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THE NEOCLASSICAL MODEL IN A MULTIPLE-COMMODITY WORLD
A Criticism on Marglin

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THE NEOCLASSICAL MODEL IN A MULTIPLE-COMMODITY WORLD

A Criticism on Marglin

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SUMÁRIO

1 INTRODUCTION

More than thirty years after Sraffa's challenge to the neoclassical theory of capital, his contention that the choice of techniques is not in general a monotonic function of the rate of interest continues to be disputed. One of the arguments that has been raised is that reswitching of techniques may be excluded provided that the production function is sufficiently smooth. This idea was first presented by Bruno, Burmeister and Sheshinski (1966) and it has been followed up by other authors, such as Starret (1969) and Stiglitz (1973), who stressed that enough substitutability is sufficient to guarantee the impossibility of reswitching.

More recently, Prof. Marglin (1984) has made extensive use of this argument in order to construct a multisectoral economic model that allows systematic comparisons between the three fundamental schools of economic thought - the neoclassical, the Post-Keynesian, and the Marxian. Apparently, one remarkable feature of this work is to show how a multiple commodity model, if adequately specified, can preserve both the traditional concept of free competition and the neoclassical inverse relationship between capital intensity and the rate of interest. By the traditional concept of free competition it is understood the notion that capital flows from less profitable sectors to more profitable ones, generating a tendency to the equalization of the profit rate throughout the economy. As a result, the position of full equilibrium is given by a situation in which it prevails an homogeneous rate of profits[1].

Marglin's results are outstanding since the possibility of having a continuous variation of techniques and yet the phenomenon of reswitching of techniques has been proved by Pasinetti (1969) while a non-monotonic relation between the rate of interest and capital per man can also be obtained in the absence of reswitching (Bruno, Burmeister and Sheshinski, 1966; Pasinetti, 1966). Moreover, in a recent work, Garegnani (1990) showed the incompatibility between the traditional concept of free competition and the neoclassical theory in terms quite general.

The objective of this paper is to show why the properties of the neoclassical model break down in the case of a competitive economy in full equilibrium even if the production functions are quite "neoclassical". As a consequence, it does not seem to be valid to assert, as Marglin does, that the three schools of economic though have the same level of theoretic self-consistency.

This paper is divided in six sections. The first three ones present Marglin's "general" model of a simple economy in steady growth, the neoclassical specification, and Marglin's theorem about the impossibility of reswitching of techniques. Section four presents a counterproof regarding the reswitching theorem and the next section discusses the compatibility between the neoclassical and the classical (Sraffian-Marxian) models. The conclusions are presented in the last section. The Appendix presents a numerical example illustrating the questions under discussion.

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1. This concept is held by classical authors, like Smith, Ricardo, and Marx, as well as by neoclassical economists, like Marshall, Wicksell, and Walras.

2 THE "GENERAL" MODEL

Marglin's simple economic system in a multiple-commodity context is composed by three basic equations (Marglin, 1987, pp. 231-233). The production equation regards the (column) vector of current outputs $\mathbf{x}_t$ as equal to the sum of current consumption $\mathbf{c}_t$ plus the requirements of inputs for the following year $\mathbf{A}\ \mathbf{x}_{t+1}$, where $\mathbf{A}$ is the (m by m) Leontieff input-output matrix:

$$\mathbf{x}_t = \mathbf{c}_t + \mathbf{A}\ \mathbf{x}_{t+1} \tag{1}$$

Since output is supposed to grow at a constant rate g, current output is related to next year's production as:

$$\mathbf{x}_{t+1} = (1 + g)\ \mathbf{x}_t \tag{2}$$

Price formation is given by the following "Sraffian" equation, which assumes that wages are post paid:

$$\mathbf{p} = w\ \mathbf{a}_0 + (1 + r)\mathbf{p}\ \mathbf{A} \tag{3}$$

where $\mathbf{p}$ is the (row) price vector, w is the nominal wage rate and r is the rate of profits.

Since in the model above prices are relative ones and there is no reference to the scale of production, Marglin adopts the following normalization procedures:

$$w = 1 \tag{4}$$

and

$$\mathbf{a}_0\ \mathbf{x}_t = 1 \tag{5}$$

Substituting equation (2) into (1) and equation (4) into (3), and dropping the subscripts yield:

$$\mathbf{x} = [\mathbf{I} - (1 + g)\ \mathbf{A}]^{-1}\ \mathbf{c} \tag{6}$$

and

$$\mathbf{p} = \mathbf{a}_0\ [\mathbf{I} - (1 + r)\mathbf{A}]^{-1} \tag{7}$$

Now examining the last three equations it can be seen that under fixed coefficients the system has m + 1 degrees of freedom. Thus, m + 1 additional equations must be specified in the neoclassical, Marxian, or Keynesian models.

3 THE NEOCLASSICAL SPECIFICATION

In order to specify the model in a neoclassical fashion, Marglin assumes a simple economy in which all households are identical in composition and preferences, and each allocates its wage between consumption in two periods of its economic life (c_1 and c_2) according to maximization of a "life-cycle" utility function U, subject to the constraint given by its wage rate w = 1 (see Marglin, pp. 23-25 and 245-252). Thus, the households' problem is:

$$\max U = U(c_1, c_2) \tag{8}$$

subject to

$$p\, c_1 + p\, c_2/(1 + r) = 1 \tag{9}$$

Since consumption demand in any one year is the sum of demand of overlapping generations of working and retired households, the aggregate consumption demand can be expressed as:

$$c = c_1 + c_2/(1 + g) \tag{10}$$

Now substituting equations (10) and (2) into (1) and pre-multiplying the result by the price vector **p** yields **px**. Substituting (9) and (5) into the price equation (3) and post-multiplying the result by **x** gives **px** as well. Equalizing both equations produces the balance between investment and savings:

$$(1 + g)p\, A\, x = p\, c_2/(1 + r) \tag{11}$$

Finally, equilibrium between supply and demand for labor in the long run gives:

$$g = n \tag{12}$$

where n is the (exogenous) rate of populational growth.

As it was pointed out before, the households' problem is to maximize their utility function $U = U(c_1, c_2)$ subject to the restriction given by (9). Then, taking the Lagrangean of the system and considering the first order conditions for maximization it is obtained the following set of additional equations:

$$U'_1 = [(\delta U/\delta c_{i1}) = \mu p \tag{13}$$

and

$$U'_2 = [(\delta U/\delta c_{i2})] = (1 + r)U'_1 \tag{14}$$

where $(\delta U/\delta c_{i1})$ denotes the marginal utility of good i consumed in the current year and $(\delta U/\delta c_{i2})$ the marginal utility of the same good to be consumed next year, and i = 1,...,m. Note that in so far as restriction (13) reflects a system of relative prices there are (m-1) equations in it while in (14) there are m restrictions.

Equation (13) states the traditional neoclassical tenet according to which relative prices must be proportional to marginal utilities while equation (14) reflects the Fisherian idea that the rate of time preference is equal to the rate of interest.

Apparently, everything is fine: while under the assumption of fixed coefficients the "general" model has m+1 degrees of freedom, the neoclassical specification adds 2m unknowns ($\mathbf{c}_1$ and $\mathbf{c}_2$) but 3m+1 restrictions, given by equations (9), (10), (12), (13) and (14). Since none of the restrictions is redundant the model seems to be determined.

With these additional equations, Marglin closes the system claiming that the existence of equilibrium "survives under conditions that are neither more nor less general than in the one good case" (Marglin, 1987, p. 252).

4 CONTINUOUS SUBSTITUTION AND THE IMPOSSIBILITY OF RESWITCHING

So far, the input-output matrix **A** and the labor-coefficients vector $\mathbf{a}_0$ are assumed to be fixed. This hypothesis, however, can be relaxed to allow for the existence of many **techniques** of production available to each industry. Then it is possible no have a large number of different **technologies**, each one characterized by a specific combination of techniques that yields a particular matrix **A** and vector $\mathbf{a}_0$.

Marglin claims that while the possibility of reswitching is established under the assumption of fixed coefficients, continuous substitution implies its impossibility. The proof is given as follows.

Suppose that it prevails constant returns to scale but that **A** and $\mathbf{a}_0$ are not given any more. Hence, the model has $m^2 + m$ additional unknowns. To close it once more, write the production function for the generic good j as:

$$1/a_{0j} = f^j(\mathbf{k}_j) \qquad (15)$$

where $\mathbf{k}_j$ stands for the vector of capital per worker coefficients:

$$\mathbf{k}_j = (1/a_{0j})\ \mathbf{A}_{.j} \tag{16}$$

and $\mathbf{A}_{.j}$ means the column vector of inputs for industry j.

Now let the elements of the matrix $[\mathbf{a}_0\ \mathbf{A}]'$ be linked together by a set of m production functions $f^j(\mathbf{k}_j)$ and marginal productivity relationships such that:

$$\begin{aligned} p_j\, f_i^j &\begin{bmatrix} = \\ \leq \end{bmatrix} p_i\,(1 + r) \\ \text{if } a_{ij} &\begin{bmatrix} > \\ = \end{bmatrix} = 0 \end{aligned} \tag{17}$$

and

$$\begin{aligned} p_j\, f_i^j &\begin{bmatrix} = \\ \leq \end{bmatrix} w \\ \text{if } a_{oi} &\begin{bmatrix} > \\ = \end{bmatrix} 0 \end{aligned} \tag{18}$$

where f^j_i represents the derivative of f^j with respect to a_{ij}/a_{0j}.

The system of equations of type (17) and (18) provides the restrictions needed to determine **A** and $\mathbf{a}_0$ uniquely and, so, to close the model.

Besides, the hypothesis of continuous substitution leads to the impossibility of reswitching of techniques, producing a concave (from the origin) wage-profit frontier, which corresponds to the envelope of all possible single wage-profit curves. Indeed, if r_1 and r_2 are two profit rates associated with the same technical coefficients and if two goods i and j are used as inputs in each other's production, it follows from (17) that

$$p^1_j(1 + r_1)/p^1_j = p^2_j(1 + r_2)/p^2_j = f^j_i$$

or

$$p^1/p^2_i = p^1_j(1 + r_2)/p^2_j(1 + r_1)$$

where p^1 and p^2 denote the prices associated with r_1 and r_2, respectively.

By the same token, it can be seen that

$$p^1/p^2_j = p_{1i}(1 + r_1)/p_{2i}(1 + r_2)$$

Substituting the latter equation into the former and considering that $(1 + r_2) > 0$ gives:

$$r_1 = r_2 = r \tag{19}$$

The same property holds when goods i and j do not enter in each other's production since it can be always constructed a chain of marginal-productivity equations linking any two goods, even when the chains are very long.

By this way Marglin proves that the inverse relationship between the quantity of a production factor and its rate of remuneration as assumed in the neoclassical model is still valid in a multi-commodity world, showing that the Cambridge Critique ultimately rests on the assumption of fixed coefficients, with the only exception of the problem of the "aggregate capital".

5 THE RESWITCHING THEOREM RESTATED

Apparently, the only remaining question regarding the neoclassical model is the problem of "aggregate capital", which is, nevertheless, irrelevant (Marglin, 1987, p. 286). The problem may be viewed as follows.

The neoclassical theory claims an identity between the profit rate and the marginal productivity of capital. In the model above, this means:

$$\delta x_j / \delta K_j = (1 + r) \qquad j = 1,2,\ldots,n \tag{20}$$

where x_j means total output of commodity j and K_j is the total capital employed in its production. The difficulty comes from the fact that the quantity of capital in sector j is a product of two vectors:

$$K_j = \mathbf{p}\ \mathbf{A}_{.j} \tag{21}$$

which means that to determine the amount of capital employed in sector j the prices of capital goods have to be known in advance, which is impossible since they depend on the profit rate. In short, the neoclassical claim that price determination is a one-way road coming from factor endowments to prices leads to a vicious circle.

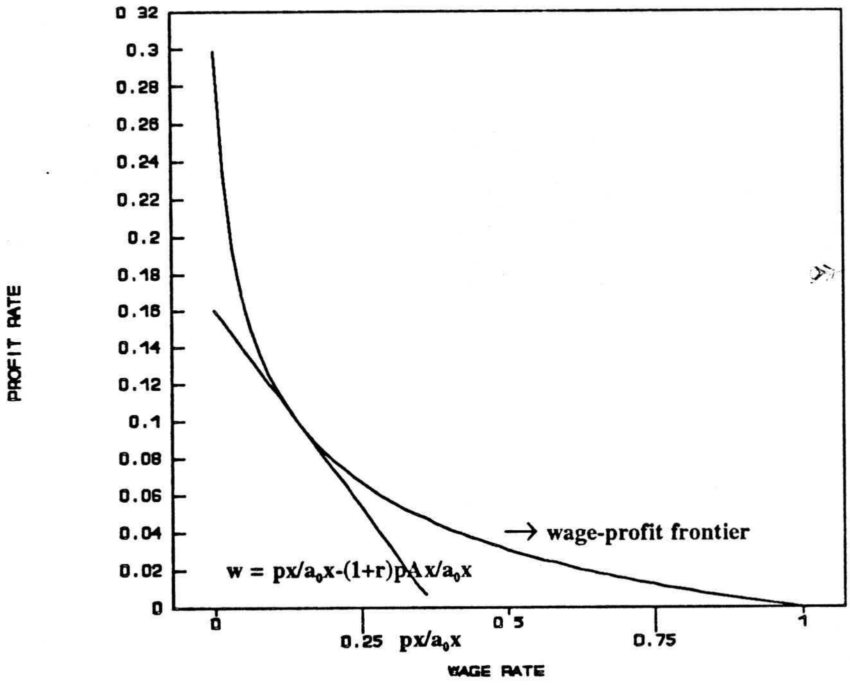
But this is not the whole story. Multiplying (3) by $\mathbf{x}$ and dividing the result by $\mathbf{a}_0\mathbf{x}$ it is possible to express the wage rate as:

$$w = px/a_0x - (1 + r)\ pAx/a_0x \tag{22}$$

where px/a_0x is the product per worker and pAx/a_0x gives the capital per worker. Thus, the capital per worker is the tangent of the wage-profit frontier, while the product per worker shows the point in which this tangent intercepts the w axis.

FIGURE 1

Capital Intensity and the Wage-Profit Frontier



Now, if prices reflect scarcity, the quantity of capital per worker has to be inversely related to its "price" - the profit rate. This implies that the profit-wage frontier should be concave (from below). But concave profit-wage frontiers require either concave or straight wage-profit curves. However, if the wage-profit curves are concave the possibility of reswitching is still present; if they are straight lines the inverse relationship between the quantity of capital and the rate of profit holds only for the envelope of the individual wage-profit curves. As it is well known, this case implies a one-commodity world in a disguised form.

It seems that Marglin either ignores or underestimates all these problems but the question of reswitching, which is apparently avoided by asserting from the very beginning the identity between the "marginal productivity of capital" and the gross rate of interest (1 + r). Nevertheless, a closer examination of the whole problem undermines Marglin's claims.

Indeed, one way to express (14) is:

$$c_2 = (1 + r) <\mu> c_1 \tag{14a}$$

where $<\mu>$ is a diagonal matrix derived from (14) and it reflects consumers' preferences. Substituting it into the budget constraint (9) and, then, the resulting equation into (3) gives:

$$\mathbf{p}\ [\mathbf{I} - (1+r)\mathbf{A}[\mathbf{I} - <\mathbf{I}+\mu>c_1\mathbf{a}_0]^{-1}] = p\ [\mathbf{I} - (1+r)B] = \mathbf{0} \tag{23}$$

But this is a system of homogeneous equations, which means that the necessary and sufficient condition for a non-trivial solution is given by the characteristic equation:

$$\det\ [\beta\mathbf{I} - \mathbf{A}\ [\mathbf{I} - <\mathbf{I} + \mu>\ c_1\mathbf{a}_0]^{-1}] = \det\ [\beta\mathbf{I} - B] = 0 \tag{24}$$

where

$$\beta = 1/(1 + r) \tag{25}$$

Notice that by the conditions of the problem **B** is non-negative. Thus, it follows from the Perron-Frobenius Theorem that its spectral radius β_{max} is real and positive and the associated eigenvector **p** is non-negative (it is positive if **B** is irreducible). Besides, since the economic system is supposed to be productive, β_{max} is less than unity, which means that r is positive.

Now it can be seen that Marglin's restriction upon the production functions requires their "derivatives" in relation to the specific capital good/labor ratio to be equal to an eigenvalue that results from elements that are determined by the production functions themselves, multiplied by the components of the corresponding eigenvector[2].

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2. Note that if the equality always holds equation (17) can be rewritten as:

$$\mathbf{F} = (1 + r) <\mathbf{P}>\ \mathbf{U}\ <\mathbf{P}>^{-1}$$

where $\mathbf{F} = [f^j_i]$ is the m by m matrix of marginal productivities, $<\mathbf{P}>$ is a matrix with prices in the principal diagonal and zeros elsewhere, and **U** is an m by m matrix of ones.
It is interesting to notice that as the equation above shows matrix **F** is similar to **U**, which means that it has only one non-zero eigenvalue, i.e. **F** is itself a matrix of rank one and it can be expressed as

$$\mathbf{F} = f^1_1\ \alpha\ \beta' \tag{A.3}$$

where $\alpha = [\alpha_i]$ is a column vector of scalars such that $\alpha_1 = 1$ and $\beta' = [\beta_i]$ is a row vector of scalars such that $\beta_1 = 1$. This shows the restrictive nature of Marglin's production functions: at current prices all the derivatives are merely multiples of the derivative of a particular production function in relation to a specific input. It is hard to give any economic meaning to this condition (other than to make reswitching impossible).

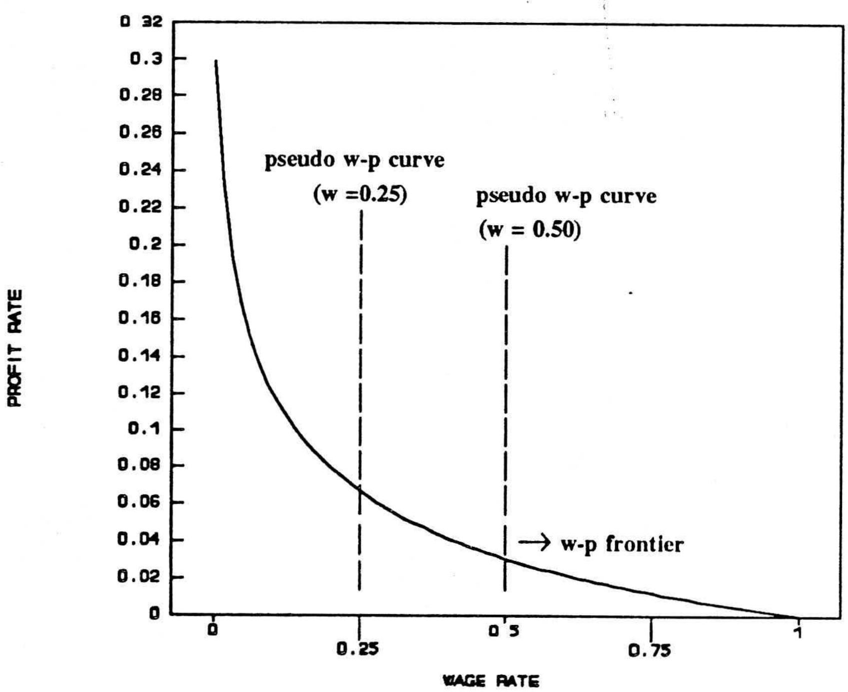
To examine the meaning of this condition, assume that any input/labor ratio a_{ij}/a_{0j} has a small increase. Since f^{j}_{i} is positive when a_{ij} is different from zero, it follows that the output of j will increase. But the coefficients a_{ij} are measured in terms of unity output, which means that all the coefficients a_{kj} (including a_{0j}) will decrease, with the possible exception of a_{ij}. For all other coefficients the decrease will be proportional to $(1+r)p_i/p_j$. But by the Perron Frobenius Theorem it is known that the maximum eigenvalue of **B** is an increasing function of its elements. Thus, if all elements of the column $\mathbf{A}_{.j}$ decrease (with the possible exception of a_{ij}) so does β_{max}, which means that the profit rate r increases.

Thus the consequence is that an increase in the capital intensity, as defined by Marglin, produces an increase in the profit rate, a result which is in opposition to the canons of the neoclassical school. Besides, since prices change as well, the final result in terms of the capital intensity for both the sector in which the changes happens as well as for the economy as a whole cannot be inferred **a priori**, and anything is possible - either an increase or a decrease in r. This shows exactly what the reswitching technique debate has done before - the inverse relationship between the profit rate and the quantity of capital is fallacious.

A second important characteristic of Marglin's production function is that it suggests the possibility of an infinity profit rate, since any increase in "capital intensity" (as defined by Marglin) leads to an augment of the profit rate. In other words, as shown in Figure 2, the production functions can be represented as vertical "pseudo wage-profit curves", i.e. given determined level of real wage the rate of profit can vary from zero to infinity, depending on the level of Marglin's "capital intensity".

Figure 2

"PSEUDO WAGE PROFIT CURVES"



Now assuming profit maximization it follows that it should be expected that the entrepreneurs will increase "capital intensity" indefinitely, which would make all the elements of the matrix **A** and the vector $\mathbf{a}_0$ to approach zero, yielding a profit rate that tends to infinity. It is interesting to notice that in this world there is no scarcity: ultimately goods can be produced without inputs.

But this is not all. Suppose that for any reason there is an increase in the real wage. According to the neoclassical tradition there should be a process of substitution of capital for labor, with the corresponding increase in the capital/labor ratio. However, even adopting Marglin's "production functions" there is little hope that this will happen systematically. Indeed, the change in the real wage will bring about changes in prices and in the rate of profits as well, changing all the derivatives of the production functions, with some increasing and some decreasing at random. Besides, since both the distributive variables r and w and prices have to be determined **before** the derivatives of the production function can be calculated any change in the real wage does not produce any change in the technical coefficients, but only in the derivatives. Thus, Marglin's restriction has no effect whatsoever upon the shape of the wage-profit curve, which preserves all its known properties. In terms of graphic representation, changes in the real wage cause changes in the vertical lines that represent Marglin's production functions, without affecting the shape of the wage-profit curve itself (see Figure 2). Since this means that the wage-profit frontier can assume any shape, the possibility of reswitching cannot be ruled out, and the existence of chains of "marginal-productivity" equations linking any two goods is not sufficient to prevent it.

Actually, from what was said above it can be concluded that Marglin's restriction on the production function is meaningless, and should be abandoned altogether.

6 PREFERENCES VERSUS WEIGHTED QUANTITIES OF LABOR?

Once the "perverse" properties of the wage-profit frontier have been reestablished, it is time to analyze the neoclassical tenet that prices reflect consumers' preferences.

Using Marglin's price equation (7) it is not difficult to see that, since the matrix $[\mathbf{I} - (1 + r)\,\mathbf{A}]$ is diagonally dominant (it is an "M" matrix), it is necessarily invertible (Graham, 1987, pp. 167-71). If follows that its inverse can be expressed as a sum of a power series:

$$[I - (1 + r) A]^{-1} = I + (1 + r) A + (1 + r)^2 A^2 + ...$$

which gives:

$$p = a_0 + (1 + r) a_0 A + (1 + r)^2 a_0 A^2 + ... \tag{26}$$

In other words, prices are conceived as weighted quantities of labor and, in principle, do not reflect preferences. This, however, is not entirely true. Indeed, manipulating (3) it is possible to rewrite the price equation as:

$$p = w\, a_0(I - A)^{-1}[I - r\, A(I - A)^{-1}]^{-1}$$

Now, multiplying both sides of the equation above by $c_1 + c_2/(1+r)$ and considering (4), (9) and (14a) gives

$$1 = a_0 (I - A)^{-1}[I - rA (I - A)^{-1}]^{-1} <I+\mu>^{-1} c_1 \tag{27}$$

This equation shows that the profit rate r depends on three factors: i) on the quantities of labor embodied in the commodities, since $a_0[I - A]^{-1}$ is the vector of quantities of labor values; ii) on the technical conditions of production, given by the matrix of technical coefficients A; iii) on the workers' preferences, given by the matrix of time preferences $<\mu>$ and by the vector of present consumption c_1. Note that it also shows the simultaneous determination character of the neoclassical model: c_1 depends on p but p depends on r which is a function of c_1.

Actually, equation (27) is a "consumption-profit frontier". It shows that, since r is a decreasing function of the elements of both $<\mu>$ and c_1, households' preferences enter price determination through their effects upon the rate of profits. Thus, if the households prefer to increase their present consumption at the expense of future consumption this results in a lower profit rate, while if they decide for a greater future consumption this leads to a higher profit rate.

Nevertheless, as equation (26) shows the resulting change in r coming from changes in preferences has no systematic effect on prices. For instance, if the relative preference for good i increases this affects r and, hence, all prices. However, r can either rises of falls and so can the price of good i p_i. The direction of the price change depends on the time structure of the labor embodied in the commodities and on the sign and magnitude of the change in the profit rate. In short, there is no definite economic law linking preferences and prices.

It should be stressed that the "consumption-profit frontier" expressed in (27) can be viewed as a special case of the traditional wage-profit frontier, which can be expressed as:

$$1 = a_0 (I - A)^{-1} [I - rA (I - A)^{-1}]^{-1} d \qquad (28)$$

where **d** stands for the (given) workers' basket. The particularity of (27) is that the workers are supposed to be proprietors of the means of productions and, as such, they are entitled to receive the whole income. Thus, the inverse relationship between r and household's consumption is nothing else but the relation between the profit rate and workers' consumption expressed in another way and it holds good only for a non-capitalist economy. As far as society is composed by workers and capitalists the dependence of r on households' preferences disappears. In this case only workers' "preferences" matters.

7 CONCLUSIONS

From the above discussion it can be concluded that continuous substitution is not a sufficient condition to avoid either reswitching of techniques or perverse capital deepening in a multiple-commodity world even when the "production functions" are "correctly" specified. Besides, it seems clear that for an economy in which the traditional concept of free competition holds prices can be conceived as weighted quantities of labor though they are affected by preferences.

In short, from all cannons of the neoclassical theory the only one that is compatible with the traditional concept of free competition is the notion that consumers' preferences are an element in the determination of the profit rate and prices. Nevertheless, this holds good only in so far as consumers are workers. Besides, there is no systematic law connecting prices and preferences.

8 BIBLIOGRAPHY

BRUNO, M., BURMEISTER, E. and SHESHINSKI, E. (1966). "The nature and implications of the reswitching of techniques". *Quarterly Journal of Economics 80*(4): 526-53.

DOUGHERTY, C. (1980). *Interest and Profit.* New York: Columbia University Press.

FISCHER, I. (1930). *The Theory of Interest.* New York: Macmillan.

GAREGNANI, P. (1990). "Quantity of capital". In: EATWELL. J., MILGATE, M. and NEWMAN, P. (eds). *The New Palgrave: Capital Theory.* New York: Norton, pp. 1-78.

GRAHAM, A. (1987). *Non Negative Matrices and Applicable Topics in Linear Algebra.* Chichester: Ellis Horwood; New York: John Willey.

HATTA, T. (1976). "The paradox in capital theory and complementarity of inputs". *Review of Economic Studies 43*(1): 127-42.

_____ (1990).. "Capital perversity". In: EATWELL. J., MILGATE, M. and NEWMAN, P. (eds). *The New Palgrave: Capital Theory.* New York: Norton, pp. 130-5.

MARGLIN, S. A. (1984). *Growth, Distribution, and Prices.* Cambridge, MA: Harvard University Press.

PASINETTI, L. L. (1966). "Changes in the rate of profit and switches of techniques. *Quarterly Journal of Economics 80*(4): 503-17.

_____ (1969). "Switches of technique and the 'rate of return' in capital theory". *Economic Journal 79*(3): 508-31.

_____ (1977). *Lectures on the Theory of Production.* New York: Columbia University Press.

STARRET. D. A. (1969). "Switching and reswitching in a general production model". *Quarterly Journal of Economics 83*(4): 673-87.

STIGLITZ, J. (1973). "The badly behaved economy with the well-behaved production function". In: J. A. MIRRLES and N. H. STERN (eds), *Models of Economic Growth.* London: Macmillan; New York: Wiley.

APPENDIX: THE WAGE PROFIT FRONTIER AND MARGLIN'S PRODUCTION FUNCTIONS. A NUMERICAL EXAMPLE

The objective of this Appendix is to present a numerical example showing that even satisfying Marglin's conditions for the production functions both reverse capital deepening and reswitching of techniques cannot be avoided. The classical specification of the price equation will be used in order to simplify the calculations and to avoid the problems arising from the simultaneous determination character of the neoclassical model. Thus, the system here is composed by equation (3) with the classical assumption that the real wage **d** is given:

$$w = \mathbf{p}\,\mathbf{d} \tag{A.1}$$

and by conditions (15) to (18), which specify the properties of the production functions.

Let suppose an economy characterized by the following input-output decomposable matrix **A** and the vectors $\mathbf{a}_0$ and **d**:

$$\mathbf{A} = \begin{bmatrix} 0.60 & 0.10 & 0.20 \\ 0.05 & 0.50 & 0.25 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

$$\mathbf{a}_0 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} 0.00 \\ 0.00 \\ 0.05 \end{bmatrix}$$

Choosing the commodity produced by industry 3 as the **numéraire** of the system the equilibrium values of the variables are:

$w = 0.05$

$r = 0.4947$

$\mathbf{p} = [1.3501 \quad 1.1946 \quad 1.0000]$

Using (17a) Marglin's restrictions on the production functions the matrix of marginal productivities **F** can be obtained:

$$\mathbf{F} = \begin{bmatrix} 1.4947 & 1.3226 & 1.1072 \\ 1.6893 & 1.4947 & 1.2512 \\ 2.0180 & 1.7856 & 1.4947 \end{bmatrix}$$

Now suppose an increase of one percent in a_{22}. Since all f^i_i are positive the output of industry 2 will increase, which means that all coefficients a_{i2} will decrease. Of course the final result will be the integral of a continuous function connecting **A**, $\mathbf{a}_0$ and **F**, but since the increase in a_{22} is small a discrete approximation can be used. Thus, short of a small error the final result of such an increase is the following technical and labor coefficients for industry 2:

$$A_{.2} = \begin{bmatrix} 0.0985 \\ 0.4976 \\ 0.0000 \end{bmatrix}$$

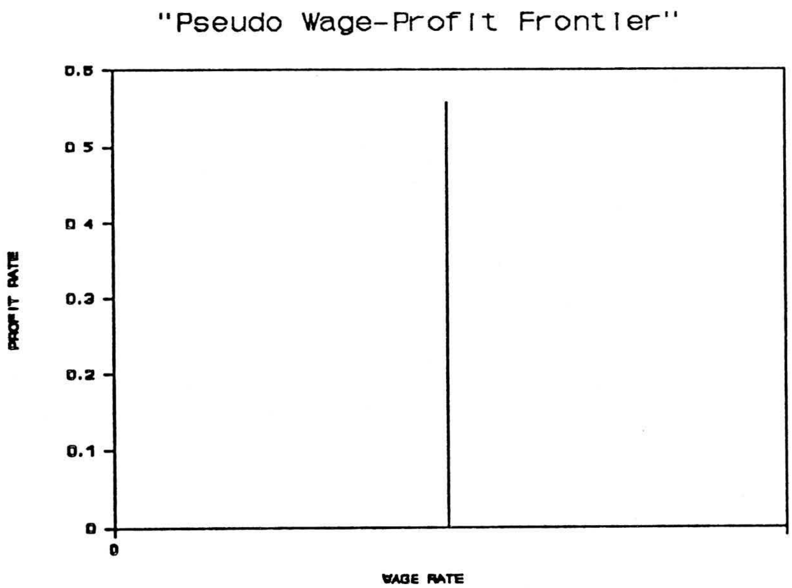
$$\mathbf{a}_{02} = 1.9705$$

Substituting these new coefficients into matrix **A** and into vector $\mathbf{a}_0$ gives:

w = 0.05
r = 0.4979
p = [1.3650 1.1778 1.0000]

Note that, as expected, there is an increase in the profit rate. If this procedure is repeated indefinitely it is possible to arrive at a "pseudo wage profit curve" like that one in Figure A.1. This illustrates what was said before -Marglin's production functions lead to a infinity profit rate.

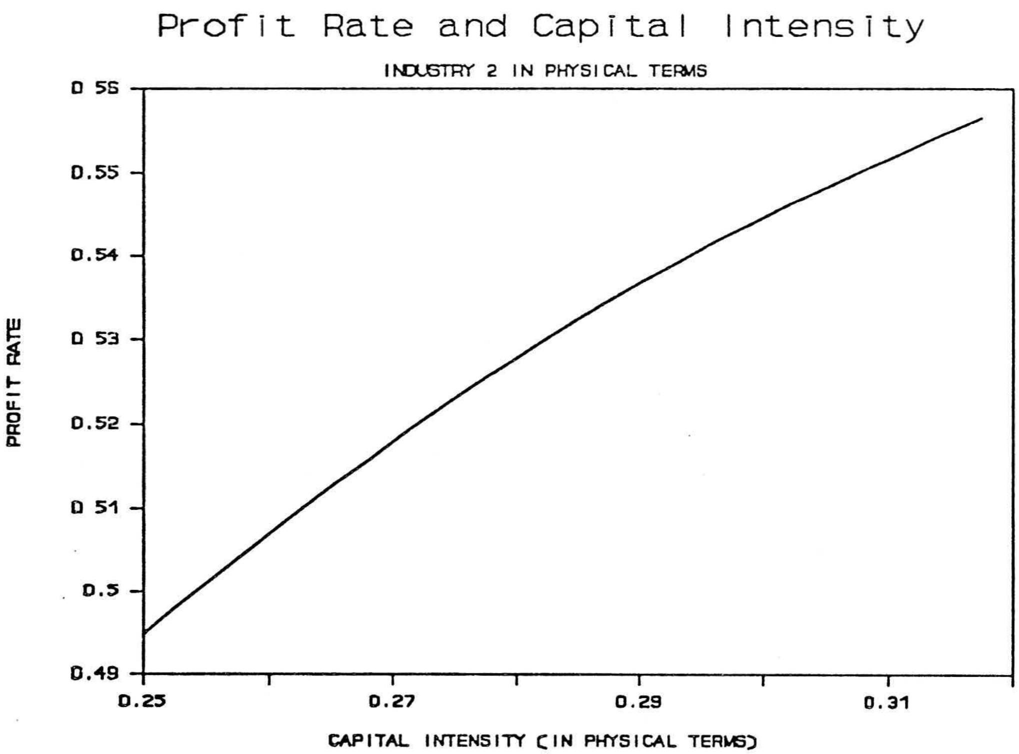
Figure A.1



An alternative exercise is to study the behavior of capital intensity as the economy moves along the "pseudo wage profit curve". Regarding this question two aspects are important. The first one is the change in capital intensity in the industry in which changes in technical coefficients are happening. The second aspect is the change in the capital/labor ratio for the economy as a whole.

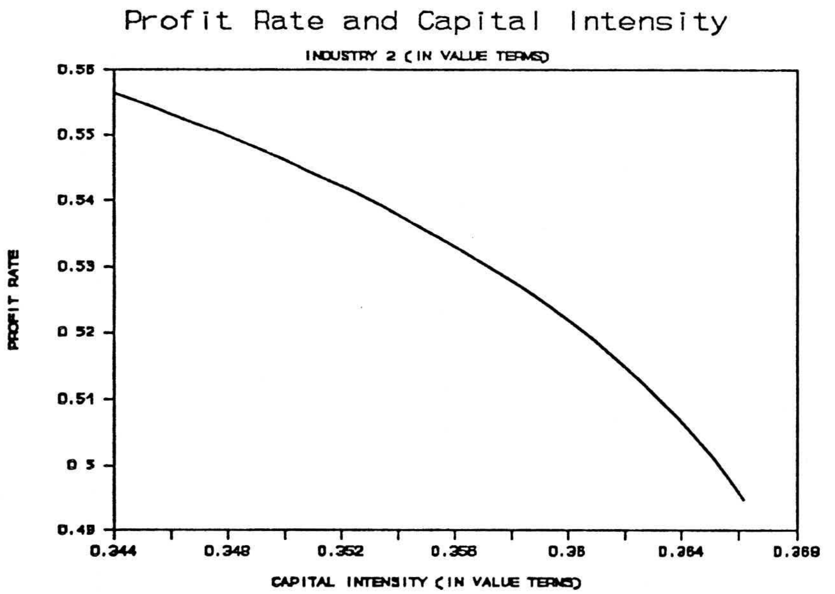
In the numerical example above capital intensity in physical terms increased - the ratio a_{22}/a_{02} rose from 0.25 to 0.2525 while all other ratios a_{ij}/a_{0j} remained the same. At the same time the profit rate increased 0.6%. Calculating the rate of profit for an increasing a_{22}/a_{02} is possible to draw Figure A.2, which shows an inverse relationship between these two variables, i. e. it indicates an inverse relation between capital intensity, measured in physical terms, and the profit rate. It does not need to be stressed here how unfavorable is this fact for the neoclassical theory.

Figure A.2



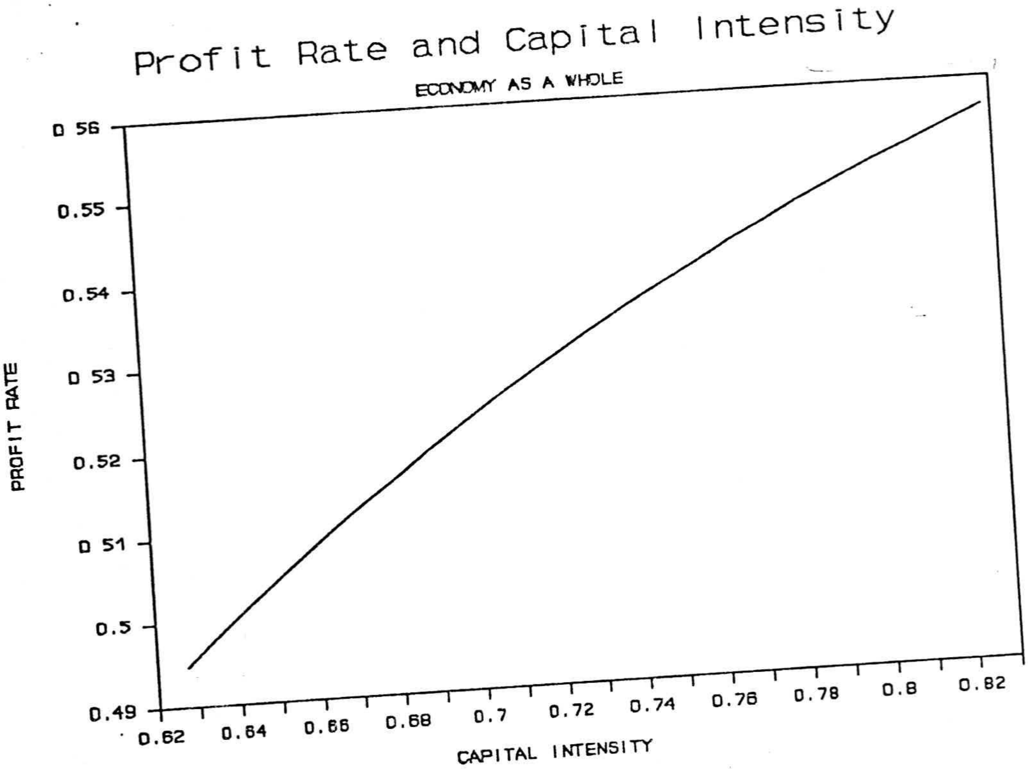
Now when prices are taken into account the whole picture changes. Figure A.3 relates capital intensity in industry 2 and the profit rate. In the special case under study, capital intensity decreases with an increasing profit rate. This is so because the price effect is such that it offsets the increase in capital intensity measured in physical terms. However, the same is not true for the economy as a whole since there is a direct relation between the profit rate and capital intensity. This is the result of the combined effects of changes in prices and output composition. Considering only the effects of output composition it can be said that if the production of industry 2 grows faster than the production of the other sectors it might be the case that capital intensity decreases. But even in this case this does not necessarily happens since it could be that industry 2 does not grow fast enough to offset the increase in capital intensity in those sectors. Actually, contrary to what would be expected from a neoclassical point of view the problem of weighting the production of the different sectors cannot be avoided. In the present example Sraffa's standard commodity was used[3] to prevent the use of any arbitrary weighting process. As it can be seen in Figure A.4, the result was that a **direct** relationship between capital intensity and the profit rate was obtained though Marglin's "well-behaved" production functions were used. In other words, reverse capital deepening shows up even in the case of Marglin's "pseudo wage-profit curves".

Figure A.3



3. This is equivalent of using the "dual" of the price system as the weights for the aggregation procedure.

Figure A.4



The same phenomenon is present when the usual wage-profit curve is considered. Indeed, in the example above the curve is convex from the origin (see Figure A.5) showing a **direct** relation between the profit rate and capital intensity (see Figure A.6). Finally, it should be stressed that this shape of the wage-profit curve is compatible with reswitching of techniques, contradicting Marglin's assertions regarding the subject.

Figure A.5

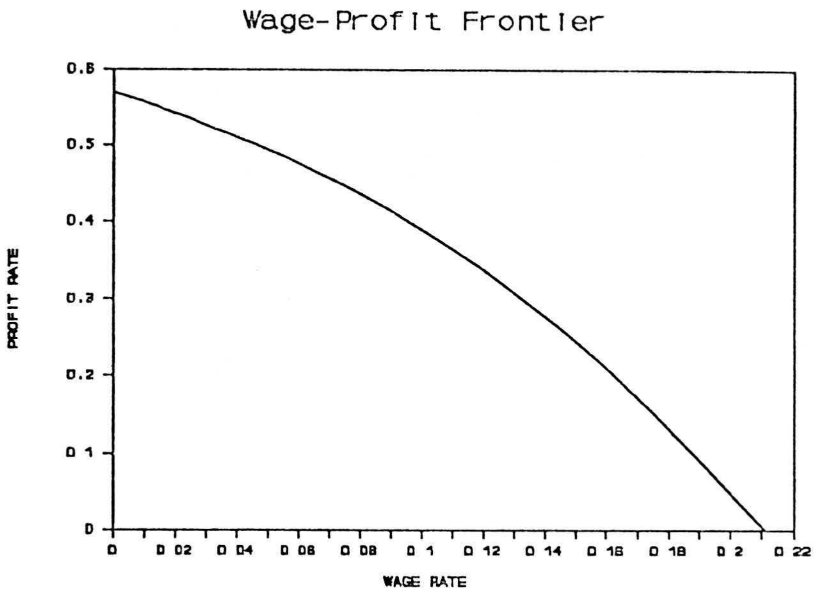


Figure A.6