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TIME CONSISTENCE AND STABLE DIGITAL CURRENCY IN GENERAL EQUILIBRIUM

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SUMÁRIO

RESUMO

Este artigo desenvolve uma estrutura de equilíbrio geral que estende o modelo de tributação de Ramsey-Friedman ao incorporar explicitamente agentes heterogêneos, mercados competitivos e um planejador central responsável pela política fiscal e pela gestão da dívida. Dentro dessa estrutura unificada, o modelo captura a interação entre os resultados de mercado descentralizados e os instrumentos de política centralizada, enquanto a adoção de preferências quasi-lineares garante que as alocações do planejador permaneçam consistentes ao longo do tempo. Com base nesses fundamentos, introduzimos uma unidade monetária virtual que surge endogenamente em equilíbrio como um serviço de compensação de transações. Essa contribuição aborda a questão em aberto de como caracterizar a moeda dentro da teoria do equilíbrio geral, avançando a discussão ao propor um sistema digital que preserva o poder de compra entre regiões e setores produtivos de maneira estável e não inflacionária, sem gerar distorções na alocação de recursos.

Palavras-Chave: Equilíbrio geral, Moeda digital estável, Modelo de tributação de Ramsey-Friedman, Consistência intertemporal

ABSTRACT

This paper develops a general equilibrium framework that extends the Ramsey-Friedman taxation model by explicitly incorporating heterogeneous agents, competitive markets, and a central planner responsible for fiscal policy and debt management. Within this unified structure, the model captures the interaction between decentralized market outcomes and centralized policy instruments, while the adoption of quasi-linear preferences guarantees that the planner's allocations remain time-consistent. Building on these foundations, we introduce a virtual monetary unit that emerges endogenously in equilibrium as a transaction-clearing service. This contribution addresses the open question of how to characterize money within general equilibrium theory, advancing the discussion by proposing a digital system that preserves purchasing power across regions and productive sectors in a stable, non-inflationary manner, without generating distortions in resource allocation.

Keywords General equilibrium, Stable Digital Currency, Ramsey-Friedman Taxation Model, Time Consistency.

JEL Code: D41, D50, D51, D53, D62, E42

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1 Introduction

Kydland and Prescott (1977) demonstrate that discretionary policy, even under a fixed social welfare function and full knowledge of policy effects, fails to maximize social welfare because economic agents rationally anticipate future policy actions. Their analysis shows that optimal control theory is inapplicable when expectations are rational, as agents' current decisions in each period depend on beliefs about future policies. However, their results hinge critically on the assumption that the consumer's problem features strictly increasing marginal utility and admits an interior solution, so that first-order conditions govern behavior and expectations of future policies are incorporated directly into present choices. If, by contrast, consumption allocations lie at the boundaries of the feasible set or if preferences generate corner solutions, the intertemporal channel that gives rise to policy inconsistency may be substantially attenuated or altogether absent.

This paper considers a framework with quasi-linear utility, where agents' optimal consumption and saving plans depend solely on past trajectories of the central planner's strategies. This specification eliminates the forward-looking channel that generates the inconsistency emphasized by Prescott (Kydland and Prescott, 1977). Consequently, the planner's policy sequence is time-consistent: the policy that is optimal ex-ante remains optimal when re-evaluated in subsequent periods. The quasi-linear formulation therefore offers a tractable and theoretically coherent environment in which recursive equilibrium coincides with Ramsey-optimal planning, demonstrating that the well-known time-inconsistency problem can be circumvented when preferences exclude the intertemporal spillovers that underpin Prescott's paradox.

The *numéraire* good represented by the linear component of the utility function may also be interpreted as a service that supports the accounting of transactions.[1] In this interpretation, its units correspond to transactions themselves rather than to particular goods, which is consistent with the assumption of constant marginal utility. Moreover, by Walras's law, the equilibrium quantity of this service equals the number of net transactions. Hence, it can be regarded as the provision of a transaction-clearing service that is remunerated per net transaction.

The framework developed here is closely related to the contribution of Magill and Quinzii (1992) and Dubey and Geanakoplos (2003), as both works seek to specify and characterize the existence of money within a general equilibrium framework. While their analysis emphasizes the role of fiat money and nominal assets in resolving indeterminacy and in generating real effects of monetary policy under incomplete markets, our approach focuses on the introduction of a virtual monetary unit embedded directly into general equilibrium without relying on market imperfections. In contrast to their model, the virtual money considered here is not supplied or controlled by central authorities but arises endogenously from

[1] We assume that the benefit from using the service is determined by the opportunity cost of employing alternative digital means of transaction, which are implicitly available within the model's underlying structure.

the settlement of transactions. Moreover, it is designed to be intrinsically stable, in the sense that it does not generate inflationary pressures, thereby providing a unit of account and a stable medium for clearing exchanges within the economy.

The growing role of artificial intelligence and the proliferation of big data algorithms influencing economic behavior raise a fundamental question: how can models be constructed with microfoundations sufficiently rich to approximate real-world dynamics while remaining analytically tractable? In this article, we advance a hybrid framework that combines competitive interaction with strategic behavior. This formulation incorporates insights from both general equilibrium theory and central planning, thereby connecting allocative mechanisms to institutional design and to the broader foundations of economic coordination.

The objective of this paper is to develop a general equilibrium framework that provides complete microfoundations for macroeconomic analysis by explicitly incorporating a central planning authority responsible for fiscal policy, a central bank that regulates the supply of public bonds and manages public debt operations, and heterogeneous markets. Within this unified structure, the model captures the interaction between decentralized market outcomes and centralized policy instruments. We assume that the central planner can estimate consumer demand based on historical prices and demand data, thereby possessing the ability to anticipate consumers' best responses to each fiscal policy. However, the multiplicity of equilibria prevents the planner from predicting which equilibrium price will be selected by the markets. Therefore, we assume that the game implements a pure simultaneous Nash equilibrium in prices and a Subgame Perfect Nash Equilibrium[2] in the remaining strategies.

Furthermore, the paper demonstrates that, under suitable conditions on agents' preferences, specifically quasi-linear utility, the planner's policy sequence is time-consistent. This property, in turn, enables the theoretical implementation of a Stable Digital Currency (SDC), which can be viewed as a medium of exchange whose circulation is endogenously determined by market-clearing conditions and which preserves purchasing power across sectors and regions. This technology thus implements a real and universal unit of account and can be further subdivided into multiple components, differentiated along both regional and sectoral dimensions.

2 Basic Notation

We follow the notation in Mas-Colell et al. (1995). The basic conventions are as follows. Define

$$T = \{0, 1, 2, \dots\} \quad \text{and} \quad N = \{0, 1, 2, \dots, \mathrm{n}\}.$$

Throughout the paper, exogenous uncertainty is represented by a finite set Θ and a tree[3] (Magill and Quinzii, 1994)

$$\Omega = \{(\omega_1, \dots, \omega_s, \dots, \omega_t) : \omega_s \in \Theta \text{ for all } s \leq t \in T\}.$$

[2] More precisely, a variant of Stackelberg equilibrium in which the central planner acts as the leader.

[3] Although one could rely on a filtration to define plans along the tree, we adopt partial constant functions for the sake of notational simplicity.

Let $(\Omega, \rho, \mathscr{S})$ be a measure space, where $\mathscr{S}$ is a σ-algebra (tribe) representing exogenous information and $\rho \in \mathrm{Prob}(\Omega)$ is the objective joint probability distribution over all periods, with $\rho(\omega) > 0$ for all $\omega \in \Omega$. The model may allow for differential information across agents, represented by a family of sub-tribes of $\mathscr{S}$ that define their individual information sets (Hervés-Beloso et al., 2009).

In this framework, aggregate variables such as prices are measurable with respect to $\mathscr{S}$. For reasons of notational simplicity, however, we shall omit this feature in the subsequent analysis. Let Z denote the set of all bounded $\mathscr{S}$-measurable real-valued functions[4] defined on Ω. The algebraic structure of Z is analogous to that of the real numbers under pointwise operations.[5] For instance, we write $z \leq z'$ whenever $z(\omega) \leq z'(\omega)$ for all $\omega \in \Omega$. Given a mapping $f : Z \to Z$ and $y = f(z)$, we denote by $f(z)(\omega)$ the value $y(\omega)$ for each $\omega \in \Omega$.

Given $t \in T$, we say that a function $z_t \in Z$ is a t-section[6] if it is bounded and satisfies $z_t(\omega) = z_t(\omega')$ for all $\omega \in \Omega$ and $\omega' \in \Omega$ such that $\omega_s = \omega'_s$ for every $s \leq t$. Observe that if $z_t \in Z^{\mathrm{n}}$ is a t-section, then the set $Z_t := z_t(\Omega)$ is finite, because Θ is finite. Hence,[7] the expected value of z_t can be written as

$$\begin{aligned}\sum_{\varsigma \in Z} \varsigma \, \rho(z_t^{-1}(\varsigma)) &= \sum_{\varsigma \in Z} \sum \left\{ \rho(\omega) z_t(\omega) : \omega \in z_t^{-1}(\varsigma) \right\} \\ &= \sum \left\{ \rho(\omega) z_t(\omega) : \omega \in \cup_{\varsigma \in Z} z_t^{-1}(\varsigma) \right\} \\ &= \sum_{\omega \in \Omega} \rho(\omega) z_t(\omega),\end{aligned}$$

since $\Omega = \bigcup_{\varsigma \in Z} z_t^{-1}(\varsigma)$. Thus, for each t-section $z_t \in Z^{\mathrm{n}}$ we may write

$$E[z_t] = \sum_{\omega \in \Omega} \rho(\omega) z_t(\omega).$$

Note that we rely on the fact that if $z_t \in Z$ is a t-section for some $t \in T$, then the sum $\sum_{\omega \in \Omega} z_t(\omega)$ is finite.

We work on the following vector spaces,

$$\begin{aligned} Z^{\mathrm{n}} &= \{(z_n)_{n \in N} : z_n \in Z \text{ for all } n \in N\} \\ Z^{\mathrm{t}} &= \{z_t \in Z : z_t \text{ is a } t\text{-section for all } t \in T\} \\ Z^{\mathrm{t} \times \mathrm{n}} &= \{z_{tn} \in Z : z_{tn} \text{ is a } t\text{-section for all } t \in T\} \\ Z^{\mathrm{n} \times \mathrm{t}} &= \{z_{nt} \in Z : z_{nt} \text{ is a } t\text{-section for all } t \in T\} \\ Z^{\mathrm{t} \times \mathrm{n} \times \mathrm{n}} &= \{z_{tnn'} \in Z : z_{tnn'} \text{ is a } t\text{-section for all } t \in T\}. \end{aligned}$$

Given random matrices $z \in Z^{\mathrm{t} \times \mathrm{n}}$ and $y \in Z^{\mathrm{n} \times \mathrm{n}}$, we denote their standard matrix product by $zy \in Z^{\mathrm{t} \times \mathrm{n}}$. We refer to any such space as a linear space L. An element $z(\omega) \in \mathbb{R}^{\mathrm{t} \times \mathrm{n}}$ is a matrix with t

[4] That is, discrete random variables.

[5] Function composition and matrix multiplication are defined pointwise in the standard way.

[6] Although one could introduce a filtration to define the plans along the tree, we restrict attention to partially constant functions for simplicity, since Θ is finite.

[7] We assume that Z contains only distinct values.

rows and n columns. For a random element $z \in Z^{\mathrm{t\times n\times n}}$, we write $z_t \in Z^{\mathrm{n\times n}}$ with entries $(z_t)_{nk} = z_{tnk}$ for each $t \in T$.

We write the letter with the symbol "$\widehat{\ }$" to define a strategy and "$\widetilde{\ }$" a best response function evaluated on the actions. For example $\hat{z} = \tilde{z}(p)$ stands for the best response strategy $\hat{z}$ evaluated at the action p and $z = \hat{z}(y)$ the strategy $\hat{z}$ evaluated on the action y. For notation purposes, let us denote the symbol without upper index as the Cartesian product, for example, write $Z = \prod_{i\in I} Z^i$ and analogously for elements and functions. Moreover, denote the symbol "+" in the upper index to stand for aggregate variables over markets excluding the central planner. For example denote $z^I = (z_i)_{i\in I}$ and $z^{I+} = \sum_{i\in I} z^i$, the aggregate variable of a vector of I agents.

We use the symbol "$\widehat{\ }$" to denote a strategy and "$\widetilde{\ }$" to represent a best-response function evaluated at given actions. For example, $\hat{z} = \tilde{z}(p)$ denotes the best-response strategy $\hat{z}$ evaluated at the action p, while $z = \hat{z}(y)$ represents the strategy $\hat{z}$ evaluated at the action y.

3 Model

We generalize the framework of Magill and Quinzii (1994) to a Ramsey-Friedman type economy (Chamley, 1985) with a finite set of consumers indexed by I and a central planner indexed by ι. Define $\mathcal{I} = I \cup \{\iota\}$. Consider j firms indexed by the set J in a multi-sector production plants. The economy evolves over an infinite horizon with periods indexed by $T = \{1, 2, \ldots\}$. In each period, there are n goods collected in the set N. The first good, indexed by η, is not a standard consumption commodity but rather a service that enables agents to settle transactions at predictable costs, thereby functioning as a transaction medium. We assume the existence of a subset of regions serving as a common index across agents, firms, and goods, but we omit explicit regional indexing throughout the analysis.

Suppose that m assets are available in each period, indexed by the set M, with portfolio holdings[8] defined in $A^\ell \subset Z^{\mathrm{m\times t}}$ for each $\ell \in \mathcal{I} \cup J$. The first two assets, supplied by the central planner, correspond to a Treasury bond and fiat money, indexed by $\{b, \mu\}$. The remaining assets consist of long-lived instruments, which may or may not be contingent on firm profits, together with short-lived assets delivering exogenous payoffs. Accordingly, we assume that $J \cup \{b, \mu\} \subset M$. For each $j \in J$, we consider the subset $M^j \subset M$ representing the firm j's capital structure, i.e., the collection of contracts designing differential payoffs linked to profits. Asset positions follow the standard sign convention: a positive quantity indicates a long (purchase) position, while a negative quantity represents a short sale.

Assume that the first good η is the *numéraire* and it is represented by a digital service provided by the central planner, which records the volume of net transactions denominated in units of a Stable Digital Currency (SDC). We assume that all agents exhibit constant and positive marginal utility with respect to the *numéraire*. The marginal benefit of each unit of the digital service corresponds to the benefit of executing a trade of any good at any point in time. Hence, this marginal benefit must be constant, as it does not depend on the specific good being traded. Finally, we assume that a real fundamental unity is based on a certain fixed unitary bundle[9] $\bar{x}$ indexed in $\bar{N} \subset N$ and $\bar{M} \subset M$ for which $\bar{x}_{t\ell} \in \mathbb{R}_{++}$ if

[8] Portfolio choices are assumed to be constant on Ω at period $t = 0$.

[9] Which defines a price index and can be discriminated over the regions and production sectors.

$\ell \in \bar{N} \cup \bar{M}$ and $\bar{x}_{t\ell} = 0$ otherwise for all $t \in T$.

Assume that the first good η serves as the *numéraire* and is represented by a digital service supplied by the central planner, which records the volume of net transactions denominated in units of a Stable Digital Currency (SDC). All agents are assumed to exhibit constant and strictly positive marginal utility with respect to the *numéraire*. The marginal benefit of each unit of the digital service corresponds to the benefit of executing a transaction of any good at any point in time. Consequently, this marginal benefit must be constant, as it does not depend on the particular good being traded.

Finally, we assume the existence of a real fundamental unit defined by a fixed reference bundle[10] $\bar{x}$, indexed in $\bar{N} \subset N$ and $\bar{M} \subset M$, such that $\bar{x}_{t\ell} \in \mathbb{R}_{++}$ if $\ell \in \bar{N} \cup \bar{M}$ and $\bar{x}_{t\ell} = 0$ otherwise, for all $t \in T$.

Good and asset prices are represented by the row vectors $p_t = [p_{tn}]_{n \in N} \in Z_+^{\mathrm{n}}$ and $q_t = [q_{tm}]_{m \in M} \in Z_+^{\mathrm{m}}$, respectively, for each period $t \in T$. Define $p = (p_t)_{t \in T} \in Z_+^{\mathrm{t} \times \mathrm{n}}$ as the stream of good prices, and analogously $q = (q_t)_{t \in T} \in Z_+^{\mathrm{t} \times \mathrm{m}}$ as the stream of asset prices. The set of admissible good prices is denoted by $P \subset Z_+^{\mathrm{t} \times \mathrm{n}}$ and the set of admissible asset prices by $Q \subset Z_+^{\mathrm{t} \times \mathrm{m}}$.

We assume that durable capital goods of each firm $j \in J$ exhibit an input retention rate[11] $0 \leq |\delta_{tn}^j| < 1$ for all $t \in T$, which is equal to zero for non-durable goods. The value $1 - |\delta_{tn}^j| > 0$ represents the depreciation rate. For each $j \in J$, define $\delta^j \in Z^{\mathrm{t} \times \mathrm{n} \times \mathrm{n}}$, with δ_t^j denoting the diagonal matrix whose entries are δ_{tn}^j for $n \in N$.

Let us assume that the central planner finances its resources through tax collection or the issuance of public debt. Regarding tax notation, the tax variables[12] at each period $t \in T$ are represented as the diagonal matrix $\tau_t^\iota \in \mathbb{R}^{\mathrm{n} \times \mathrm{n}}$ with a typical element $\tau_{tk\ell}^\iota = 0$ if $k \neq \ell$ and $\tau_{tkk}^\iota \in \mathbb{R}_+$ representing the *ad valorem* tax of good k at time t. Denote by $\tau^\iota = (\tau_t^\iota)_{t \in T} \in \Gamma^\iota \subset \mathbb{R}^{\mathrm{t} \times \mathrm{n} \times \mathrm{n}}$ the array of taxes and τ_{tnn}^ι by τ_{tnn}^ι for each $n \in N$ to simplify. Therefore, the amount $p_t \tau_t^\iota c_t^i \in Z_+$ represents the total tax Agent i pays for consuming c_t^i. Assume that taxes are levied on endowments and net purchase but not on net selling.

Let us assume that the central planner finances its resources through tax collection or the issuance of public debt. With respect to tax notation, the tax variables[13] in each period $t \in T$ are represented by the diagonal matrix $\tau_t^\iota \in \mathbb{R}^{\mathrm{n} \times \mathrm{n}}$, where $\tau_{tk\ell}^\iota = 0$ if $k \neq \ell$ and $\tau_{tkk}^\iota \in \mathbb{R}_+$ denotes the *ad valorem* tax on good k at time t.

We denote $\tau^\iota = (\tau_t^\iota)_{t \in T} \in \Gamma^\iota \subset \mathbb{R}^{\mathrm{t} \times \mathrm{n} \times \mathrm{n}}$ as the array of taxes and, for simplicity, write τ_{tnn}^ι for each $n \in N$. Accordingly, the amount $p_t \tau_t^\iota c_t^i \in Z_+$ represents the total tax paid by agent i for consuming c_t^i. Finally, we assume that taxes are levied on endowments and net purchases, but not on net sales.

Let $Y = \Gamma^\iota \times R^\iota \times X^J$ denote the set of endogenous variables that directly induce global interdependence in individual choices. A typical element is $y = (\tau^\iota, r^\iota, x^J)$, representing the tax system, the federal funds rate, fiat money payoffs, and the profit strategies of firms.

In each period $t \in \mathbb{N}$, bond returns are denoted by $r_{bt}^\iota \in \mathbb{R}^{\mathrm{n}}$ and fiat money payoffs by $r_{\mu t}^\iota \in \mathbb{R}$. We collect these as $r^\iota = (r_{bt}^\iota, r_{\mu t}^\iota)$. If m represents an equity instrument, then the payoff to firm $j \in M$ is given by its net profits.

[10] This bundle defines a price index and can be disaggregated across regions and production sectors.

[11] Since durable capital inputs are recorded in negative amounts, we adopt negative rates by convention.

[12] Note that taxes are non-random parameters.

[13] Taxes are treated as non-random parameters.

Finally, we assume that fiat money yields non-positive payoffs,[14] while the *numéraire* is normalized to have unit price. In this framework, the "inflation rate" is represented by the relative fiat money payoff, namely $|r_{t\mu}|/p_{t\mu}$ for each $t \in T$.

In this model, the central planner possesses sufficient information about the economy to infer consumers' demand, but lacks the information required to determine equilibrium prices due to the possibility of multiplicity. Consequently, it behaves as a price taker, analogously to consumers, while setting the supply of government bonds and fiat money,[15] as well as defining taxes and social expenditures[16]. We adopt the concept of Subgame Perfect Nash Equilibrium, as presented in Mas-Colell et al. (1995), to characterize agents' best responses.

3.1 Firms' problem

We assume that physical capital can be endowed by firms (Hillebrand, 2012) and functions as an asset, thereby transferring resources across consecutive periods. At the same time, the equivalent quantity of physical capital may also serve as an input in production. We assume that input-output operations, payoff distribution, and savings decisions take place within the same period, although they are not determined simultaneously.[17]

Each firm j's allocation is represented by a matrix $x^j = (c^j, a^j) \in X^j$, where X^j denotes the set of feasible firm choices,

$$X^j \subset Z^{\mathrm{n}\times\mathrm{t}} \times Z^{\mathrm{m}\times\mathrm{t}},$$

with $c_t^j \subset Z^{\mathrm{n}}$ denoting the input-output random vector in each period $t \in T$. Positive components represent outputs, while negative components represent inputs. The production column vector is written as $c_t^j = (c_{nt}^j)_{n\in N}$ for each $j \in I$.

Each firm $j \in J$ also selects $a^j \in A^j \subset Z^{\mathrm{m}\times\mathrm{t}}$, as the vector of asset holdings, where A^j is assumed to be bounded and embodies the unit net supply of capital structure, that is

$$A^j = \left\{ a^j \in Z^{\mathrm{m}\times\mathrm{t}} : \sum_{m\in M^j} a_{mt}^j = 1 \text{ for all } t \in T \right\}. \tag{1}$$

We define $X^j = C^j \times A^j$ for each $j \in J$ with a typical element $x^j = (c^j, a^j)$.

Firm j's strategies in each period $t \in T$ are represented by a function $\hat{c}^j : Y \to C^j$, where $\hat{c}_t^j(y)(\omega) \in \mathbb{R}^{\mathrm{n}}$ denotes the input-output vector at period t given the fiscal policy and market production actions $y = (\tau^\iota, r^\iota, x^\jmath) \in Y$ and the event $\omega \in \Omega$. We assume that $\hat{c}_t^j$ is a t-section on Y for each $t \in T$ and $j \in J$. Equivalently,[18] $\hat{c}_t^j(y) = \hat{c}_t^j(\dot{y})$ for all $y, \dot{y} \in Y$ such that $y_s = \dot{y}_s$ for every $s \leq t$. Define $\widehat{C}^j$ as the set of all such strategies $\hat{c}^j$.

Firms' saving strategies are defined analogously by a function $\hat{a}^j : Y \to A^j$ of bonds and assets,

[14] Fiat money serves merely as a residual instrument, used only when the SDC does not provide sufficient liquidity.

[15] The central planner controls bond supply by adjusting the corresponding interest rates.

[16] Which generates positive externalities.

[17] In addition, we assume that firms' profits are distributed *pro rata* according to the allocation of equity and third-party capital.

[18] Pointwise in Ω.

satisfying $\hat{a}^j_t(y) = \hat{a}^j_t(\dot{y})$ whenever $y_s = \dot{y}_s$ for all $s \leq t$. Let $\widehat{A}^j$ denote the set of all such functions. We then write $\widehat{X}^j = \widehat{C}^j \times \widehat{A}^j$ for all $j \in J$.

The production set of firm $j \in J$ is subject to the following constraints. For every $t \in T$, denote by $\hat{e}^j_{t-1} : C^j \to Z^{\mathrm{n}}$ the endowment stream of goods, where $\hat{e}^j_{t-1}(c^j)$ is the stock of durables carried over from period $t-1$ after depreciation. In our specification, the period-t capital employed equals the depreciated stock from period $t-1$ used as an input; hence we set $\hat{e}^j_{t-1}(c^j) = \delta^j_t c^j_{t-1}$ for all $j \in J$. For each $t \in T$, define $\tilde{r}^j_t : P \times Q \times Y \times C^j \to Z$ by[19]

$$\tilde{r}^j_{tm}(p, q, \dot{y}, c^j) = \begin{cases} q_{tm} + p_t(c^j_t + \delta^j_t c^j_{t-1}) & \text{if } m = j \\ q_{tm} + p_t(\dot{c}^j_t + \delta^j_t \dot{c}^j_{t-1}) & \text{if } m \neq j \end{cases}$$

The production plants are represented as a stochastic production function[20] $f^j : X^j \to Z^{\mathrm{t} \times \mathrm{n}}$.

Define firm j's period-t net profit function $\pi^j_t : P \times Q \times Y \times X^j \to Z$ for each $(p, q, \dot{y}, x^j) \in P \times Q \times Y \times X^j$ by

$$\pi^j_t(p, q, \dot{y}, x^j) = \tilde{r}^j_t(p, q, \dot{y}, c^j) a^j_{t-1} - q_t a^j_t,$$

where $c^j_{t-1} \in Z^{\mathrm{m}}$ embodies the stock of capital available from the previous period and $c^j_t \in Z^{\mathrm{m}}$ incorporates the capital employed in the current period (both for contemporaneous production and for carryover to the next period) for each $t \in T$ and $j \in J$. We assume that firms distribute all profits as dividends before determining reinvestment through equity purchases. The expected, time-discounted objective incorporating risk preferences is, for each $(p, q, \dot{y}, x^j) \in P \times Q \times Y \times X^j$,

$$u^j(p, q, \dot{y}, x^j) := \sum_{t \in T} \sum_{\omega \in \Omega} \beta^t \rho(\omega) u^j_t\big(\pi^j_t(p, q, \dot{y}, x^j)(\omega)\big),$$

where $u^j_t : \mathbb{R} \to \mathbb{R}$ represents the period-t Bernoulli utility capturing risk attitudes. If u^j_t is linear, then firm j is risk neutral.

Since managers of firm j hold specific contractual claims in M^j, any modification of the capital structure may alter their individual payoffs. We therefore assume that managers commit to maximizing the firm's profits prior to pursuing their private benefits. In this setting, they treat the firm's returns as exogenously given.

The supply best response $\tilde{x}^j : P \times Q \to \widehat{X}^j$ is then defined as

$$\tilde{x}^j(p, q)(\dot{y}) = \operatorname{argmax} \left\{ u^j(p, q, \dot{y}, x^j) : x^j \in X^j \text{ and } f^j(x^j) \in Z^{\mathrm{t} \times \mathrm{n}}_+ \right\}.$$

Write it shortly as $\tilde{x}^j(p, q) = (\tilde{c}^j(p, q), \tilde{a}^j(p, q))$ for each $(p, q) \in P \times Q$.

Remark 3.1. Suppose firm j produces a perishable good $n \in N$ employing only durable capital and labor. Let $k^j \in Z^{\mathrm{t}}_-$, $l^j \in Z^{\mathrm{t}}_-$, and $c^j_n \in Z^{\mathrm{t}}_+$ denote, respectively, the capital, labor, and output-n components of

[19] We assume the firm's profits are distributed *pro-rata* according to the allocation of equity and third-party capital.

[20] Recall that any pure asset acquired by firms is in non negative amount. The acquired capital inputs as durable goods are also given in negative amounts.

c^j. Assume the production technology is given by $\hat{f}_t^j : Z \times Z \to Z_+$, such that

$$c_{nt}^j = f_t^j(k_t^j, l_t^j), \quad t \in T,$$

while all other input allocations are negligible for firm j since they yield no profitable outcome. Capital is the only durable input, with retention rate $\delta_k^j < 0$ satisfying $|\delta_k^j| < 1$, so that

$$\delta_t^j c_{t-1}^j = \delta_k^j k_{t-1}^j, \qquad c^j \in C^j.$$

Suppose firm j is characterized by a simple capital structure given by $M_j = \{j\}$. Then its equity position satisfies $a_{mt}^j = 1$ for $m = j$. Assume further that the firm does not invest in other financial assets, so that $a_{mt}^j = 0$ for all $m \neq j$ and $t \in T$. Then the period-t *pro-rata* allocation of profits is given by

$$\begin{aligned}
\pi_t^j(p, q, \dot{y}, x^j) &= \tilde{r}_t^j(p, q, \dot{y}, c^j)a_{t-1}^j - q_t a_t^j \\
&= q_{tm} + p_t(c_t^j + \delta_t^j c_{t-1}^j) - q_{tm} \\
&= p_{tn}c_{nt}^j - p_{tl}|l_t^j| - p_{tk}|k_t^j| + p_{tk}\delta_k^j k_{t-1}^j \\
&= p_{tn}f_t^j(k_t^j, l_t^j) - p_{tl}|l_t^j| + p_{tk}(\delta_k^j k_{t-1}^j - |k_t^j|)
\end{aligned}$$

These profits are analogous to those in a standard production framework, except that they explicitly incorporate the previous period's capital, which can either be reemployed or liquidated in the current period.

3.2 Consumers' problem

In this sequential game with complete information, we interpret consumers' plans as strategies contingent on exogenous uncertainty, as well as on the choices of firms and the central planner. The availability of complete information implies that rational consumers anticipate the possibility that the central planner's policies may lack time consistency (Kydland and Prescott, 1977). Consequently, plans formulated in the initial period may fail to be implemented at some future date t, since doing so would require forecasting subsequent policy actions.

Suppose, however, that consumers[21] execute in the initial period their plans for date t while considering only the fiscal policy actually observed at that date. Under this assumption, we demonstrate that constant marginal utility preferences render such behavior consistent with optimal decision-making. Moreover, once the central planner incorporates this behavioral rule into its strategy design, the resulting equilibrium ensures that all agents behave consistently. The following characterization of consumption and savings strategies further elucidates this pattern of consumer behavior.

For each $i \in I$, consumer i's consumption set is defined by $C^i \subset Z_+^{\mathrm{n}\times\mathrm{t}}$. Denote the consumption column vector at date t by $c_t^i = (c_{nt}^i)_{n\in N}$. Similarly, consumer i's asset choice set is given by $A^i \subset Z^{\mathrm{m}\times\mathrm{t}}$, where A^i is bounded. The asset column vector at date t is written as $a_t^i = (a_{mt}^i)_{m\in M}$. Hence, the overall

[21] Analogous to the way they treat their plans contingent on exogenous uncertainty.

set of consumer choices is $X^i = C^i \times A^i$ for all $i \in I$, with a typical element denoted by $x^i = (c^i, a^i)$.

Consumer i's strategies at each period $t \in T$ are represented by a function $\hat{c}^i : Y \to C^i$, where $\hat{c}^i_{nt}(y)(\omega) \in \mathbb{R}_+$ denotes the consumption of good $n \in N$ at period $t \in T$, conditional on the fiscal policy and production actions $(\tau^\iota, r^\iota, x^\jmath) \in Y$ and the event $\omega \in \Omega$. We assume that $\hat{c}^i_t$ is a t-section[22] on Y for all $t \in T$ and $i \in I$. Let $\widehat{C}^i$ denote the set of all such consumption strategies $\hat{c}^i$.

Analogously, consumers' saving strategies are defined by a function $\hat{a}^i : Y \to A^i$, with $\hat{a}^i_t$ being a t-section for every $t \in T$. Let $\widehat{A}^i$ be the set of all such saving strategies. The overall set of consumer strategies is then given by $\widehat{X}^i = \widehat{C}^i \times \widehat{A}^i$.

Denote by $\hat{e}^i_t : C^i \to Z^{\mathrm{n}}$ the endowment stream of goods for each $i \in I$. For every $t \in T$, the quantity $\hat{e}^i_{t-1}(c^i)$ represents the endowment at date t of non-durable goods together with depreciated durable goods inherited from period $t-1$. Accordingly, we define

$$\hat{e}^i_{t-1}(c^i) = e^i_t + \delta^i_t c^i_{t-1}, \quad \forall i \in I,$$

where $0 \leq \delta^i_t < 1$ is the retention rate and $e^i_t \in Z^{\mathrm{n}}$ is an exogenous endowment.

Since taxes are levied on endowments and on net purchases (but not on net sales) suppose agent i receives $\bar{e}^i_t := \hat{e}^i_{t-1}(c^i)$ at period t and consumes c^i_t. Then the net purchase is

$$\bar{c}^i_t = c^i_t - \bar{e}^i_t.$$

If $\bar{c}^i_{nt} \geq 0$ for some good n, the agent pays $p_{tn}\tau^\iota_{tnn}\bar{c}^i_{nt}$ on net purchases plus $p_{tn}\tau^\iota_{tnn}\bar{e}^i_{nt}$ on endowments. Hence, the total tax liability is

$$p_{tn}\tau^\iota_{tnn}(\bar{c}^i_{nt} + \bar{e}^i_{nt}) = p_{tn}\tau^\iota_{tnn}c^i_{nt}.$$

If instead $\bar{c}^i_{nt} < 0$, the agent sells $|\bar{c}^i_{nt}|$ and retains c^i_{nt} as endowment.[23]

Therefore, in both cases, the total tax vector for all goods is given by

$$p_t\tau^\iota_t c^i_t \in Z^{\mathrm{n}}_+.$$

Suppose preferences are represented by a utility function $u^i_{nt} : \dot{C}^i_{nt} \subset \mathbb{R}_+ \to \mathbb{R}_+$ that is strictly increasing, concave, twice differentiable, and admits a differentiable inverse. For each $t \in T$, define the (Von Neumann-Morgenstern) period utility index $u^i_t : C^i_t \to \mathbb{R}$ by[24]

$$u^i_t(c^i_t) = \sum_{\omega \in \Omega} \rho(\omega) \sum_{n \in N} u^i_{nt}(c^i_{nt}(\omega)) \quad \text{for all } c^i_t \in C^i_t.$$

The lifetime utility $u^i : C^i \to \mathbb{R}$ is given by

$$u^i(c^i) := \sum_{t \in T} \beta^t u^i_t(c^i_t) \quad \text{for all } c^i \in C^i.$$

[22] Equivalently, $\hat{c}^i_t(y) = \hat{c}^i_t(y')$ for each $y, y' \in Y$ such that $y_s = y'_s$ for all $s \leq t$.
[23] There is no taxation on net sales.
[24] Formally, u^i_t is a V.N.M. expected utility index.

We define asset pay-offs[25] $\hat{r}^i : P \times Q \times Y \to Z^{\mathrm{t}\times\mathrm{m}}$ for each $t \in T$ as

$$\hat{r}^i_{tm}(p,q,y) = \begin{cases} p_t r^\iota_{tm} \in Z & \text{if } m \in \{b,\mu\} \\ p_t r_{tm} \in Z & \text{if } m \text{ is short-lived} \\ q_{tm} + p_t(c^j_t + \delta^j_t c^j_{t-1}) a^j_m \in Z & \text{if } m \in M^j \end{cases} \tag{2}$$

where $r_{tm} \in Z^{\mathrm{n}}$ is exogenously specified and $\sum_{m \in M^j} a^j_m = 1$. For each $i \in I$, define the wealth function $\hat{w}^i : P \times Q \times Y \times X^i \to Z$ as

$$\hat{w}^i_t(p,q,y,x^i) = \hat{r}^i_t(p,q,y) a^i_{t-1} + p_t \hat{e}^i_{t-1}(c^i) \quad \text{for all } t \in T.$$

The budget correspondence of agent $i \in I$ is then $\hat{b}^i : P \times Q \times Y \to X^i$, defined in the standard way[26] by

$$\hat{b}^i(p,q,y) = \Big\{ x^i \in X^i : (p_t + p_t \tau^\iota_t) c^i_t + q_t a^i_t \leq \hat{w}^i_t(p,q,y,x^i)\ \forall t \in T \Big\}.$$

The best response strategy $\tilde{x}^i : P \times Q \to \widehat{X}^i$ is defined for each $(p,q) \in P \times Q$ as

$$\tilde{x}^i(p,q)(y) = \operatorname{argmax}\{u^i(c^i) : x^i \in \hat{b}^i(p,q,y,x^i)\} \text{ for all } y \in Y.$$

Write shortly $\tilde{x}^i(p,q) = (\tilde{c}^i(p,q), \tilde{a}^i(p,q))$ for each $(p,q) \in P \times Q$. The indirect real valued utility function $\tilde{v}^i : P \times Q \times Y \to \mathbb{R}$ is defined by

$$\tilde{v}^i(p,q,y) = \max\{u^i(c^i) : x^i \in \hat{b}^i(p,q,y,x^i)\} \text{ for all } (p,q,y) \in P \times Q \times Y. \tag{3}$$

3.3 Central planner problem

In this model, markets are assumed to track the actions of the central planner. Hence, in a subgame-perfect Nash equilibrium[27], the planner anticipates agents' demand best responses to forecast tax revenues, while still treating prices as parametric. Let $C^\iota \subset Z^{\mathrm{n}\times\mathrm{t}}_+$ and $A^\iota \subset Z^{\mathrm{m}\times\mathrm{t}}$. The planner's choice set is

$$X^\iota \subset C^\iota \times A^\iota \times \Gamma^\iota \times R^\iota,$$

where a generic policy vector $x^\iota = (c^\iota, a^\iota, \tau^\iota, r^\iota)$ comprises: endogenous public expenditures $c^\iota \in C^\iota$, an asset supply $a^\iota \in A^\iota$, a non-stochastic tax schedule $\tau^\iota \in \mathbb{R}^{\mathrm{t}\times\mathrm{n}\times\mathrm{n}}_+$, and a payoff vector $r^\iota_t = (r^\iota_{tb}, r^\iota_{t\mu}) \in Z^{\mathrm{n}}_+ \times Z$ specifying, respectively, the returns on public bonds and fiat money.

The central planner's strategies at each period $t \in T$ are contingent only on firms' actions and are represented by a function $\hat{x}^\iota : Y \to X^\iota$, where $\hat{x}^\iota_t(y)(\omega) \in \mathbb{R}^{\mathrm{n}}$ denotes the vector of public expenditures and investments at date $t \in T$, conditional on market production choices[28] $x^J \in X^J$ and on the event

[25] Recall that we are assuming that managers $i \in I$ treat the firm's returns as exogenously given.

[26] Defined pointwise in Ω.

[27] Which in this context is analogous to a Stackelberg equilibrium.

[28] We adopt Y as the domain instead of X^J solely for notational economy. Clearly, $\hat{x}^\iota(y)$ does not depend on the fiscal variables (τ^ι, r^ι).

$\omega \in \Omega$. We assume that $\hat{x}_t^\iota$ is a t-section on Y for each $t \in T$. Let $\widehat{X}^\iota$ denote the set of all such strategies $\hat{x}^\iota$.

We define the central planner asset pay-offs $\hat{r}^\iota : P \times Q \times Y \to Z^{\mathrm{t}\times\mathrm{m}}$ for each $t \in T$ as

$$\hat{r}_{tm}^\iota(p, q, x^J, r^\iota) = \begin{cases} p_t r_{tm}^\iota \in Z & \text{if } m \in \{b, \mu\} \\ p_t r_{tm} \in Z & \text{if } m \text{ is short-lived} \\ q_{tm} + p_t(c_t^j + \delta_t^j c_{t-1}^j) a_m^j \in Z & \text{if } m \in M^j \end{cases} \tag{4}$$

where $r_{tm} \in Z^{\mathrm{n}}$ is exogenously specified and $\sum_{m \in M^j} a_m^j = 1$.

Write the central planner wealth $w_t^\iota : P \times Q \times Y \times X^J \to Z$ as[29]

$$w_t^\iota(p, q, x^J, x^\iota) = \hat{r}_t^\iota(p, q, x^J, r^\iota) a_{t-1}^\iota + p_t \tau_t^\iota \tilde{c}_t^{I+}(p, q)(\tau^\iota, r^\iota, x^J) + p_t \hat{e}_{t-1}^\iota(c^\iota)$$

where $\tilde{c}^I = (\tilde{c}^i)_{i \in I}$ is the market demand best response to fiscal policy[30] and $\hat{e}^\iota$ represents good endowments. The central planner's budget constraint set is given by

$$\hat{b}^\iota(p, q, x^J) = \{x^\iota \in X^\iota : p_t c_t^\iota + q_t a_t^\iota \leq w_t^\iota(p, q, x^J, x^\iota) \text{ for all } t \in T\}$$

where $c_t^\iota \in Z_+$ is the central planner expenditure at date t. The central planner's expenditures generate positive externalities[31] that contribute to social welfare, represented by a function $u^\iota : C^\iota \to \mathbb{R}_+$. Accordingly, $u^\iota(c^\iota)$ measures the social benefit[32] derived from public expenditures c^ι. It should be noted that the optimal level of public expenditure is endogenously determined and therefore tends to decline as the magnitude of positive externalities decreases.

The central planner problem is then given by

$$v^\iota(p, q, x^J) = \max \left\{ \sum_{i \in I} \tilde{v}^i(p, q, \tau^\iota, r^\iota, x^J) + u^\iota(c^\iota) : x^\iota \in \hat{b}^\iota(p, q, x^J) \right\}. \tag{5}$$

The fiscal policy best response function of the central planner $\tilde{x}^\iota : P \times Q \to \widehat{X}^\iota$ given a vector of price (p, q) and the consumers strategy function $\tilde{c}^I$ is given by

$$\tilde{x}^\iota(p, q)(\dot{y}) = \operatorname{argmax} \left\{ \sum_{i \in I} \tilde{v}^i(p, q, \tau^\iota, r^\iota, \dot{x}^J) + u^\iota(c^\iota) : x^\iota \in \hat{b}^\iota(p, q, \dot{x}^J) \right\}. \tag{6}$$

Given $\hat{x}^\iota \in \tilde{x}^\iota(p, q)$, write $\hat{x}^\iota = (\hat{c}^\iota, \hat{a}^\iota, \hat{\tau}^\iota, \hat{r}^\iota)$ as the central planner optimal strategy.

3.4 Equilibrium

We assume that the aggregate shareholdings of all individuals exhibit unitary net supply, since we normalize each firm's capital structure by (1). Therefore,write that the initial aggregate asset endowments

[29] We omit the dependence of $\tilde{c}^I$ on w_t^ι to simplify.
[30] Note that $a^\iota < 0$ when central planer is a seller.
[31] Examples include *primary education*, social security, and public health programs.
[32] This benefit can be interpreted as a positive externality.

$\bar{a}^*_0 \in \mathbb{R}^{\mathrm{m}}_+$ by

$$\bar{a}^*_{m0} = \begin{cases} 0 & \text{if } m \text{ is a short-lived asset,} \\ 1 & \text{if } m \text{ is a long-lived asset.} \end{cases}$$

We define the equilibrium for $\mathscr{E}$ by the following objects

1. a market price $(\bar{p}, \bar{q}) \in P \times Q$
2. a fiscal policy and firms actions $\bar{y} = (\bar{\tau}^\iota, \bar{r}^\iota, \bar{x}^J) \in \Gamma^\iota \times R^\iota \times X^J$;
3. a central planner strategy $\hat{x}^\iota = (\hat{c}^\iota, \hat{a}^\iota, \hat{\tau}^\iota, \hat{r}^\iota) \in \widehat{X}^\iota$;
4. a demand strategy $\hat{x}^i = (\hat{c}^i, \hat{a}^i) \in \widehat{X}^i$ for each $i \in I$;
5. a production strategy $\hat{x}^j = (\hat{c}^j, \hat{a}^j) \in \widehat{X}^j$ for each $j \in J$

such that pointwise in Ω

1. $\hat{x}^\ell \in \tilde{x}^\ell(\bar{p}, \bar{q})$ for each $\ell \in \mathcal{I} \cup J$;
2. $\bar{y} = \big(\hat{\tau}^\iota(\bar{y}), \hat{r}^\iota(\bar{y}), \hat{x}^J(\bar{y})\big)$;
3. $\sum_{\ell \in \mathcal{I}} \hat{a}^\ell_t(\bar{y}) = \bar{a}^*_0$ for all $t \in T$;
4. $\sum_{\ell \in \mathcal{I}} \hat{c}^\ell_t(\bar{y}) = \sum_{\ell \in \mathcal{I} \cup J} \hat{e}^\ell_{t-1}(\hat{c}^\ell(\bar{y})) + \sum_{j \in J} \hat{c}^j_t(\bar{y})$.

Write shortly the vector of equilibrium as $(\bar{p}, \bar{q}, \bar{y}, \hat{x})$.

Remark 3.2. Consider $(\bar{p}, \bar{q}, \bar{y}, \hat{x})$ an equilibrium of $\mathscr{E}$ and write $\bar{x} = \hat{x}(\bar{y})$. Then the market clearing conditions are consistent with the Walras' law. Indeed, the budget restrictions are given by

$$\bar{p}_t \bar{c}^i_t + \bar{q}_t \bar{a}^i_t = \hat{r}^i_t(\bar{p}, \bar{q}, \bar{y}) \bar{a}^i_{t-1} + \bar{p}_t \hat{e}^i_{t-1}(\bar{c}^i) - \bar{p}_t \bar{\tau}^i_t \bar{c}^i_t \text{ for all } i \in I \tag{7}$$

$$\bar{p}_t \bar{c}^\iota_t + \bar{q}_t \bar{a}^\iota_t = \hat{r}^\iota_t(\bar{p}, \bar{q}, \bar{x}^J, \bar{r}^\iota) \bar{a}^\iota_{t-1} + \bar{p}_t \bar{\tau}^\iota_t \bar{c}^{I+}_t + \bar{p}_t \hat{e}^\iota_{t-1}(\bar{c}^\iota) \tag{8}$$

Since $\bar{a}^*_m = 0$ if m is a short-lived asset and $\sum_{m \in M^j} \bar{a}^j_{mt} = 1$ then by (2) and (4)

$$\begin{aligned} \sum_{\ell \in \mathcal{I}} \hat{r}^\ell_t(\bar{p}, \bar{q}, \bar{y}) \bar{a}^\ell_{t-1} &= \hat{r}^\iota_t(\bar{p}, \bar{q}, \bar{x}^J, \bar{r}^\iota) \bar{a}^\iota_{t-1} + \sum_{i \in I} \sum_{j \in J} \sum_{m \in M^j} \hat{r}^i_{tm}(\bar{p}, \bar{q}, \bar{y}) \bar{a}^i_{m,t-1} \\ &= q_t \bar{a}^*_0 + \bar{p}_t \sum_{j \in J} \sum_{m \in M^j} \sum_{\ell \in \mathcal{I}} (\bar{c}^j_t + \delta^j_t \bar{c}^j_{t-1}) \bar{a}^j_{mt} \bar{a}^\ell_{m,t-1} \\ &= q_t \bar{a}^*_0 + \bar{p}_t \sum_{j \in J} (\bar{c}^j_t + \delta^j_t \bar{c}^j_{t-1}) \end{aligned}$$

Adding up (7) and (8) we get

$$\sum_{\ell \in \mathcal{I}} (\bar{p}_t \bar{c}^\ell_t + \bar{q}_t \bar{a}^\ell_t) = q_t \bar{a}^*_0 + \bar{p}_t \sum_{j \in J} (\bar{c}^j_t + \delta^j_t \bar{c}^j_{t-1}) + \sum_{\ell \in \mathcal{I}} \bar{p}_t \hat{e}^\ell_{t-1}(\bar{c}^\ell).$$

That is,

$$\bar{p}_t \left(\sum_{\ell \in \mathcal{I}} \bar{c}^\ell_t - \sum_{j \in J} \bar{c}^j_t - \sum_{\ell \in \mathcal{I} \cup J} \hat{e}^\ell_{t-1}(\bar{c}^\ell) \right) + \bar{q}_t \left(\sum_{\ell \in \mathcal{I}} \bar{a}^\ell_t - \bar{a}^*_0 \right) = 0$$

4 Main results

In this section, we establish two main results. The first demonstrates the time consistency of the equilibrium allocation. The second underscores the feasibility of implementing a Stable Digital Currency (SDC) as a digital unit of account anchored to a representative consumption bundle, thereby preserving the purchasing power of a given class of individuals in the economy.[33]

4.1 Time consistence

We now show that agents will not revise their planned choices at any future date. This condition is crucial to ensure that plans can indeed be executed once the corresponding period is realized. In short, we must demonstrate that agents display time consistency in their decisions (Kydland and Prescott, 1977, 1980; Feng, 2015). The next result formalizes this claim.

The following lemma ensures that, actually, any strategy $\hat{x}^i \in \tilde{x}^i(p,q) \in \widehat{X}$ is a t-section on Y for all $(p,q) \in P \times Q$.

Lemma 4.1. *Fix $i \in I$. Consider $\hat{x}^i = (\hat{c}^i, \hat{a}^i) \in \tilde{x}^i(p,q)$. Then for each $(p,q) \in P \times Q$ the optimal strategy $\hat{c}^i_t$ is a t-section on Y for all $t \in T$.*

Proof. See Section 6.1 in Appendix.

□

Theorem 4.1. *Suppose that $\mathscr{E}$ could be represented as a game with complete information. Then all agents have time consistency for each $(p,q) \in P \times Q$.*

Proof. See Section 6.2 in appendix. □

4.2 Stable Currency

Regarding the implementation of a Stable Digital Currency (SDC), the central question that arises when a technology replaces current digital currency in virtually all transactions is: why introduce an SDC if a central bank digital currency (CBDC) can perform this role? The answer is informed by historical precedents involving real units of account, such as the gold standard monetary system (Barro, 1979). The volatility in the price of a durable reference good employed as a unit of account generates inefficiencies in production planning and investment decisions under uncertainty about future prices. The Perfect Foresight assumption (Radner, 1972) highlights the fundamental link between price stability and economic efficiency.

Additionally, the adoption of a Stable Digital Currency (SDC) carries important implications for the conduct and effectiveness of monetary policy. In particular, when public debt instruments are denominated in SDC units, a marginal increase in the interest rate implicitly affects the valuation of a broad spectrum of commodity markets. This contrasts with the current monetary framework, in which nominal currency appreciation is primarily driven by bond market dynamics. Under the latter, interest rate hikes

[33] Hence implying zero inflation.

may succeed in mitigating inflation but frequently at the cost of inducing economic downturns. The introduction of an SDC could enhance the efficiency of monetary policy by allowing central banks to focus exclusively on open market operations through the management of public debt issuance and absorption. Since the SDC establishes a unit of account indexed to real bundles, its circulation arises solely from the net settlement of private transactions, *implying that its supply is entirely market-driven and lies outside the discretionary control of the central authority.* Moreover, an SDC would not generate liquidity constraints, as it would coexist with the nominal sovereign currency. In addition, the central authority could employ bond issuance as a liquidity instrument.

A further advantage of an SDC concerns the distributional effects of inflation on purchasing power. Because inflation tends to be more pronounced in markets with low price elasticity (Ramsey, 1928), it effectively acts as an endogenous tax levied by traders, one that disproportionately burdens lower-income households. The welfare costs of inflation (de Holanda Barbosa and da Cunha, 2003) far outweigh any short-term gains from expansionary monetary policy, whose real effects are likely negligible, if not null, in the long run (Lucas Jr, 1972; Erceg and Levin, 2003). If inflation is defined as the observed change in the price of the reference bundle, then the implementation of an SDC would effectively imply zero inflation.

Importantly, the SDC would not be mandated by the central authority for use in private contractual arrangements, nor would it entail full indexation of the economy. Instead, its adoption could emerge organically across the economic system, facilitated by modern trading technologies that virtually eliminate transaction costs and allow for fine-grained spatial and temporal discrimination of purchasing power indices (Handbury et al., 2013). This would enable the construction of a robust, dynamic index supported by a big data infrastructure, with real-time updates reflecting changes in purchasing power. The accuracy of this index would improve proportionally with the volume of digital transaction data available. *Crucially, such a system can be implemented with currently available technologies while preserving agents' anonymity.*

Finally, the introduction of an SDC would also allow for the deregulation of the cryptocurrency market by providing a macroeconomic safeguard against speculative attacks and systemic shocks stemming from defaults in privately issued cryptocurrencies (Fernández-Villaverde and Sanches, 2019).

More precisely, consider an equilibrium $(\bar{p}, \bar{q}, \bar{y}, \hat{x})$ in which the price of good transaction service is one.[34] Then for each agent $i \in I$ the budget restriction $\hat{b}^i$ is given as[35]

$$\begin{aligned} p_{t0}(c^i_{0t} - \hat{e}^i_{0,t-1}(c^i)) = \hat{r}^i_t(p, q, y) a^i_{t-1} - q_t a^i_t \\ + \sum_{n \neq 0} p_{tn}(\hat{e}^i_{nt}(c^i) - c^i_{nt} - \tau^\iota_{tnn} c^i_{nt}) \end{aligned}$$

since $p_{t0} = 1$ for all $t \in T$ then $c^i_{0t} - \hat{e}^i_{0,t-1}(c^i)$ denotes the aggregate volume of net transactions executed by agent i in each period $t \in T$ given in units of *numéraire*. This amount actually represents the total supply of a currency given in units of the digital services as an account unity. For the central planner, the

[34] The homogeneity of degree one allows us to consider this assumption.

[35] Recall that assets do not have payment in units of *numéraire* or transaction services.

budget restriction $\hat{b}^\iota$ is given as

$$\begin{aligned} p_{t0}(c^\iota_{0t} - \hat{e}^\iota_{0,t-1}) &= \hat{r}^\iota_t(p, q, x^J, r^\iota)a^\iota_{t-1} - q_t a^\iota_t \\ &+ \sum_{n\neq 0} p_{tn}\Big(\hat{e}^\iota_{n,t-1}(c^\iota) - \tau^\iota_{tnn}\tilde{c}^{\iota\prime+}_{tn}(p,q)(\dot{y}) - c^\iota_{nt}\Big) \end{aligned}$$

since $p_{t0} = 1$ for all $t \in T$ then $c^\iota_{0t} - \hat{e}^\iota_{0,t-1}$ denotes the aggregate volume of net transactions executed by central planner for each period $t \in T$ given in units of *numéraire*.

To examine the details of the operationalization of a Stable Digital Currency, define at each period $t \in T$

$$1 \text{ SDC unity} = \Bigg(\sum_{\ell\in\bar{N}\cup\bar{M}} p_{t\ell}\bar{x}_{\ell t}\Bigg) \text{ NC units.}$$

Consider the deflator given as

$$\varsigma(p) = \frac{1}{\sum_{\ell\in\bar{N}\cup\bar{M}} p_{t\ell}\bar{x}_{\ell t}}\ \frac{\text{SDC units}}{\text{NC unity}}$$

The homogeneity of degree zero allow us to normalize prices by multiplying all budget equations by $\varsigma(p)$ in such a way that the set of prices is then given in SDC units by

$$P = \Bigg\{p' \in Z_+^{\text{t}\times\text{n}} : \sum_{\ell\in\bar{N}\cup\bar{M}} p'_{t\ell}\bar{x}_{\ell t} = 1 \text{ for all } t \in T\Bigg\}.$$

where $(p', q') = \varsigma(p)(p, q)$ and the price of trade services is then given by

$$p'_{t0} = \Bigg(\sum_{\ell\in\bar{N}\cup\bar{M}} p_{t\ell}\bar{x}_{\ell t}\Bigg)^{-1} = \varsigma(p). \tag{9}$$

Since the reference bundle consists of a large number of goods and assets, Equation (9) implies that transaction services can be provided at very low unit costs.

Because the return $\hat{r}^\ell$ for $\ell \in \mathcal{I} \cup J$ is homogeneous of degree one in prices, the demand functions $\{\tilde{x}^\ell : \ell \in \mathcal{I}\}$ are homogeneous of degree zero in prices. Moreover, as firms' problems are homogeneous of degree one in profits, the supply functions $\{\tilde{x}^\ell : \ell \in J\}$ are likewise homogeneous of degree zero in prices. We thus obtain the following theorem.

Theorem 4.2. *Suppose that* $(\bar{p}, \bar{q}, \bar{y}, \hat{x})$ *is an equilibrium for* $\mathscr{E}$*. Then*

$$(\varsigma(p)\bar{p}, \varsigma(p)\bar{q}, \bar{y}, \hat{x})$$

is also an equilibrium for $\mathscr{E}$*.*

The supply of an SDC must be determined by the number of transactions executed in each period, independently of its connection to the reference bundle $\bar{x}$. The reason is that optimal allocations of certain goods within $\bar{x}$ corresponding solely to the consumption of current endowments do not generate demand for money as a medium of exchange, since no transaction occurs. In the limiting case of autarky, or equivalently a pure exchange economy, money becomes entirely superfluous, as its demand collapses to zero.

Regarding the operationalization of an SDC, the key consideration is the design of a mechanism through which the transaction service is seamlessly integrated into market exchanges, ensuring efficiency and scalability while preserving minimal transaction costs. More precisely, the total supply of the SDC is *endogenously* given by

$$|c^{\iota}_{0t} - \hat{e}^{\iota}_{0,t-1}| + \sum_{i \in I} |c^{i}_{0t} - \hat{e}^{i}_{0,t-1}(c^{i})|.$$

5 Conclusion

This paper has proposed a unified general equilibrium framework that extends the Ramsey–Friedman taxation model by embedding heterogeneous agents, competitive markets, and a central planner responsible for fiscal policy and debt management. The quasi-linear specification of preferences plays a crucial role in guaranteeing time consistency of the planner's strategy, thereby overcoming the classic inconsistency problem identified by Kydland and Prescott (1977). By eliminating the forward-looking channel that usually generates policy inconsistency, the model provides a tractable and coherent environment in which recursive equilibrium coincides with Ramsey-optimal planning.

A central contribution of the paper lies in the introduction of a virtual monetary unit that arises endogenously as a transaction-clearing service. In contrast to traditional fiat money, as analyzed by Magill and Quinzii (1992), this virtual currency does not depend on market imperfections, is not supplied by a central authority, and remains intrinsically stable in the sense of being non-inflationary. Its equilibrium supply is fully determined by market activity, which ensures that purchasing power is preserved across regions and productive sectors without introducing distortions into the allocation of resources. This result contributes to the broader effort to specify the existence of money in general equilibrium and illustrates how a digital system can be designed to sustain a stable unit of account.

Overall, the analysis highlights that incorporating a stable digital monetary mechanism into general equilibrium provides both theoretical and policy insights. On the theoretical side, it resolves indeterminacy issues while maintaining tractability and consistency. On the policy side, it offers a blueprint for digital monetary systems whose circulation is market-driven, rather than centrally controlled, thereby aligning transaction-clearing with efficiency and stability. This framework thus advances the literature on money in general equilibrium by bridging foundational theory with the design of digital systems capable of supporting modern economic coordination.

6 Appendix

Auxiliary Remarks

Remark 6.1. Given $z \in L$ and $z' \in L$, write $\mathring{z} = z - z'$ as a differential direction. Denote generically by $\mathring{z}(t, n, \omega) \in Z^{\mathrm{t \times n}}$ as a direction which is zero in all but the tn-coordinate and $\omega \in \Omega$. Write $\mathring{z}(t, n, \omega) \in Z^{\mathrm{t \times n}}$ as the unitary direction when $\mathring{z}_{sk}(t, n, \omega)(w) = 0$ for $(s, k, w) \neq (t, n, \omega)$ and $\mathring{z}_{tn}(t, n, \omega)(\omega) = 1$.

First, consider $f : L \to L'$ a linear function. Then

$$Df(z)(\mathring{z}) = f(\mathring{z}) \tag{10}$$

Second, consider $f : L \times L' \to L''$ a bilinear function. Then[36]

$$Df(z, z')(\mathring{z}, \mathring{z}') = f(z, \mathring{z}') + f(\mathring{z}, z') \tag{11}$$

Remark 6.2. Note that given $z \in Z$, $\dot{z} \in Z^{\mathrm{t}\times\mathrm{n}}$ and $f : Z \subset Z^{\mathrm{t}\times\mathrm{n}} \to L$ we have[37]

$$Df(z)(\dot{z})(\omega) = \sum\{\dot{z}_{tn}(\omega)Df(z)(\mathring{z}(t, n, \omega)) : (t, n, \omega) \in T \times N \times \Omega\}$$

because

$$\dot{z} = \sum\{\dot{z}_{tn}(\omega)\mathring{z}(t, n, \omega) : (t, n, \omega) \in T \times N \times \Omega\}.$$

6.1 Proof of Lemma 4.1

Proof. Let $\lambda^i \in Z^{\mathrm{t}}_+$. Consider the Lagrangian

$$\begin{aligned}
\ell^i(x^i, y) = u^i(c^i) &+ \sum_{s\in T}\sum_{w\in\Omega} \lambda^i_s(w)\hat{w}^i_s(p, q, y, x^i)(w) \\
&- \sum_{s\in T}\sum_{w\in\Omega} \lambda^i_s(w)\big((p_s(w) + p_s(w)\tau^\iota_s)c^i_s(w) + q_s(w)a^i_s(w)\big) \\
&= u^i(c^i) + \sum_{s\in T}\sum_{w\in\Omega} \lambda^i_s(w)\hat{r}^i_s(p, q, y)(w)a^i_{s-1}(w) \\
&+ \sum_{s\in T}\sum_{w\in\Omega} \lambda^i_s(w)p_s(w)(e^i_s + \delta^i_s c^i_{s-1}(w)) \\
&- \sum_{s\in T}\sum_{w\in\Omega} \lambda^i_s(w)\big((p_s(w) + p_s(w)\tau^\iota_s)c^i_s(w) + q_s(w)a^i_s(w)\big)
\end{aligned} \tag{12}$$

Fix $(t, n, \omega) \in T \times N \times \Omega$ arbitrarily and let

$$\mathring{x}^i(t, n, \omega) = (\mathring{c}^i(t, n, \omega), \mathring{a}^i) \in Z^{\mathrm{n}\times\mathrm{t}} \times Z^{\mathrm{m}\times\mathrm{t}}$$

[36]See Bartle (1976) for more details.
[37]Pointwise in Ω.

be a unitary direction with $\mathring{a}^i = 0$. The F.O.C. of evaluated at the optimum $\hat{x}^i(y) = (\hat{c}^i(y), \hat{a}^i(y)) \in X^i$ implies by (3.2) that

$$\begin{aligned}
0 &= \partial_1 \ell^i(\hat{x}^i(y), y)(\mathring{x}^i(t,n,\omega)) \\
&= \partial_1 u^i(\hat{c}^i(y))(\mathring{c}^i(t,n,\omega)) + \sum_{s\in T}\sum_{w\in\Omega} \lambda_s^i(w) p_s(w)\delta_s^i \mathring{c}_{s-1}^i(t,n,\omega)(w) \\
&- \sum_{s\in T}\sum_{w\in\Omega} \lambda_s^i(w)(p_s(w) + p_s(w)\tau_s^\iota)\mathring{c}_s^i(t,n,\omega)(w) \\
&= \sum_{s\in T}\sum_{w\in\Omega}\sum_{k\in N} \beta^s \rho(w)\partial_1 u_{ks}^i(\hat{c}_{ks}^i(y)(w))\mathring{c}_{ks}^i(t,n,\omega)(w) \\
&+ \sum_{s\in T}\sum_{w\in\Omega}\sum_{k\in N} \lambda_s^i(w)\delta_{sk}^i p_{sk}(w)\mathring{c}_{k,s-1}^i(t,n,\omega)(w) \\
&- \sum_{s\in T}\sum_{w\in\Omega}\sum_{k\in N} \lambda_s^i(w)\big(p_{sk}(w) + p_{sk}(w)\tau_{sk}^\iota\big)\mathring{c}_{ks}^i(t,n,\omega)(w) \\
&= \beta^t\rho(\omega)\partial_1 u_{nt}^i(\hat{c}_{nt}^i(y)(\omega)) + \lambda_{t+1}^i(\omega)\delta_{t+1,n}^i p_{t+1,n}(w) \\
&- \lambda_t^i(\omega)\big(p_{tn}(\omega) + p_{tn}(\omega)\tau_{tnn}^\iota\big).
\end{aligned} \tag{13}$$

To conclude the proof, note that $u_{nt}^i(c_{nt}^i) = \alpha_t c_{nt}^i$ for $n = \eta$, then Equation (13) becomes[38]

$$\alpha_t\beta^t\rho(\omega) = \lambda_t^i(\omega)(p_{t\eta}(\omega) + p_{t\eta}(\omega)\tau_{t\eta\eta}^\iota) = \lambda_t^i(\omega) \text{ for all } t \in T.$$

Thus we get the result since (13) implies for each $(t,n) \in T \times N$

$$\begin{aligned}
\hat{c}_{nt}^i(y)(\omega) &= (\partial_1 u_{nt}^i)^{-1}\left(\frac{\lambda_t^i(\omega)(p_{tn}(\omega) + p_{tn}(\omega)\tau_{tnn}^\iota) - \lambda_{t+1}^i(\omega)\delta_{t+1,n}^j p_{t+1,n}(w)}{\beta^t\rho(\omega)}\right) \\
&= (\partial_1 u_{nt}^i)^{-1}\big(\alpha_t(p_{tn}(\omega) + p_{tn}(\omega)\tau_{tnn}^\iota) - \alpha_{t+1}\beta\delta_{t+1,n}^j p_{t+1,n}(w)\big)
\end{aligned}$$

therefore, $\tilde{c}^i(p,q)(y)$ is a t-section on $y = (\tau^\iota, r^\iota, x^\jmath)$ for all $t \in T$.

□

6.2 Proof of Theorem 4.1

Proof. Consider $(p,q) \in P \times Q$ fixed. Suppose that Agent i has no time consistency.[39] Then there exist:

1. a fiscal policy and production-investment $\bar{y} = (\bar{\tau}^\iota, \bar{r}^\iota, \bar{x}^\jmath)$;
2. an optimal strategy $\hat{x}^i = (\hat{c}^i, \hat{a}^i) \in \tilde{x}^i(p,q)$;
3. another strategy $\dot{x}^i = (\dot{c}^i, \dot{a}^i) \in \hat{b}^i(p,q,\bar{y})$ such that[40] for some t $\in T$:

[38] Recall that the *numéraire* is not taxed, $p_{t\eta}(\omega) = 1$ for all $t \in T$ and it has full depreciation.
[39] We do not consider the case of firms since they do not have budget restrictions.
[40] This allows to implement a feasible continuation of the allocation $\dot{x}^i$ which coincides with x^i for $t <$ t.

$$\dot{a}^i_{\mathrm{t}} = \hat{a}^i_{\mathrm{t}}(\bar{y})$$
$$p_t\dot{c}^i_t + q_t\dot{a}^i_t \leq \hat{r}^i_t(p,q,\bar{y})\dot{a}^i_{t-1} + p_t\hat{e}^i_{t-1}(\dot{c}^i) - p_t\tau^\iota_t\dot{c}^i_t \text{ for all } t > \mathrm{t}$$
$$\sum_{t>\mathrm{t}}\beta^t u^i_t(\dot{c}^i_t) > \sum_{t>\mathrm{t}}\beta^t u^i_t(\hat{c}^i_t(\bar{y})).$$

Define $\ddot{x}^i_t = \hat{x}^i_t(\bar{y})$ for all $t \leq \mathrm{t}$ and $\ddot{x}^i_t = \dot{x}^i_t$ for all $t > \mathrm{t}$. Since $\hat{x}^i \in \tilde{x}^i(p,q)$ then, for $t \leq \mathrm{t}$ we get

$$p_t\ddot{c}^i_t + q_t\ddot{a}^i_t \leq \hat{r}^i_t(p,q,\bar{y})\ddot{a}^i_{t-1} + p_t\hat{e}^i_{t-1}(\ddot{c}^i) - p_t\tau^\iota_t\ddot{c}^i_t.$$

Therefore $\ddot{x}^i \in \hat{b}^i(p,q,\bar{y})$ since $\dot{a}^i_{\mathrm{t}} = \hat{a}^i_{\mathrm{t}}(\bar{y})$. But this is a contradiction since $\hat{x}^i$ is optimal and $u^i(\ddot{c}^i) > u^i(\hat{c}^i(\bar{y}))$.

Suppose now that the central planner has no time consistency. Then there exist:

1. a period $\mathrm{t} \in T$
2. an optimal strategy[41] $x^\iota \in \tilde{x}^\iota(p,q)$;
3. an optimal strategy[42] $\hat{x}^i \in \tilde{x}^i(p,q)$ for all $i \in I$;
4. a production-investment allocation $\bar{x}^J \in X^J$;
5. an allocation $\bar{x}^\iota = (\bar{c}^\iota, \bar{a}^\iota, \bar{\tau}^\iota, \bar{r}^\iota) \in \hat{x}^\iota(p,q)(\bar{y})$
6. an allocation $\dot{x}^\iota = (\dot{c}^\iota, \dot{a}^\iota, \dot{\tau}^\iota, \dot{r}^\iota) \in X^\iota$;
7. vectors $\bar{y} = (\bar{\tau}^\iota, \bar{r}^\iota, \bar{x}^J)$ and $\dot{y} = (\dot{\tau}^\iota, \dot{r}^\iota, \bar{x}^J)$,

such that

$$\dot{a}^\iota_{\mathrm{t}} = \hat{a}^\iota_{\mathrm{t}}(\bar{y})$$
$$p_t\dot{c}^\iota_t + q_t\dot{a}^\iota_t \leq \hat{r}^\iota_t(p,q,\bar{x}^J,\dot{r}^\iota)\dot{a}^\iota_{t-1} + p_t\dot{\tau}^\iota_t\hat{c}^{I+}_t(\dot{y}) + p_t\hat{e}^\iota_{t-1}(\dot{c}^\iota) \text{ forall } t > \mathrm{t}$$
$$\sum_{t>\mathrm{t}}\beta^t\Big(u^\iota_t(\dot{c}^\iota_t) + \sum_{i\in I}u^i_t(\hat{c}^i_t(\dot{y}))\Big) > \sum_{t>\mathrm{t}}\beta^t\Big(u^\iota_t(\bar{c}^\iota_t) + \sum_{i\in I}u^i_t(\hat{c}^i_t(\bar{y}))\Big).$$

Define $\ddot{x}^\iota_t := (\ddot{c}^\iota_t, \ddot{a}^\iota_t, \ddot{r}^\iota_t, \ddot{\tau}^\iota_t) = \bar{x}^\iota_t$ for all $t \leq \mathrm{t}$ and $\ddot{x}^\iota_t = \dot{x}^\iota_t$ for all $t > \mathrm{t}$. Write $\ddot{y}_t = (\bar{\tau}^\iota_t, \bar{r}^\iota_t, \bar{c}^J_t)$ for all $t \leq \mathrm{t}$ and $\ddot{y}_t = (\dot{\tau}^\iota_t, \dot{r}^\iota_t, \bar{c}^J_t)$ for all $t > \mathrm{t}$. Lemma 4.1 implies that consumers' best response functions are t-sections, that is, they satisfy

$$\hat{c}^i_t(\ddot{y}) = \hat{c}^i_t(\bar{y}) \text{ for all } t \leq \mathrm{t} \text{ and all } i \in I$$

since $\ddot{y}_s = \bar{y}_s = (\bar{\tau}^\iota_s, \bar{r}^\iota_s, \bar{c}^J_s)$ for all $s \leq t \leq \mathrm{t}$. Therefore $\bar{x}^\iota \in \hat{b}^\iota(p,q,\bar{x}^J)$ implies

$$p_t\ddot{c}^\iota_t + q_t\ddot{a}^\iota_t \leq \hat{r}^\iota_t(p,q,\bar{x}^J,\ddot{r}^\iota)\ddot{a}^\iota_{t-1} + p_t\ddot{\tau}^\iota_t\hat{c}^{I+}_t(\ddot{y}) + p_t\hat{e}^\iota_{t-1}(\ddot{c}^\iota) \text{ for all } t \leq \mathrm{t}.$$

[41]Recall that $\hat{x}^\iota = (\hat{c}^\iota, \hat{a}^\iota, \tau^\iota, r^\iota)$.
[42]Recall that $\hat{x}^i = (\hat{c}^i, \hat{a}^i)$ for all $i \in I$.

since $\hat{r}_t^\iota$ is also a t-section. Thus, $\ddot{x}^\iota \in b^\iota(p, q, \bar{x}^\jmath)$ and

$$\sum_{i \in I} \tilde{v}^i(p, q, \ddot{y}) + u^\iota(\ddot{c}^\iota) > \sum_{i \in I} \tilde{v}^i(p, q, \bar{y}) + u^\iota(\bar{c}^\iota)$$

But this is a contradiction with (6). □

References

Barro, R. J. (1979). Money and the price level under the gold standard. *The Economic Journal 89*(353), 13–33.

Bartle, R. G. (1976). *The Elements of Real Analysis*. Wiley and Sons. 3933690309.26003.

Chamley, C. (1985). Efficient taxation in a stylized model of intertemporal general equilibrium. *International Economic Review*, 451–468.

de Holanda Barbosa, F. and A. B. da Cunha (2003). Inflation tax and money essentiality. *Economics Letters 78*(2), 187–195.

Dubey, P. and J. Geanakoplos (2003). Monetary equilibrium with missing markets. *Journal of Mathematical Economics 39*(5-6), 585–618.

Erceg, C. J. and A. T. Levin (2003). Imperfect credibility and inflation persistence. *Journal of monetary economics 50*(4), 915–944.

Feng, Z. (2015). Time-consistent optimal fiscal policy over the business cycle. *Quantitative Economics 6*(1), 189–221.

Fernández-Villaverde, J. and D. Sanches (2019). Can currency competition work? *Journal of Monetary Economics 106*, 1–15.

Handbury, J., T. Watanabe, and D. E. Weinstein (2013). How much do official price indexes tell us about inflation? Technical report, National Bureau of Economic Research.

Hervés-Beloso, C., V. F. Martins-da Rocha, and P. K. Monteiro (2009). Equilibrium theory with asymmetric information and infinitely many states. *Economic Theory 38*(2), 295–320.

Hillebrand, M. (2012). On the optimal size of social security in the presence of a stock market. *Journal of Mathematical Economics 48*(1), 26–38.

Kydland, F. E. and E. C. Prescott (1977). Rules rather than discretion: The inconsistency of optimal plans. *Journal of political economy 85*(3), 473–491.

Kydland, F. E. and E. C. Prescott (1980). Dynamic optimal taxation, rational expectations and optimal control. *Journal of Economic Dynamics and control 2*, 79–91.

Lucas Jr, R. E. (1972). Expectations and the neutrality of money. *Journal of economic theory 4*(2), 103–124.

Magill, M. and M. Quinzii (1992). Real effects of money in general equilibrium. *Journal of Mathematical Economics 21*(4), 301–342.

Magill, M. and M. Quinzii (1994). Infinite horizon incomplete markets. *Econometrica: Journal of the Econometric Society*, 853–880.

Mas-Colell, A., M. D. Whinston, J. R. Green, et al. (1995). *Microeconomic theory*, Volume 1. Oxford university press New York.

Radner, R. (1972). Existence of equilibrium of plans, prices, and price expectations in a sequence of markets. *Econometrica: Journal of the Econometric Society*, 289–303.

Ramsey, F. P. (1928). A mathematical theory of saving. *The Economic Journal 38*(152), 543–559.