as cyclical regime switch sequences — can be investigated for node-wise and global sufficiency. Note also that we applied the Law of Iterated Expectations. Therefore, the theorem would not apply if expectations violate this law.[5]

[5] An example expectations that violate the Law of Iterated Expectations would be the case where agents expect the current regime to last one additional period: $\Re(\Im_t, X_{t-1}, \varepsilon_t) = (s_t, s_t, s_{t+2}^e, s_{t+3}^e, ...)$ and $\Re(\Im_{t+1}) = (s_t, s_t, s_{t+3}^e, s_{t+4}^e, ...)$. In such a case, agents expect today something different from what they expect to expect tomorrow if expectations today are confirmed.

3.4 The endogenous probability of regime-switch

We now turn to the third component of *RegGae*. To model the fact that switching to a financial crisis regime depends on the state of the economy, we assume that the probability of a switch is a function of the system's variables up to the previous period X_{t-1}, and the history of regime realizations up to s_{t-1}, denoted $\Im_{t-1} \equiv (..., s_{t-2}, s_{t-1})$:

$$Pr[s_t = s|X_{t-1}, \Im_{t-1}] \equiv p(s|X_{t-1}, \Im_{t-1}) \tag{20}$$

where a Markov process is a special case.

This modeling strategy where crises are probabilistic events is consistent with the current understanding of financial crises in the academic and policy-making field. It is understood that it is possible to identify vulnerabilities and other conditions that make financial crises more likely and more severe but it is not possible to predict when a crisis will be triggered. For example, credit growth and indebtedness, which can be modeled with DSGEs, are usually considered vulnerabilities with superior early warning properties as crises predictors (BCBS, 2010; Gonzalez et al. 2017). As credit is booming and households and firms are more indebted, crises are more likely. However, the exact timing of financial crises cannot predicted as the immediate crisis trigger may be a political or other exogenous event.

In many OBCC models, this crisis probability function emerges naturally from the model and do not need to be assumed (Laseen et al., 2017; Bianchi and Mendoza, 2018). In these models, the constraint slackness is a metric for crisis probability. This is because, the smaller the slackness, the more likely there will be a shock larger than the slackness — which would bind the constraint and tip the economy into a crisis.

Central banks, multilateral organizations, and other financial stability watchdogs already monitor proxies for function $p(\cdot)$ through a variety of means. Examples are the IMF's Vulnerability Exercise (Basu, Chamon, Crowe, 2017) and the "Financial System Stability Monitor" of the US's Office of Financial Research.[6] Function $p(\cdot)$ can be interpreted as the *vulnerabilities*, that is, the *unconditional* probability of a regime switch, unconditional on relevant factors not modeled in the DSGE. In actual policy-making, this unconditional probability is confronted with environmental *risks*, that is, outside knowledge about the environment and judgment to form a subjective *conditional* probability of a regime switch. Outside factors include political and external *risks*. *RegGae* allows modeling this idea precisely with a DSGE where credit and indebtedness are among the variables by assuming $p(crisis|\cdot, \Im_{t-1})$ is an increasing function of credit and indebtedness

[6] https://www.financialresearch.gov/financial-vulnerabilities

(the toy model presented in section 4 will be such an example). In the example of this paper, we will consider a case where a crisis happens with a logistic probability function and lasts one period. However, expectations would not conform with the "true" crisis probability.

Note that the probability of crises depends on the system in the previous period, not contemporaneously. As it has been shown, it would make no sense if the system's variables after a crisis determined the probability of triggering it (Barthelemy and Marx, 2017).

RegGae's setup provides a natural way to search for the optimal policy mix. The model's solution $\mathbb{S}(\Re)$ combined with the process governing the regime switch, $p(s_t|X_{t-1}, \Im_{t-1})$, can be simulated multiple times. With some welfare metric, the expected welfare of each tuple $< \mathbb{S}, p >$ can be approximated by Law of Large Numbers. The social planner can then choose the value of the parameters $\Upsilon(s_t)$ of the reaction functions to be implemented by the policy-authority in order to maximize expected welfare and thereby ensure optimal coordination of monetary and macroprudential policy. In a linear model, the parameters $\Upsilon(s_t)$ are those rows of matrices $A(s_t)$ and $B(s_t)$ under control by the policy authority. These may include (depending on the model) the parameters of the monetary and macroprudential policy rules — including the intermediate targets for inflation and financial stability. The model's solution depends on the policy rules and, to make this dependence explicit, we will write $\mathbb{S}(\Re, \Upsilon(s_t))$.

Welfare depends on the sequence of realizations of the system, $\{x_{t+k}\}_{k=1}^{\infty}$. The optimization problem to be solved is:

$$\max_{\Upsilon(s_t)} E_t[U(\{x_{t+k}\}_{k=1}^{\infty})|\Im_t, p(\cdot), \mathbb{S}(\Re, \Upsilon(s_t))] \tag{21}$$

The constraints are built in the expectations of the social planner, including the BK conditions for s-infinite regimes. We assume that the social planner knows the true process governing the regime switch, $p(\cdot)$, defined in equation (20). This models the idea that the conditions and vulnerabilities contained in x_t affecting the crisis probability are known by the social planner. The social planner also knows agents' expectation formation protocol, $\Re(\cdot)$, which embeds the cognitive biases of agents, including of the macroprudential and monetary policy authority. The social planner acts once and for all by choosing $\Upsilon(s_t)$, thereby affecting agents expectations, actions, and the model's solution $\mathbb{S}(\Re, \Upsilon(s_t))$. The optimal choice takes agents' imperfectly rational expectations into account in the sense that the optimum depends on $\Re$. Thus, while agents form imperfectly rational expectations, their actions are affected by the rational expectations of the social planner through the choice of the parameters of the policy-makers reaction functions $\Upsilon(s_t)$.

4 Illustrating *RegGae* in action

We illustrate *RegGae*'s functionalities with a toy model: a standard, small neo-keynesian DSGE augmented with an (arbitrary) credit equation and macroprudential policy rule. The choice of model shows that even a simple DSGE can be used for basic macroprudential stress testing and policy analysis with *RegGae*. The toy model has two regimes $s_t \in R = \{\text{normal, crisis}\}$ where it is assumed that the steady state level of credit is related to the steady state output: lower credit yields lower output. This models the idea that, in a financial crisis, financial intermediation is severely disrupted and the financial sector "shuts down", thereby reducing potential output. Credit is assumed to be the only variable that matters for the transition probability from normal regime into crisis. This is because credit is considered to be the single most powerful predictor of financial crises (BCBS, 2010; Gonzales et al., 2017). The crisis probability function $p(\cdot)$ is such that a crisis lasts for one period (one year) although a new crisis may materialize immediately after another. The assumption that crises last one year is commensurate to the 2008 crisis in the US, which lasted approximately from summer 2008 to spring 2009. The crisis duration of 1 year is also adopted by Bianchi and Mendoza (2018), although their work defines financial crisis differently.

In line with Assumption (2), agents expect normal periods to last forever and crises to last one single year. Therefore, there are two history node types $\iota \in \mathbb{I}$ in this model: normal years and crisis years. The full solution is in Appendix B.

Since the objective now is to simply illustrate the framework, we are not concerned with the empirical validity of the specific DSGE used in this example, we do not microfound it, and we allow ourselves to calibrate it in a caricatural fashion suitable for qualitative analysis only. We use this "toy" DSGE in the spirit presented by Blanchard (2018) according to whom "[d]ifferent types of general equilibrium models are needed for different purposes. For exploration and pedagogy, the criteria should be transparency and simplicity and, for that, toy models are the right vehicles." In a sense, the weaknesses of this specific toy DSGE strengthens the case for using *RegGae* as they show that *RegGae* is able to equip for macroprudential policy analysis even simple models.

4.1 The toy model under *RegGae*

The toy DSGE under *RegGae* has 8 elements along the same lines of Aikman et al. (2018), namely, six structural equations, a regime-switch rule, and an expectation formation protocol. The model has 6 variables — output, inflation, interest rate, credit, macroprudential policy instrument, and an AR(1) disturbance on credit — and 5 exogenous shocks (standard normal). Now we introduce the 8 elements of the toy model under *RegGae*:

1. A (credit-augmented) dynamic IS curve:

$$y_t = \rho_y y_{t-1} + (1-\rho_y)E_t y_{t+1} - \frac{1}{\sigma}[(i_t - \bar{i}) - (E_t\pi_{t+1} - \bar{\pi})] + \alpha(c_t - \bar{c}) + \sigma_y \varepsilon_t^y \quad (22)$$

where the bars indicate the steady-state values of the variables. Variable y_t denotes log-output, i_t is the nominal interest rate, c_t denotes real log-credit, σ is a parameter for the impact of the *ex-ante* real interest rate on output, α is a credit parameter, the transmission from credit to output, and σ_y is the shock term's standard deviation (the variance of ε_t^y is 1). The variables are in levels, not in gaps from steady states. We normalize the steady state $\bar{y}$ at ln(1) in normal times but [ln(.95)] in a crisis (meaning that a crisis regime GDP, if a crisis lasted forever, would be [5] percent short of the normal GDP). We use levels in order to distinguish different steady-states and to compute utility. Credit enters the IS and Phillips curves additively, implying credit rationing (for a constant interest rate, more credit would add to GDP and inflation).

2. A dynamic Phillips curve:

$$(\pi_t - \bar{\pi}) - \omega(\pi_{t-1} - \bar{\pi}) =$$
$$\beta[(E_t\pi_{t+1} - \bar{\pi}) - \omega(\pi_t - \bar{\pi})] + \theta(y_t - \bar{y}) + \phi(c_t - \bar{c}) + \sigma_\pi \varepsilon_t^\pi \quad (23)$$

where π_t denotes inflation, the parameter ω captures inflation inertia while θ and ϕ capture the impact of output and credit on inflation, respectively. σ_π is the standard deviation of the shock term (as the variance of ε_t^π is 1). We assume that credit may pressure inflation (through ϕ) independently from its effect on output. The steady-state of inflation $\bar{\pi}$ in normal times is chosen to be 2 percent (approximately equivalent to the inflation target of the Bank of England and the ECB) but only 0.125 percent in crises to mimic the typical desinflationary process of financial crises. Think of the inflation steady-state under a crisis not as a desired inflation target but the level of the system's nominal anchor.

3. A monetary policy rule:

$$i_t - \bar{i} = \rho_i(i_{t-1} - \bar{i}) + (1 - \rho_i)[\gamma_\pi(\pi_t - \bar{\pi}) + \gamma_y(y_t - \bar{y}) + \gamma_c(c_t - \bar{c})] + \sigma_i \varepsilon_t^i \quad (24)$$

where ρ_i is a smoothness parameter for monetary policy while the gammas are weights attributed by the monetary policy authority to each intermediate policy objective. σ_i is the shock standard deviation. The steady-state of the interest rate $\bar{i}$ is 4 percent in normal times but only 0.25 percent in crisis, implying that the long-run real interest rate also drops in financial crises. These levels are twice the respective levels of inflation.

With this specification, we allow for the possibility that the central bank may choose to fight financial instability via interest rates "leaning-against-the-wind", depending on the value of γ_c. This assumption has been a trend in the literature on monetary policy after the 2008-9 financial crisis (Svensson, 2016 and 2017). Different values for the coefficients of this reaction function can be tested to determine optimal values.

4. A credit equation:

$$c_t - \bar{c} = \eta(c_{t-1} - \bar{c}) - \psi(i_t - \bar{i}) + \lambda(y_t - \bar{y}) - \mu(m_t - \bar{m}) + \varepsilon_t^c \quad (25)$$

where m_t is the level of a macroprudential policy lever, such as the value of Basel III's countercyclical capital buffer (CCyB) or the cap on loan-to-value (LTV) ratios of the collateral constraint or on debt-service-to-income ratios (DSTI) for new mortgage loans or anything else usable countercyclically to constrain credit. We choose the steady state of credit to be [ln .60] in normal times but [ln .50] in crises to mimic credit crunches associated with financial crises. The parameters are η, ψ, λ, μ, and σ_c. We assume that ε_t^c follows a stationary, zero-mean AR(1) process to provide some persistence in credit shocks (sixth equation).

5. A macroprudential policy rule:

$$m_t - \bar{m} = \xi(m_{t-1} - \bar{m}) + \kappa(c_t - \bar{c}) + \sigma_m \varepsilon_t^m \quad (26)$$

where the dependence of the macroprudential tool on the deviation of credit from its long-term trend is a natural specification of the "Basel rule" proposed for the CCyB (BCBS, 2010). While the macroprudential authority, who decides the value of κ, may choose to fight inflation with macroprudential policy (due to the transmission from macroprudential policy to credit and, from there, to inflation via the Phillips curve), we constrain the regulatory authority from reacting to inflation with the macroprudential instrument. While it has been a trend to consider using monetary policy to fight financial instability, it is not generally discussed the idea of using macroprudential policy to fight inflation (although this is a theoretical possibility). In this context, some policy levers have dual identity (macroprudential and monetary) as they affect both financial stability and monetary stability. Such is the case of bank reserve requirements.

We choose the steady state $\bar{m}$ to be 7 percent in the normal regime to mimic the level of Common Equity Tier 1 (CET1) capital as a percentage of risk weighted assets (RWA) corresponding to Basel III's minimum level plus the conservation buffer. Any excess of m_t beyond 7 percent would represent the deployment of Basel III's CCyB. We set the crisis level of $\bar{m}$ to be 2 percent of RWA. With a hand-waving at this point, we state that levels below 7 percent may represent the authorization by the regulator that banks consume their Basel III's conservation buffer — we choose not to impose the occasionally binding constraint that requires the CCyB to be between 0 and 2.5% of RWA for simplicity.

6. The AR(1) process for the credit shock (stationary):

$$\varepsilon_t^c = \rho_{\varepsilon^c}\varepsilon_{t-1}^c + \epsilon_t^c \text{ where } \epsilon_t^c \sim N(0,1) \tag{27}$$

7. A regime-switch probability rule: it is assumed that crises last one period but a new crisis may happen immediately after another. Therefore, all history nodes are attainable. The probability function that a crisis will happen in t is logistic along the same lines as Ajello et al (2015) and Aikman et al (2018): [check this specification]

$$Pr[s_t = \text{crisis}|c_{t-1}] = p(c_{t-1}) = \frac{\exp[\zeta_0 + \zeta_1(c_{t-1} - \bar{c}_0)]}{1 + \exp[\zeta_0 + \zeta_1(c_{t-1} - \bar{c}_0)]} \tag{28}$$

where $\bar{c}_0$ is a parameter that equals the steady state value of c_t in normal times, [ln(.6)]. This is in line with the understanding that credit in excess of long term trend is a crisis predictor (BCBS, 2010; Schularick and Taylor, 2012). Parameters ζ_i are such that the normal regime steady-state crisis probability is [1] percent. This calibration of crises duration (one year) is broadly in line with the calibration in Svensson (2017), and Bianchi and Mendoza's (2018), where values for crises duration and were chosen to match data. However, crisis frequency is somewhat larger than the literature has documented to caricature these episodes and replicate stylized facts.

8. An expectation formation protocol: agents expect that normal times will last forever and that crises will last one period (one year), as in Bianchi and Mendoza (2018). Therefore, agents always expect normal regime from the next period onward. The expectation formation protocol is

$$\Re(\Im_t, X_{t-1}, \varepsilon_t) = \{s_t, \text{normal, normal, ...}\} \; \forall \; \Im_t \tag{29}$$

Expectations in all attainable history nodes culminate in an s-infinite regime (the normal regime). By Theorem 1, the BK conditions holding in the normal regime (only) is sufficient to imply global existence and uniqueness of a solution.

We assume the parameters take the values of Table 1. Most of the dynamics parameters are similar in both regimes, except for the policy rules, which are turned-off in the crisis regime (in the sense that BK conditions are violated during the crisis regime). However, their levels (steady-states) differ markedly according to the regime.

4.2 Simulation and Macroprudential Stress Testing

We solve the toy model applying the formulas in Appendix B and obtain the system's law of motion. To portray the data generation pattern of the solution of the toy model under *RegGae*, first we simulate it without shocks ε_t to visualize the anatomy of a financial crisis (Figure 1). The simulation helps to elaborate the narrative for the evolution of the economy during and after financial crises. In the particular simulation of Figure 1, one crisis happened in the [5th] year (bottom left chart of Figure 1). During crisis, GDP, inflation, and credit collapse. We set monetary and macroprudential to "switch-off", to ease markedly during crises. This behavior reproduces the anecdotal behavior of the economy after crisis. Because the Taylor Principle is violated in the crisis regime by the choice of reaction function parameters, the BK conditions do not hold in the crisis regime — this is unproblematic for existence and uniqueness by Theorem 1. Naturally, a more empirically valid model would have more regimes such as a boom phase, a crash, and a post-crisis phase with a low interest rate regime.

We also conducted event-history analysis to understand the typical behavior before and after crises. To that end, out of a simulation of 5,000 periods, we collected all [245] crisis episodes [(4.9%)] together with the tree years before and after the crisis year. Averaging out across episodes yields the analysis in Figure 2.

[Expand interpretation of Figure 2 here]

Figure 1: The anatomy of a toy financial crisis under *RegGae*

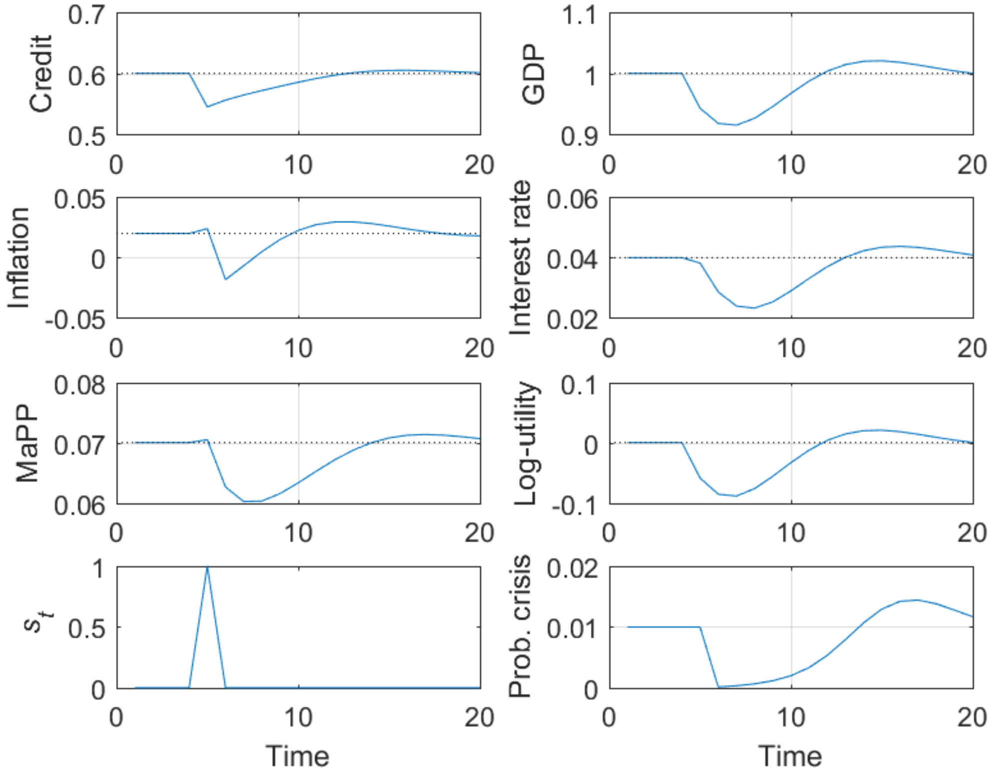


Figure 2: Toy-RegGae: event-history analysis

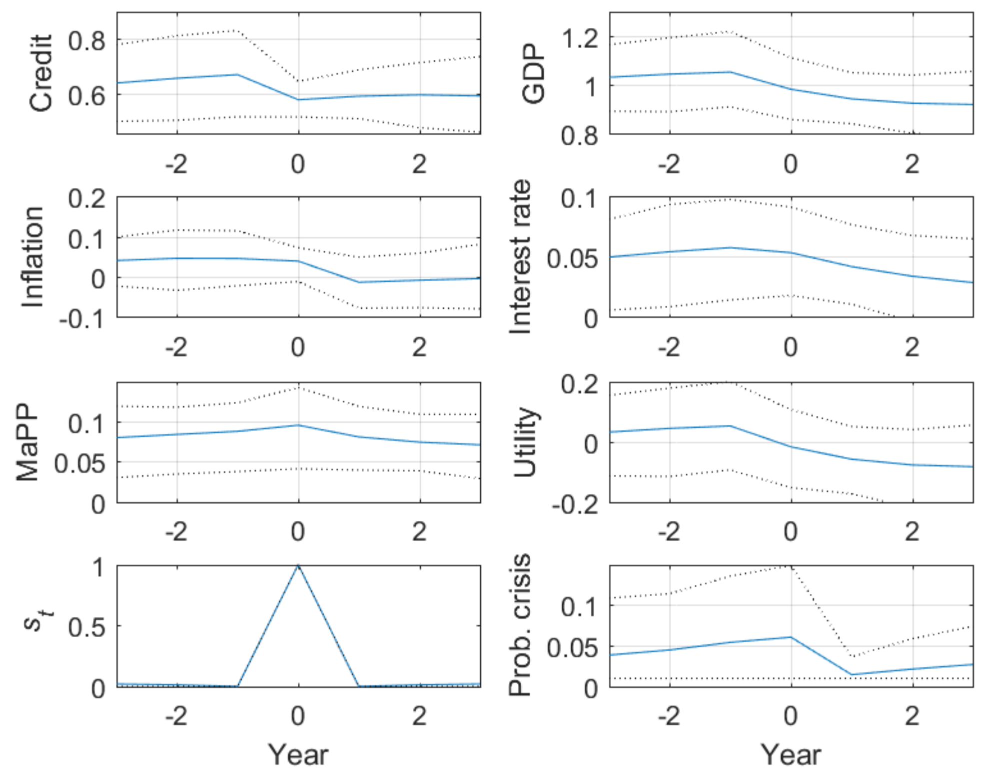
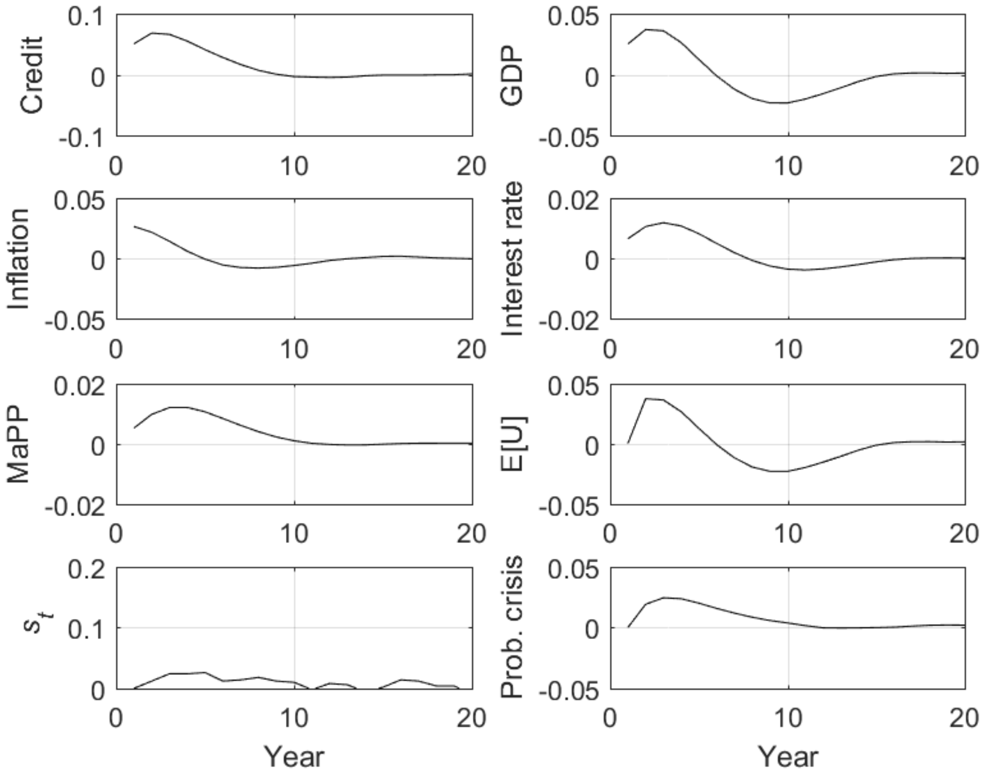


Figure 3: GIRFs under *RegGae* (shock to ϵ_t^c of 1 s.d.)

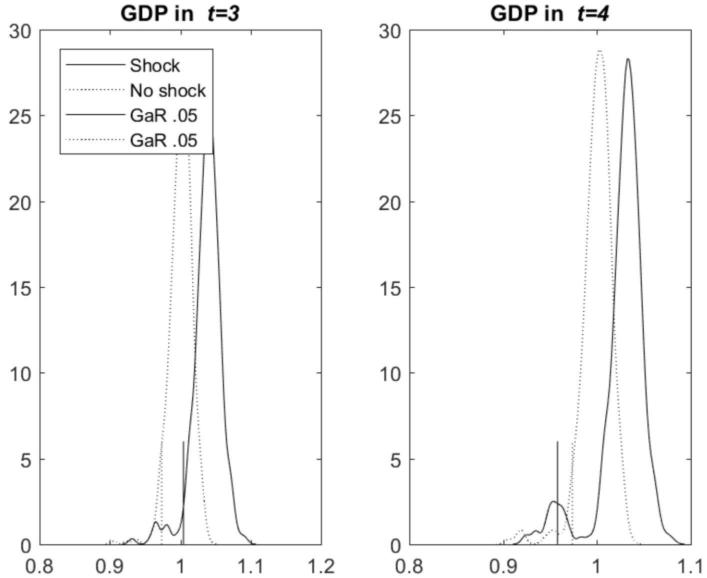


Next we conducted a macroprudential stress test by subjecting the economy to a positive credit shock, which increases the probability of a financial crisis. We also computed the effect of the credit boom on GDP-at-risk (GaR) and other value-at-risk metrics (Figures 4 to 8). To see the shock average impact, we computed the generalized impulse response functions (GIRF) of the credit shock by implementing the shock 5,000 times (Figure 3). The GIRF for this toy economy follow closely the IRF of the normal regime-only economy under a credit boom.[7] On first sight, it looks like there are only benefits in a credit boom: credit grows thereby boosts GDP. It also leads to inflation and to tightening of monetary and macroprudential policies. Because crises are rare events, their negative effect is not noticeable by looking at averages. [Problem: Explain positive GIRF for utility]

With this IRF simulation exercise, *RegGae* makes it easy to visualize the tail-fattening of the outcome distribution when a credit boom is underway. The magnitude of the tail-fattening can work as metric of financial vulnerability along the same lines of IMF (2017). To that end, we look at the left-shift in the GaR that results from the credit shock (Figure 4). The chosen GaR threshold in the literature is 5%, which tends to capture most crisis

[7] It is assumed that crises cannot happen in $t = 1$ and, because of this GDP "bonus" without crisis risk, the utility GIRF in $t = 1$ was set to zero.

Figure 4: *GDP-at-risk (GaR)*: Density of GDP IRF



episodes. In our toy example, as expected, the credit boom caused the distribution tail to fatten and GaR to drop: without the credit shock, there was a 5% probability that GDP would be below [.97] in a 3 year horizon but, if such a shock happened, the 5% probability cutoff would shift left to [.94], a shift of [3] percentage points. This measure works as a vulnerability metric as it is the impact of a plausible shock, the result of the macroprudential stress test. In the 4-year horizon, the left shift of the GaR 5% cutoff would be even higher, [12.8] percentage points (from [.98] to [.93]). This measure can minimized with the choice of the optimal policy rule.

5 Conclusion

Macroprudential policy and macroprudential stress tests can be conducted even with simple DSGE models. To that end, *RegGae* provides a *how-to* explanation of elements needed and formulas to be applied to augment a DSGE into a macroprudential policy- and stress-test-ready DSGE. *RegGae*'s framework can be applied to *any* specific DSGE, including non-linear models. Therefore, existing DSGEs being used in central banks for monetary policy can be seamlessly augmented to inform macroprudential policy as well. It suffices to collect the Jacobians and steady-state vectors of the DSGE (one set per regime), rearrange the matrices in the format of equation (2), figure out a reasonable expectation formation protocols and apply the formulas in this article.[8] With the crisis probability distribution evolving in parallel to the DSGE process, *RegGae* makes viable the derivation of values-at-risk metrics and the simulation of shifts in values-at-risk (vul-

[8]The Jacobian and steady-states can be obtained with software packages such as Dynare.

Figure 5: Density of interest rate IRF

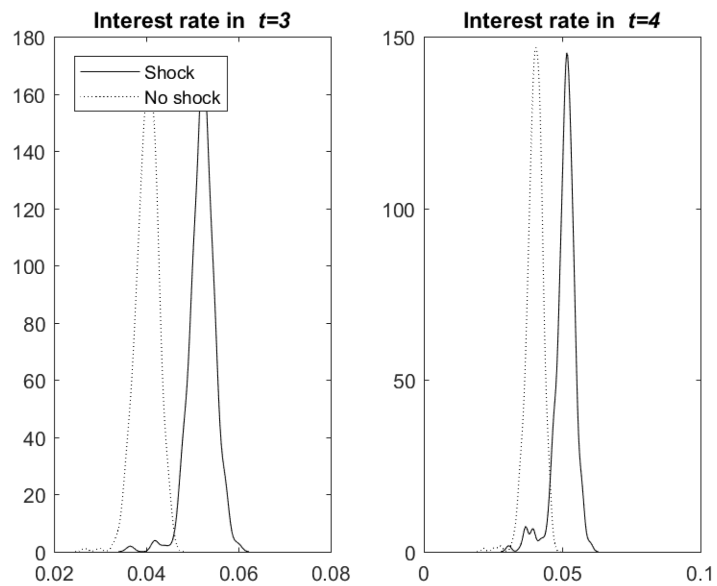


Figure 6: *Credit-at-risk*: Density of credit IRF

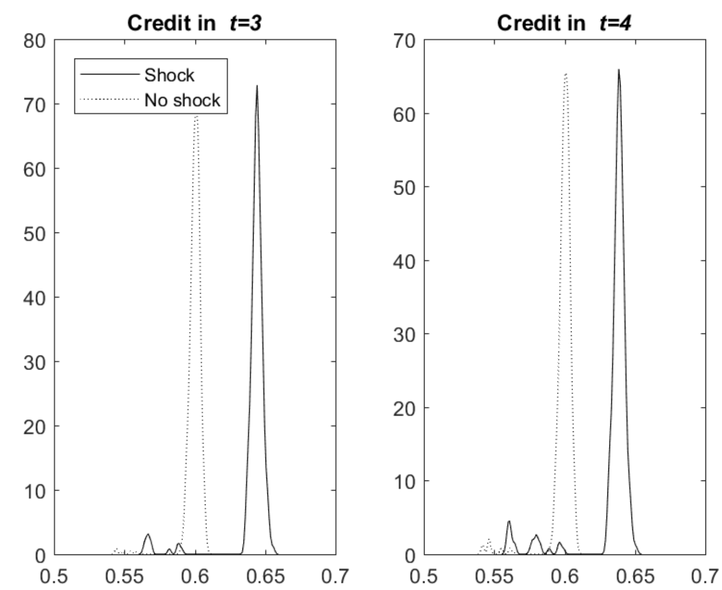


Figure 7: *MaPP-at-risk*: Density of Macroprudential Policy IRF

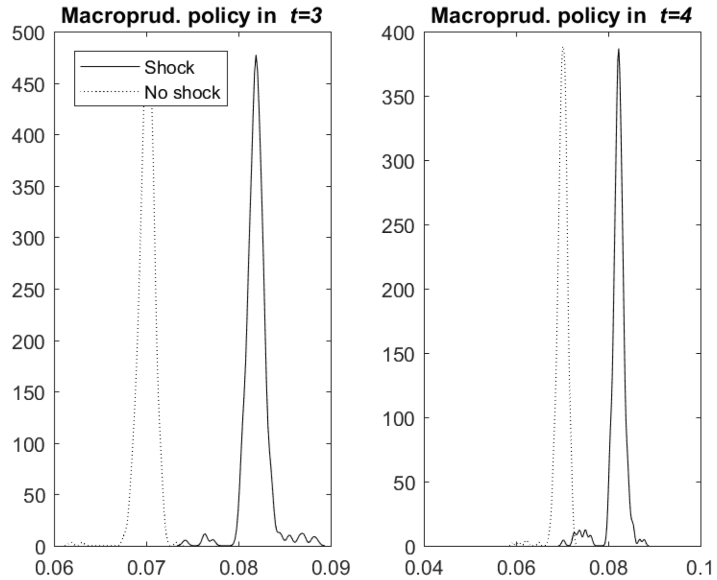


Figure 8: *Inflation-at-risk*: Density of inflation IRF

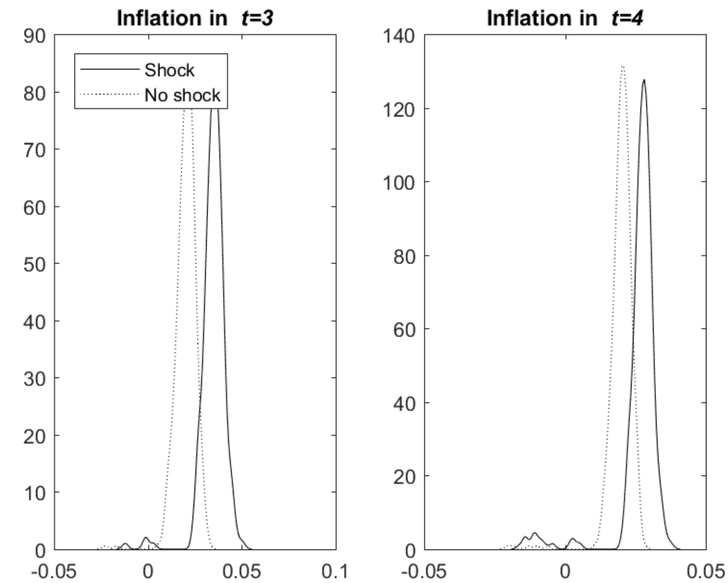


Table 1: Parameter Values

Parameter	Normal	Crisis
ω	0.01	0.01
σ	1	1
β	0.98	0.98
θ	0.1	0.1
α	0.09	0.15
ϕ	0.05	0.05
ρ_y	0.85	.8
ρ_i	0.767	0.7
ρ_{ε^c}	0.75	0.75
γ_π	1.01	0.975
γ_y	0.05	0.025
γ_c	0.00001	0.00001
η	0.65	0.25
ψ	0.05	0.05
λ	0.25	0.25
μ	1	1
ξ	0.5	0.6
κ	0.1	0.295
$\bar{y}$	ln(1)	ln(.8)
$\bar{\pi}$	0.02	0.00125
$\bar{i}$	0.04	0.0025
$\bar{c}$	ln(.6)	ln(.5)
$\bar{m}$	0.07	0.02
s.d.(ε_t^y)	0.0025	0.000075
s.d.(ε_t^π)	0.0075	0.000075
s.d.(ε_t^i)	0.001	0.00001
s.d.(ϵ_t^c)	0.05	0.001
s.d.(ε_t^m)	0.0005	0.00001
ζ_0	-4.6	...
ζ_1	45	...

nerabilities) caused by any of model's shocks.

The challenge to use *RegGae* is rather empirical and computational than technical. On the empirical side, it is difficult to calibrate reasonable parameter values for crises regimes, let alone estimate them. This is because financial crises are rare, temporary and therefore elusive. In addition, finding a tuple $< \mathbb{S}, p >$ requires devising an appropriate crisis probability function $p(\cdot)$. This is a direction where the early warning research could usefully evolve and, in this sense, *RegGae* unifies the research on DSGEs with that on early warning methods.

Performing the optimization of section 3.5 may require non-usual computational capacity for the search algorithm. Indeed, the lack of analytic solution for the optimization problem implies the need to simulate the model thousands of times to obtain good estimates of the expected welfare of each policy rule.

Despite these challenges, *RegGae* presents a promising avenue for research on financial stability and for assisting policy making. It does so by providing DSGE models with new uses by repurposing them to a new application.

Appendices

A Solution of *RegGae* (Proof of Propositions 1 and 2)

In this section, we derive *RegGae's* solution $\mathbb{S}(\Re, \Upsilon)$, which are the formulas in Propositions 1 and 2. We proceed with the Blanchard-Kahn method of eigenvalue/eigenvector manipulation. Starting from equation (5), diagonalize each $A(s_t)$ such that (with some abuse of notation):

$$A(s_t) \equiv P_t^{-1} \Lambda_t P_t \tag{30}$$

where Λ_t is a diagonal matrix of eigenvalues and P_t is the inverse of the eigenvectors' matrix. Eigenvalues are ordered in increasing absolute values while eigenvectors are ordered accordingly.

Plugging (30) into (5) and premultiplying by P_t results in:

$$P_t \begin{bmatrix} x_t^p \\ E_t[x_{t+1}^j] \end{bmatrix} = \Lambda_t P_t \begin{bmatrix} x_{t-1}^p \\ x_t^j \end{bmatrix} + P_t B(s_t) + P_t C(s_t) \varepsilon_t \tag{31}$$

Partition the matrix P_t into $\begin{bmatrix} P_{p,t} \\ P_{j,t} \end{bmatrix}$ corresponding to the p (number of predetermined variables) upper rows and j (number of "jumper", non-predetermined variables) lower rows of P_t. Taking only the lower j rows of P_t allows us to extract the following relationship from (31):

$$P_{j,t} \begin{bmatrix} x_t^p \\ E_t[x_{t+1}^j] \end{bmatrix} = \Lambda_{j,t} P_{j,t} \begin{bmatrix} x_{t-1}^p \\ x_t^j \end{bmatrix} + P_{j,t} B(s_t) + P_{j,t} C(s_t) \varepsilon_t \tag{32}$$

where $\Lambda_{j,t}$ is a diagonal matrix of dimension j that, because of the ordering adopted, contains the j largest eigenvalues (in absolute value) of Λ_t.[9]

Premultiplying (32) by $\Lambda_{j,t}^{-1}$ and rearranging terms yields:

$$P_{j,t} \begin{bmatrix} x_{t-1}^p \\ x_t^j \end{bmatrix} = \Lambda_{j,t}^{-1} P_{j,t} \begin{bmatrix} x_t^p \\ E_t[x_{t+1}^j] \end{bmatrix} - \Lambda_{j,t}^{-1} P_{j,t} B(s_t) - \Lambda_{j,t}^{-1} P_{j,t} C(s_t) \varepsilon_t \tag{33}$$

[9] Later we will assume that all and only the eigenvalues in $\Lambda_{j,t}$ lie outside the unit circle (the Blanchard-Kahn conditions)

Forwarding (33) by one period yields:

$$P_{j,t+1}\begin{bmatrix} x^p_t \\ x^j_{t+1}\end{bmatrix} = \Lambda^{-1}_{j,t+1}P_{j,t+1}\begin{bmatrix} x^p_{t+1} \\ E_{t+1}[x^j_{t+2}]\end{bmatrix} - \Lambda^{-1}_{j,t+1}P_{j,t+1}B(s_{t+1}) - \Lambda^{-1}_{j,t+1}P_{j,t+1}C(s_{t+1})\varepsilon_{t+1} \quad (34)$$

We proceed by solving for the forward looking variables under s-infinite histories, then for the pre-determined variables and finally for the finite histories.

A.1 Forward-looking variables at s-infinite histories

To calculate the forward-looking variables under s-infinite expectations, take expectations as of t of (33) and (34) conditional on $s_{t+k} = s$ for some s for $k = 1, 2, ...$ denoted by $\vec{s}$. Thus, the expectations of equations (33) and (34) as of t become:

$$P_{j,s}\begin{bmatrix} x^p_{t-1} \\ x^j_t\end{bmatrix} = \Lambda^{-1}_{j,s}P_{j,s}\begin{bmatrix} E_t[x^p_t|\vec{s}] \\ E_t[x^j_{t+1}|\vec{s}]\end{bmatrix} - \Lambda^{-1}_{j,s}P_{j,s}B(s) - \Lambda^{-1}_{j,s}P_{j,s}C(s)\varepsilon_t \quad (35)$$

and

$$P_{j,s}\begin{bmatrix} E_t[x^p_t|\vec{s}] \\ E_t[x^j_{t+1}|\vec{s}]\end{bmatrix} = \Lambda^{-1}_{j,s}P_{j,s}\begin{bmatrix} E_t[x^p_{t+1}|\vec{s}] \\ E_t[x^j_{t+2}|\vec{s}]\end{bmatrix} - \Lambda^{-1}_{j,s}P_{j,s}B(s) - \Lambda^{-1}_{j,s}P_{j,s}C(s)E_t[\varepsilon_{t+1}] \quad (36)$$

Plug (36) into (35):

$$P_{j,s}\begin{bmatrix} x^p_{t-1} \\ x^j_t\end{bmatrix} = \Lambda^{-1}_{j,s}\left[\Lambda^{-1}_{j,s}P_{j,s}\begin{bmatrix} E_t[x^p_{t+1}|\vec{s}] \\ E_t[x^j_{t+2}|\vec{s}]\end{bmatrix} - \Lambda^{-1}_{j,s}P_{j,s}B(s) - \Lambda^{-1}_{j,s}P_{j,s}C(s)E_t[\varepsilon_{t+1}]\right] - \Lambda^{-1}_{j,s}P_{j,s}B(s) - \Lambda^{-1}_{j,s}P_{j,s}C(s)\varepsilon_t \quad (37)$$

$$\Rightarrow P_{j,s}\begin{bmatrix} x^p_{t-1} \\ x^j_t\end{bmatrix} = \Lambda^{-2}_{j,s}P_{j,s}\begin{bmatrix} E_t[x^p_{t+1}|\vec{s}] \\ E_t[x^j_{t+2}|\vec{s}]\end{bmatrix} - \sum_{k=1}^{2}\Lambda^{-k}_{j,s}P_{j,s}\big[B(s) + C(s)E_t[\varepsilon_{t+k-1}]\big] \quad (38)$$

Repeating recursively *ad infinitum* the substitution of (36) (forwarded one period at a time) into (38) yields:

$$P_{j,s}\begin{bmatrix} x^p_{t-1} \\ x^j_t\end{bmatrix} = \lim_{T\to\infty}\Lambda^{-T}_{j,s}P_{j,s}\begin{bmatrix} E_t[x^p_{t+T-1}|\vec{s}] \\ E_t[x^j_{t+T}|\vec{s}]\end{bmatrix} - \sum_{k=1}^{\infty}\Lambda^{-k}_{j,t}P_{j,s}\big[B(s) + C(s)E_t[\varepsilon_{t+k-1}]\big] \quad (39)$$

Let the Blanchard-Kahn (1980) conditions hold for all s-infinite regimes, i.e., all elements of $\Lambda_{j,s}$ are larger than 1 in modulus. This allows crossing out the term with the limit of future expectations. Let also the "present value" of all the expectations of fundamental shocks (discounted by the inverse eigenvalues) multiplied $P_{j,s}C(s)$ converge to zero. This allows crossing out the expectations of future shocks but leaving behind the term with current period shock. Then (39) becomes:

$$\Rightarrow P_{j,s}\begin{bmatrix} x^p_{t-1} \\ x^j_t \end{bmatrix} = -\sum_{k=1}^{\infty}\Lambda^{-k}_{j,s}P_{j,s}B(s) - \Lambda^{-1}_{j,s}P_{j,s}C(s)\varepsilon_t$$

$$= -[(I_j - \Lambda^{-1}_{j,s})^{-1} - I_j]P_{j,s}B(s) - \Lambda^{-1}_{j,s}P_{j,s}C(s)\varepsilon_t \qquad (40)$$

where I_j is an identity matrix of dimension j.

Adopting the notation where a generic matrix L is partitioned into $L \equiv \begin{bmatrix} L_p \\ L_j \end{bmatrix} \equiv \begin{bmatrix} L_{pp} & L_{pj} \\ L_{jp} & L_{jj} \end{bmatrix}$ and plugging the partition of $P_{j,s}$ in (40) imply

$$\Rightarrow P_{j,s}\begin{bmatrix} x^p_{t-1} \\ x^j_t \end{bmatrix} \equiv P_{jp,s}x^p_{t-1} + P_{jj,s}x^j_t = -[(I_j - \Lambda^{-1}_{j,s})^{-1} - I_j]P_{j,s}B(s) - \Lambda^{-1}_{j,s}P_{j,s}C(s)\varepsilon_t \quad (41)$$

Solving (41) for x^j_t pins down the contemporaneous value of the jumpers as a function of the pre-determined variables under s-infinite expectations:

$$x^j_t = -(P_{jj,s})^{-1}\{P_{jp,s}x^p_{t-1} + [(I_j - \Lambda^{-1}_{j,s})^{-1} - I_j]P_{j,s}B(s) + \Lambda^{-1}_{j,s}P_{j,s}C(s)\varepsilon_t\} \qquad (42)$$

Equation (42) can be recast as:

$$x^j_t = V_s x^p_{t-1} + Z_s + W_s\varepsilon_t \qquad (43)$$

where

$$V_s \equiv -(P_{jj,s})^{-1}P_{jp,s} \qquad (44)$$

$$Z_s \equiv -(P_{jj,s})^{-1}[(I_j - \Lambda^{-1}_{j,s})^{-1} - I_j]P_{j,s}B(s) \qquad (45)$$

$$W_s \equiv -(P_{jj,s})^{-1}\Lambda^{-1}_{j,s}P_{j,s}C(s) \qquad (46)$$

Equations (43)-(46) are the solution for the jumpers under s-infinite expectations. This solution is identical to the single-regime solution. It also pins down the expectation as of t of future value of jumpers:

$$E_t[x^j_{t+k}|\vec{s}] = V_s E_t[x^p_{t+k-1}|\vec{s}] + Z_s + W_s E_t[\varepsilon_{t+k}] \text{ for } k > 0 \tag{47}$$

These results will be used in the next steps.

A.2 Pre-determined variables

Let the forward-looking variables at time t be

$$x^j_t \equiv V_t x^p_{t-1} + Z_t + W_t \varepsilon_t \tag{48}$$

for some values of matrices V_t, Z_t and W_t. Then, taking the p upper rows of the DSGE in the format of equation (4) and replacing the forward-looking variables with the solution (48), the following holds:

$$x^p_t = A_{pp}(s_t)x^p_{t-1} + A_{pj}(s_t)[V_t x^p_{t-1} + Z_t + W_t\varepsilon_t] + B_p(s_t) + C_p(s_t)\varepsilon_t \tag{49}$$

$$\Rightarrow x^p_t = [A_{pp}(s_t) + A_{pj}(s_t)V_t]x^p_{t-1} + A_{pj}(s_t)Z_t + B_p(s_t) + [A_{pj}(s_t)W_t + C_p(s_t)]\varepsilon_t \tag{50}$$

$$\Rightarrow x^p_t = M_t x^p_{t-1} + Q_t + S_t\varepsilon_t \tag{51}$$

where

$$M_t \equiv A_{pp}(s_t) + A_{pj}(s_t)V_t \tag{52}$$

$$Q_t \equiv A_{pj}(s_t)Z_t + B_p(s_t) \tag{53}$$

$$S_t \equiv A_{pj}(s_t)W_t + C_p(s_t) \tag{54}$$

These formulas show that the motion of pre-determined variables during t depends on previous expectations (embedded in x^p_{t-1}), the current regime s_t and current beliefs about present and future regimes (embedded in V_t, Z_t and W_t).

If at t the realization of s_t is such that agents have s-infinite expectations, then matrices V_t, Z_t and W_t are given by formulas (44)-(46) and the solution for current period is complete. We proceed now to solve for the forward-looking variables in nodes with finite expectations.

A.3 Forward-looking variables at finite expectations

In history nodes with finite expectations at time t, agents expect that the regime will switch eventually. Then, we can find the solution for V_t, Z_t and W_t of equation (48) as functions of expected matrices of the signal equation in the next period $t+1$. Let

$$E_t[V_{t+1}] \equiv V^*_{t+1} \text{ for some specific value } V^*_{t+1} \tag{55}$$

$$E_t[Z_{t+1}] \equiv Z^*_{t+1} \text{ for some specific value } Z^*_{t+1} \tag{56}$$

$$E_t[W_{t+1}] \equiv W^*_{t+1} \text{ for some specific value } W^*_{t+1} \tag{57}$$

$$\Rightarrow E_t[x^j_{t+1}] = V^*_{t+1}x^p_t + Z^*_{t+1} + W^*_{t+1}E_t[\varepsilon_{t+1}] \tag{58}$$

Because of Assumption (2) (Deterministic Expectations), it is also true that

$$E_t[s_{t+1}] = s^*_{t+1} \text{ for some specific value } s^*_{t+1} \in R \tag{59}$$

Plug (58) in the lower j rows of the DSGE (equation 31) (suppress the * superscript to unclutter the notation):

$$P_{j,t}\begin{bmatrix} x^p_t \\ V_{t+1}x^p_t + Z_{t+1} + W_{t+1}E_t[\varepsilon_{t+1}] \end{bmatrix} = \Lambda_{j,t}P_{j,t}\begin{bmatrix} x^p_{t-1} \\ x^j_t \end{bmatrix} + P_{j,t}B(s_t) + P_{j,t}C(s_t)\varepsilon_t \tag{60}$$

$$\begin{aligned} \Rightarrow x^j_t = &-(P_{jj,t})^{-1}P_{jp,t}x^p_{t-1} + \\ &(P_{jj,t})^{-1}\Lambda^{-1}_{j,t}\{(P_{jp,t} + P_{jj,t}V_{t+1})x^p_t + P_{jj,t}Z_{t+1} - P_{j,t}B(s_t) \\ &+ P_{jj,t}W_{t+1}E_t[\varepsilon_{t+1}] - P_{j,t}C(s_t)\varepsilon_t\} \end{aligned} \tag{61}$$

Plug x^p_t of equation (50) in (61):

$$\begin{aligned} \Rightarrow x^j_t = &-(P_{jj,t})^{-1}P_{jp,t}x^p_{t-1} + (P_{jj,t})^{-1}\Lambda^{-1}_{j,t} \\ &\Big\{(P_{jp,t} + P_{jj,t}V_{t+1}) \\ &\{[A_{pp}(s_t) + A_{pj}(s_t)V_t]x^p_{t-1} + A_{pj}(s_t)Z_t + B_p(s_t) + [A_{pj}(s_t)W_t + C_p(s_t)]\varepsilon_t\} \\ &+ P_{jj,t}Z_{t+1} - P_{j,t}B(s_t) + P_{jj,t}W_{t+1}E_t[\varepsilon_{t+1}] - P_{j,t}C(s_t)\varepsilon_t\Big\} \end{aligned} \tag{62}$$

$$\Rightarrow x_t^j = \Big\{ -(P_{jj,t})^{-1}P_{jp,t} + (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})[A_{pp}(s_t) + A_{pj}(s_t)V_t]\Big\}x_{t-1}^p + (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}\Big\{(P_{jp,t}+P_{jj,t}V_{t+1})[A_{pj}(s_t)Z_t+B_p(s_t)]+P_{jj,t}Z_{t+1}-P_{j,t}B(s_t)+P_{jj,t}W_{t+1}E_t[\varepsilon_{t+1}]\Big\}+ (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}\Big\{(P_{jp,t} + P_{jj,t}V_{t+1})[A_{pj}(s_t)W_t + C_p(s_t)] - P_{j,t}C(s_t)\Big\}\varepsilon_t \quad (63)$$

Comparing equation (63) with (48), we find V_t, Z_t and W_t and can solve for each of them. For V_t:

$$\Rightarrow V_t = -(P_{jj,t})^{-1}P_{jp,t} + (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})[A_{pp}(s_t) + A_{pj}(s_t)V_t] \quad (64)$$

$$\Rightarrow V_t = -(P_{jj,t})^{-1}P_{jp,t} + (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})A_{pp}(s_t) + (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})A_{pj}(s_t)V_t \quad (65)$$

$$\Rightarrow V_t = [I_j - (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})A_{pj}(s_t)]^{-1} [-(P_{jj,t})^{-1}P_{jp,t} + (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})A_{pp}(s_t)] \quad (66)$$

For Z_t:

$$\Rightarrow Z_t = (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}\Big\{(P_{jp,t}+P_{jj,t}V_{t+1})[A_{pj}(s_t)Z_t+B_p(s_t)]+P_{jj,t}Z_{t+1}-P_{j,t}B(s_t)+P_{jj,t}W_{t+1}E_t[\varepsilon_{t+1}]\Big\} \quad (67)$$

$$\Rightarrow Z_t = (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})A_{pj}(s_t)Z_t + (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}\{(P_{jp,t} + P_{jj,t}V_{t+1})B_p(s_t) + P_{jj,t}Z_{t+1} - P_{j,t}B(s_t) + P_{jj,t}W_{t+1}E_t[\varepsilon_{t+1}]\} \quad (68)$$

$$\Rightarrow Z_t = [I_j - (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})A_{pj}(s_t)]^{-1} (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}\{(P_{jp,t} + P_{jj,t}V_{t+1})B_p(s_t) + P_{jj,t}Z_{t+1} - P_{j,t}B(s_t) + P_{jj,t}W_{t+1}E_t[\varepsilon_{t+1}]\} \quad (69)$$

For W_t

$$\Rightarrow W_t = (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}\Big\{(P_{jp,t} + P_{jj,t}V_{t+1})[A_{pj}(s_t)W_t + C_p(s_t)] - P_{j,t}C(s_t)\Big\} \quad (70)$$

$$\Rightarrow W_t = (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})A_{pj}(s_t)W_t \\ + (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}[(P_{jp,t} + P_{jj,t}V_{t+1})C_p(s_t) - P_{j,t}C(s_t)] \quad (71)$$

$$\Rightarrow W_t = [I_j - (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}(P_{jp,t} + P_{jj,t}V_{t+1})A_{pj}(s_t)]^{-1} \\ (P_{jj,t})^{-1}\Lambda_{j,t}^{-1}[(P_{jp,t} + P_{jj,t}V_{t+1})C_p(s_t) - P_{j,t}C(s_t)] \quad (72)$$

Formulas in equations (43)-(46), (51)-(54), (66),(69) and (72) define the full solution $\mathbb{S}(\Re, \Upsilon)$ of any *RegGae*, if a solution exists. Existence of a solution for all histories $\Im_t$ depends on the expectation formation protocol $\Re$. In particular, note that to obtain the formulas, the Blanchard-Kahn rank condition were necessary only at s-infinite histories. Therefore, if for all histories $\Im_t$ the expectation protocol yields s-infinite expectations at some future time $t + k$ ($k \in \mathbb{N}$), then the Blanchard-Kahn conditions of the s-infinite regimes are sufficient to ensure node-wise existence and uniqueness (Theorem 1). Otherwise, if the expectation protocol yields eternal switches for some history $\Im_t$ then, node-wise existence and stability require other conditions. In the next Appendix, we apply the formulas to find the specific solution for the example in the text.

B Solution of the toy example

Let $s = 0$ denote the normal regime and $s = 1$ denote the financial crisis regime. The protocol is such that the normal regime is 0-infinite. If one crisis does happen, agents expect it to last for one period followed by 0-infinite (normal) regime. Therefore, by Proposition 1, the BK conditions holding in the normal regime are sufficient for existence and uniqueness. The probability function $p(\cdot)$ is such that crises last exactly one period although a new crisis may start immediately after the pr evious. Given this protocol and this regime probability function, there are two history node types ι: normal periods and crisis periods. We proceed to find the solution for each of these period types.

B.1 Normal periods

At this period type, agents have 0-infinite expectations of normality. Formulas in equations (43)-(46) can be applied to find the forward-looking variables:

$$x_t^j = V_0 x_{t-1}^p + Z_0 + W_0 \varepsilon_t \tag{73}$$

where

$$V_0 \equiv -(P_{jj,0})^{-1} P_{jp,0} \tag{74}$$

$$Z_0 \equiv -(P_{jj,0})^{-1}[(I_j - \Lambda_{j,0}^{-1})^{-1} - I_j] P_{j,0} B(0) \tag{75}$$

$$W_0 \equiv -(P_{jj,0})^{-1} \Lambda_{j,0}^{-1} P_{j,0} C(0) \tag{76}$$

According to formulas (50)-(54), the pre-determined variables move according to:

$$x_t^p = M_0 x_{t-1}^p + Q_0 + S_0 \varepsilon_t \tag{77}$$

where

$$M_0 \equiv A_{pp}(0) + A_{pj}(0) V_0 \tag{78}$$

$$Q_0 \equiv A_{pj}(0) Z_0 + B_p(0) \tag{79}$$

$$S_0 \equiv A_{pj}(0) W_0 + C_p(0) \tag{80}$$

B.2 Crisis periods

The crisis period type is not infinite (as agents expect a switch back to normality). To find forward-looking variables we proceed backwards by chaining future matrices V_0, Z_0

and W_0 into the formulas (66),(69) and (72) to find the current matrices, denoted V_{10}, Z_{10}, and W_{10}. It is assumed that ε_t is zero-mean i.i.d. Therefore, $E_t[\varepsilon_{t+1}] = 0$, which allows us to simplify the expression for Z_{10}:

$$V_{10} = [I_j - (P_{jj,1})^{-1}\Lambda_{j,1}^{-1}(P_{jp,1} + P_{jj,1}V_0)A_{pj}(1)]^{-1} \\ [-(P_{jj,1})^{-1}P_{jp,1} + (P_{jj,1})^{-1}\Lambda_{j,1}^{-1}(P_{jp,1} + P_{jj,1}V_0)A_{pp}(1)] \quad (81)$$

$$Z_{10} = [I_j - (P_{jj,1})^{-1}\Lambda_{j,1}^{-1}(P_{jp,1} + P_{jj,1}V_0)A_{pj}(1)]^{-1} \\ (P_{jj,1})^{-1}\Lambda_{j,1}^{-1}[(P_{jp,1} + P_{jj,1}V_0)B_p(1) + P_{jj,1}Z_0 - P_{j,1}B(1)] \quad (82)$$

$$W_{10} = [I_j - (P_{jj,1})^{-1}\Lambda_{j,1}^{-1}(P_{jp,1} + P_{jj,1}V_0)A_{pj}(1)]^{-1} \\ (P_{jj,1})^{-1}\Lambda_{j,1}^{-1}[(P_{jp,1} + P_{jj,1}V_0)C_p(1) - P_{j,1}C(1)] \quad (83)$$

Then, the motion of pre-determined variables evolve according to

$$x_t^p = M_{10}x_{t-1}^p + Q_{10} + S_{10}\varepsilon_t \quad (84)$$

where

$$M_{10} \equiv A_{pp}(1) + A_{pj}(1)V_{10} \quad (85)$$

$$Q_{10} \equiv A_{pj}(1)Z_{10} + B_p(1) \quad (86)$$

$$S_{10} \equiv A_{pj}(1)W_{10} + C_p(1) \quad (87)$$

C Assessing the quality of the regime-wise linear solution of *RegGae*

In this section, we assess the regime-wise linear solution of *RegGae* is sufficiently close to the exact (non-linear) solution. The example used was taken and adapted from the worked-out example of section 4.3 of Fernandez-Villaverde et al. (2016). The time indices were adapted to the end-of-period convention used in this article.

Let a regime switching model under *RegGae* be:

$$\max_{\{c_t,k_t\}} E_0 \sum_{t=1}^{\infty} \beta^t \ln c_t \tag{88}$$

$$c_t + k_t = e^{z_t} s_t k_{t-1}^{\alpha} \tag{89}$$

$$z_t = \rho z_{t-1} + \sigma\varepsilon_t \text{ where } |\rho| < 1 \text{ and } \varepsilon_t \sim N(0,1) \tag{90}$$

$$s_t = \{s_{\text{high}}\ s_{\text{low}}\} \tag{91}$$

$$\Re(\Im_t, X_{t-1}, \varepsilon_t) = \{s_{\text{high}}, s_{\text{high}}, ...\}\ \forall\ \Im_t \tag{92}$$

$$p(\text{low}|\Im_{t-1}) = \frac{e^{\zeta_0+\zeta_1(k_{t-1}-\bar{k})}}{1+e^{\zeta_0+\zeta_1(k_{t-1}-\bar{k})}} \tag{93}$$

The first order conditions are taken regime-wise under the assumption that agents are in constant-parameter worlds. The following equations constitute the non-linear system $g(\cdot)$:

$$\frac{1}{c_t} = \beta E_t \frac{\alpha e^{z_t} s_t k_t^{\alpha-1}}{c_{t+1}} \tag{94}$$

$$c_t + k_t = e^{z_t} s_t k_{t-1}^{\alpha} \tag{95}$$

$$z_t = \rho z_{t-1} + \sigma\varepsilon_t \tag{96}$$

Regime-wise linearization (first order perturbation)

We are looking for decision rules for consumption and capital:

$$c_t = c(k_{t-1}, z_t)$$

$$k_t = k(k_{t-1}, z_t)$$

Substitute the decision rules into (94) and (95) to obtain:

$$\frac{1}{c(k_{t-1}, z_t)} = \beta E_t \frac{\alpha e^{z_t} s_t k(k_{t-1}, z_t)^{\alpha-1}}{c(k(k_{t-1}, z_t), z_{t+1})} \tag{97}$$

$$c(k_{t-1}, z_t) + k(k_{t-1}, z_t) = e^{z_t} s_t k_{t-1}^{\alpha} \tag{98}$$

What follows is an adaptation almost literal from Fernandez-Villaverde et al. (2016). The decisions rules are approximated by perturbation solutions on the state variable k_t plus the perturbation parameter η introduced as ηz_t:

$$c_t = c(k_{t-1}; \eta)$$

$$k_t = k(k_{t-1}; \eta)$$

If $\eta = 0$ we can find the regime-specific deterministic steady-state by solving:

$$\frac{1}{c} = \beta \frac{\alpha s k^{\alpha-1}}{c} \tag{99}$$

$$c + k = s k^{\alpha} \tag{100}$$

which are

$$\bar{k}(s) = (\alpha\beta s)^{\frac{1}{1-\alpha}} \tag{101}$$

$$\bar{c}(s) = s(\alpha\beta s)^{\frac{\alpha}{1-\alpha}} - (\alpha\beta s)^{\frac{1}{1-\alpha}} \tag{102}$$

Taking a first order expansion regime-wise around the steady-state yields the equivalent of equation (2) for this model:

$$A_1(s_t) \begin{bmatrix} k_t \\ z_t \\ E_t[c_{t+1}] \end{bmatrix} = A_2(s_t) \begin{bmatrix} k_{t-1} \\ z_{t-1} \\ c_t \end{bmatrix} + [A_1(s_t) - A_2(s_t)] \begin{bmatrix} \bar{k}(s_t) \\ 0 \\ \bar{c}(s_t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \sigma \end{bmatrix} \varepsilon_t \tag{103}$$

where matrices $A_1(s_t)$ and $A_2(s_t)$ contain the Jacobian of the system formed by equations (96)-(98). We calibrate the DSGE as in the original textbook:

$$\beta = 0.99$$

$$\rho = 0.95$$

$$\sigma = 0.01$$

$$\alpha = 0.33$$

$$s_{\text{high}} = 1 \text{ and } s_{\text{low}} = 0.95$$

From the regime-specific steady states, we can see that, under the low regime, both capital stock and consumption are below the high regime:

$$\bar{c}(s_{\text{high}}) = 0.388069$$

$$\bar{k}(s_{\text{high}}) = 0.188299$$

$$\bar{c}(s_{\text{low}}) = 0.359468$$

$$\bar{k}(s_{\text{low}}) = 0.174422$$

Linear solution under ***RegGae***

In this section we apply the formulas in Propositions 1 and 2. When $s_t = s_{\text{high}}$ and agents expect it to continue on forever, the linear solution is:

$$\begin{bmatrix} k_t \\ z_t \\ c_t \end{bmatrix} = \begin{bmatrix} 0.3300 & 0.1876 \\ 0 & 0.9500 \\ 0.6801 & 0.3599 \end{bmatrix} \begin{bmatrix} k_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 0.1262 \\ 0 \\ 0.2600 \end{bmatrix} + \begin{bmatrix} 0.0020 \\ 0.0100 \\ 0.0038 \end{bmatrix} \varepsilon_t \tag{104}$$

When $s_t = s_{\text{low}}$ and agents expect s_{high}-infinity to resume next period, the law of motion during t is:

$$\begin{bmatrix} k_t \\ z_t \\ c_t \end{bmatrix} = \begin{bmatrix} 0.3300 & 0.1656 \\ 0 & 0.9500 \\ 0.6801 & 0.3416 \end{bmatrix} \begin{bmatrix} k_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 0.1106 \\ 0 \\ 0.2471 \end{bmatrix} + \begin{bmatrix} 0.0005 \\ 0.0100 \\ 0.0048 \end{bmatrix} \varepsilon_t \tag{105}$$

Exact (non-linear) solution

When $s_t = s_{\text{high}}$ and agents expect it to continue forever, it is a known result that consumption and capital are a constant share of output:

$$k_t \equiv k(k_{t-1}, z_{t-1}, \varepsilon_t, s_{\text{high}}) = \alpha\beta e^{\rho z_{t-1} + \sigma\varepsilon_t} k_{t-1}^{\alpha} \tag{106}$$

$$c_t \equiv c(k_{t-1}, z_{t-1}, \varepsilon_t, s_{\text{high}}) = (1 - \alpha\beta) e^{\rho z_{t-1} + \sigma\varepsilon_t} k_{t-1}^{\alpha} \tag{107}$$

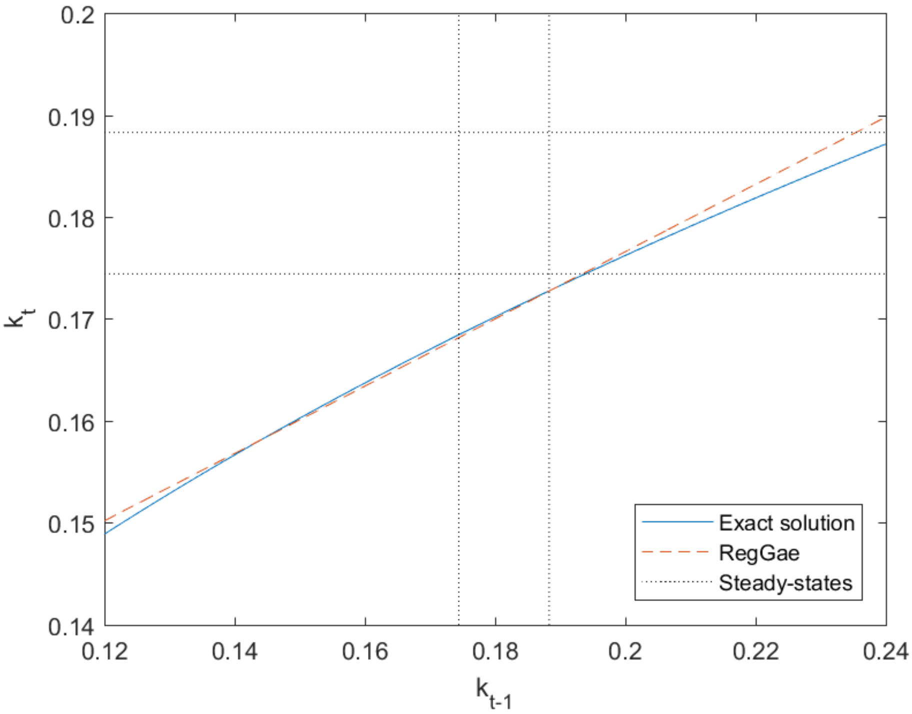
When $s_t = s_{\text{low}}$ and agents expect the high regime to resume next period and continue forever, plug the solution expected next period (107) and budget constraint in the low state (108) in current period's Euler equation (94) and solve for k_t:

$$c_t + k_t = e^{z_t} 0.95 k_{t-1}^{\alpha} \tag{108}$$

$$\frac{1}{e^{z_t} 0.95 k_{t-1}^{\alpha} - k_t} = \beta E_t \frac{\alpha e^{z_t} 0.95 k_t^{\alpha-1}}{(1 - \alpha\beta) e^{z_{t+1}} k_t^{\alpha}} \tag{109}$$

$$\Rightarrow \frac{1}{e^{z_t} 0.95 k_{t-1}^{\alpha} - k_t} = \beta \frac{\alpha e^{z_t} 0.95 k_t^{\alpha-1}}{(1 - \alpha\beta) e^{\rho z_t + \frac{\sigma^2}{2}} k_t^{\alpha}} \tag{110}$$

Figure 9: RegGae solution: capital



where we used the expectation of the log-normal distribution in the RHS denominator.

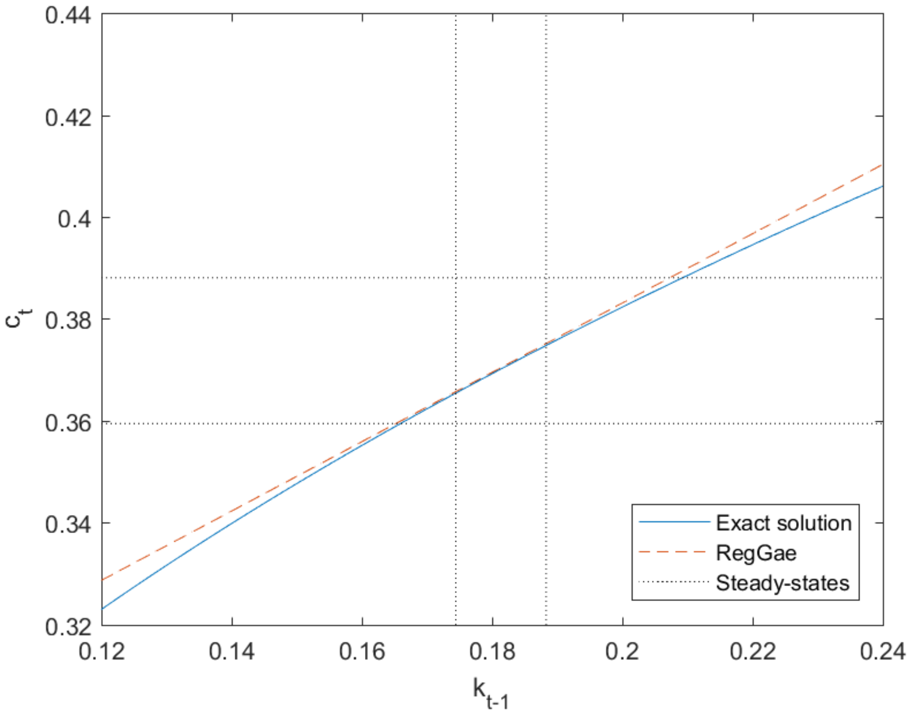
$$\Rightarrow (1-\alpha\beta)e^{\rho z_t + \frac{\sigma^2}{2}} k_t^{\alpha} = \beta\alpha e^{z_t} 0.95 k_t^{\alpha-1}(e^{z_t} 0.95 k_{t-1}^{\alpha} - k_t) \tag{111}$$

$$\Rightarrow [(1-\alpha\beta)e^{\rho z_t + \frac{\sigma^2}{2}} + \beta\alpha e^{z_t} 0.95] k_t^{\alpha} = \beta\alpha e^{2z_t} 0.95^2 k_t^{\alpha-1} k_{t-1}^{\alpha} \tag{112}$$

$$\Rightarrow k_t = \frac{\beta\alpha e^{2z_t} 0.95^2 k_{t-1}^{\alpha}}{(1-\alpha\beta)e^{\rho z_t + \frac{\sigma^2}{2}} + \beta\alpha e^{z_t} 0.95} \tag{113}$$

which defines $k_t \equiv k(k_{t-1}, z_{t-1}, \varepsilon_t, s_{\text{low}})$ and, by the resource constraint, $c_t \equiv c(k_{t-1}, z_{t-1}, \varepsilon_t, s_{\text{low}})$. Figures 9 and 10 depict the graphical representation of the linear and exact solutions for the model when $s_t = s_{\text{low}}$ where we can see that *RegGae's* regime-wise linear solution is sufficiently close to the exact (non-linear) solution even for values distant from the (regime-specific) steady-states.

Figure 10: RegGae solution: consumption



References

ADJEMIAN, Stéphane, et al. Dynare: Reference Manual, Version 4. *Dynare Working Papers*, 1, CEPREMAP, 2011. Downloaded from
<http://www.dynare.org>.

AIKMAN, David; GIESE, Julia; KAPADIA, Sujit; MCLEAY, Michael. *Targeting financial stability: macroprudential or monetary policy?* London: Bank of England, Staff Working Paper No. 734, June 2018.

Aikman, David, Michael Kiley, Seung Jung Lee, Michael G. Palumbo, Missaka Warusawitharana, 2017. Mapping heat in the U.S. financial system, *Journal of Banking & Finance*, Elsevier, vol. 81(C), pages 36-64.

AJELLO, Andrea; LAUBACH, Thomas; LOPEZ-SALIDO, David; NAKATA, Taisuke. Financial Stability and Optimal Interest-Rate Policy. Federal Reserve Board, February 2015. Available at <www.frbsf.org/economic-research/files/1A-Ajello-Laubach-Lopez-Salido-Nakata.pdf> Downloaded on July 11, 2018.

ANDERSON, Ron; DANIELSSON, Jon; BABA, Chikako; DAS, Udaibir S.; KANG, Heedon; SEGOVIANO, Miguel. *Macroprudential Stress Tests and Policies: Searching for Robust and Implementable Frameworks*, Washington D.C., International Monetary Fund, Working Paper WP/18/197, 2018.

ANGELINI, Paolo; NERI, Stefano; PANETA, Fabio. The Interaction between Capital Requirements and Monetary Policy. *Journal of Money, Credit and Banking*, Vol. 46, No. 6, September 2014.

BARTHÉLEMY, Jean; MARX, Magali. Solving Rational Expectations Models. In CHEN, Shu-Heng; KABOUDAN, Mak; DU, Ye-Rong *The Oxford Handbook on Computational Economics and Finance*, Oxford: Oxford University Press, February 2018. Available at < https://sites.google.com/site/jeanbarthelemyeconomist/research-papers>. Downloaded on May 11, 2018.

BARTHÉLEMY, Jean; MARX, Magali. Solving endogenous regime switching models. *Journal of Economic Dynamics and Control*, Volume 77, Pages 1-25, April 2017.

BASU, Suman S.; CHAMON, Marcos; CROWE, Christopher. *A Model to Assess the Probabilities of Growth, Fiscal, and Financial Crises.* Washington D.C., International Monetary Fund, Working Paper WP/17/282, 2017.

BCBS (BASEL COMMITTEE FOR BANKING SUPERVISION). *Guidance for national authorities operating the countercyclical capital buffer.* Basel, Basel Bank for International Settlements, 2010. Available at: <https://www.bis.org/publ/bcbs187.htm>. Downloaded on: April 28, 2018.

BENES, Jaromir; KUMHOF, Michael; LAXTON, Douglas. *Financial Crises in DSGE Models: Selected Applications of MAPMOD*, Washington D.C., International Monetary Fund, Working Paper WP/14/56, 2014a.

BENES, Jaromir; KUMHOF, Michael; LAXTON, Douglas. *Financial Crises in DSGE Models: A Prototype Model.* Washington D.C., International Monetary Fund, Working Paper WP/14/57, 2014b.

BLANCHARD, Olivier. On the future of macroeconomic models. *Oxford Review of Economic Policy*, Volume 34, Issue 1-2, Pages 43-54, 5 January 2018.

BLANCHARD, Olivier; KAHN, Charles. The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, Vol. 48, No. 5, pp. 1305-1311, July 1980.

BORIO, Claudio; ZHU, H. Capital regulation, risk-taking and monetary policy: A missing link in the transmission mechanism? *Journal of Financial Stability*, 8, 236-251, 2012.

DAVIG, Troy; LEEPER, Eric M. Generalizing the Taylor Principle, *American Economic Review*, 97 (3): 607-635, June 2007.

DEJONG, David; DAVE, Chetan. *Structural Macroeconometrics.* 2nd Edition, Princeton, NJ: Princeton University Press, 2011.

DEL NEGRO, Marco; EGGERTSSON, Gauti; FERRERO, Andrea; KIYOTAKI, Nobuhiro. The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities. *American Economic Review*, 107(3): 824-57, 2017.

FARMER, Roger E.A.; WAGGONER, Daniel F.; ZHA, Tao. Understanding Markov-switching rational expectations models. *Journal of Economic Theory*, 144 1849-1867, 2009.

FARMER, Roger E.A.; WAGGONER, Daniel F.; ZHA, Tao. Minimal state variable solutions to Markov-switching rational expectations models. *Journal of Economic Dynamics and Control* . 35, pp. 2150-2166, 2011.

FERNANDEZ-VILLAVERDE, J.; RUBIO-RAMIREZ, J.F.; SCHORFHEIDE, F. Solution and Estimation Methods for DSGE Models. In: TAYLOR, John B.; UHLIG, Harald. *Handbook of Macroeconomics.* Amsterdam: North-Holland, Volume 2A, 2016.

GENNAIOLI, Nicola; SHLEIFER, Andrei; VISHNY, Robert. Neglected Risks: The Psychology of Financial Crises. *American Economic Review: Papers & Proceedings*, 105(5): 310–314, 2015.

GONZALEZ, Rodrigo Barbone; MARINHO, Leonardo Sousa Gomes; LIMA, Joaquim Ignacio Alves de Vasconcellos. Re-anchoring countercyclical capital buffers: Bayesian estimates and alternatives focusing on credit growth. *International Journal of Forecasting*, Volume 33, Issue 4, Pages 1007-1024, 2017.

Guerrieri, Luca, Matteo Iacoviello, OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily, *Journal of Monetary Economics*, 70 (2015) 22–38.

HAMILTON, James. A New Approach to the Economic Analysis of Nonstationary Time

Series and the Business Cycle, *Econometrica*, Vol. 57, no. 2 pp. 357-384, March 1989.

HE, Zhiguo; KRISHNAMURTHY, Arvind. A Macroeconomic Framework for Quantifying Systemic Risk. Cambridge (MA): *NBER Working Paper Series*, Working Paper 19885, February 2014.

IMF (International Monetary Fund), *Financial Conditions and Growth at Risk*, Global Financial Stability Report, Chapter 3, October 3, 2017.

KOOP, Gary; PESARAN, M. Hashem; POTTER, Simon M. Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics* 74, 119-147, 1996.

Laséen, Stefan; PESCATORI, Andrea, TURUNEN, Jarkko. Systemic risk: A new trade-off for monetary policy? *Journal of Financial Stability*, Volume 32, Pages 70-85, October 2017.

LAUREYS, Lien; MEEKS, Roland. *Monetary and macroprudential policies under rules and discretion.* London: Bank of England, CCBS (Centre for Central Banking Studies), lecture handout, 13 February 2018.

MAIH, Junior. *Efficient Perturbation Methods for Solving Regime-Switching DSGE Models.* Oslo: Norges Bank, working paper 1, January 16, 2015. Available at: <https://www.norges-bank.no/en/Published/Papers/Working-Papers/2015/12015/>. Downloaded on: April 27, 2018.

MARTIN, Alberto; VENTURA, Jaume. The Macroeconomics of Rational Bubbles: A User's Guide. Cambridge (MA): *NBER Working Paper Series*, NBER Working Paper 24234, NBER, January 2018.

OXFORD REVIEW OF ECONOMIC POLICY (various authors). *Rebuilding macroeconomic theory.* Oxford: Oxford University Press, Volume 31, Issue 1-2, Spring and Summer 2018.

SCHULARICK, Moritz; TAYLOR, Alan M. Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870–2008. *American Economic Review*, 102(2): 1029–1061, 2012.

STIGLITZ, Joseph. Where modern macroeconomics went wrong. *Oxford Review of Economic Policy*, Volume 34, Issue 1-2, Pages 70-106, 5 January 2018.

SVENSSON, Lars E. O. *Cost-Benefit Analysis of Leaning Against the Wind: Are Costs Larger Also with Less Effective Macroprudential Policy?* Washington D.C., International Monetary Fund, Working Paper WP/16/3, 2016.

SVENSSON, Lars E. O. Cost-Benefit Analysis of Leaning Against the Wind. *Journal of Monetary Economics* 90, 193-213, October 2017.

WOODFORD, Michael. Inflation Targeting and Financial Stability, *Sveriges Riksbank Economic Review* 1, 2012.

WOODFORD, Michael. Monetary Policy Analysis when Planning Horizons are Finite. Cambridge (MA): *NBER Working Paper Series*, Working Paper 24692, June 2018. Available at: <http://www.nber.org/papers/w24692>. Downloaded on: June 18, 2018.