ISSN 2318-2377

UFMG

Cedeplar

TEXTO PARA DISCUSSÃO N° 621

JEAN-BAPTISTE FOURIER AT THE MOSCOW CONJUNCTURE INSTITUTE: HARMONIC ANALYSIS OF BUSINESS CYCLES

Marco Paulo Vianna Franco
Leonardo Costa Ribeiro
Eduardo da Motta e Albuquerque

Abril de 2020

http://www.cedeplar.ufmg.br

Textos para Discussão

A série de Textos para Discussão divulga resultados preliminares de estudos desenvolvidos no âmbito do Cedeplar, com o objetivo de compartilhar ideias e obter comentários e críticas da comunidade científica antes de seu envio para publicação final. Os Textos para Discussão do Cedeplar começaram a ser publicados em 1974 e têm se destacado pela diversidade de temas e áreas de pesquisa.

Ficha catalográfica

F825j
2020

Franco, Marco Paulo Vianna.
Jean-Baptiste Fourier at the Moscow Conjuncture Institute: Harmonic Analysis of Business Cycles / Marco Paulo Vianna Franco; Leonardo Costa Ribeiro; Eduardo da Motta e Albuquerque. - Belo Horizonte: UFMG / CEDEPLAR, 2020.

31 p. : il. - (Texto para discussão, 621)
Inclui bibliografia.
ISSN 2318-2377

1. Ciclos econômicos. 2. Análise harmônica. 3. Análise espectral. I. Ribeiro, Leonardo Costa. II. Albuquerque, Eduardo da Motta e. III. Universidade Federal de Minas Gerais. Centro de Desenvolvimento e Planejamento Regional. IV. Título. V. Série.
CDD: 330

Elaborado por Fabiana Pereira dos Santos CRB-6/2530 - Biblioteca da FACE/UFMG. – FPS/053/2020

UNIVERSIDADE FEDERAL DE MINAS GERAIS
FACULDADE DE CIÊNCIAS ECONÔMICAS
CENTRO DE DESENVOLVIMENTO E PLANEJAMENTO REGIONAL

JEAN-BAPTISTE FOURIER AT THE MOSCOW CONJUNCTURE INSTITUTE: HARMONIC ANALYSIS OF BUSINESS CYCLES[**]

Marco Paulo Vianna Franco
Fundação João Pinheiro, Belo Horizonte, Brazil

Leonardo Costa Ribeiro
Cedeplar-UFMG, Belo Horizonte, Brazil

Eduardo da Motta e Albuquerque
Cedeplar-UFMG, Belo Horizonte, Brazil

CEDEPLAR/FACE/UFMG
BELO HORIZONTE
2020

[*] We would like to thank CNPq (grant numbers 302857/2015-0 and 401054/2016-0) for the financial support for this research.

SUMÁRIO

ABSTRACT

This article proposes a historical assessment of harmonic analysis of business cycles. For that purpose, it presents Jean-Baptiste Fourier's main idea and addresses its reception at the Moscow Conjuncture Institute, mediated by Henry L. Moore and Albert L. Vainshtein, which led to Eugen Slutsky's well-known 1927 article on the random causes of cyclical processes. In addition, more recent approaches to business cycles, such as real business cycle theory and spectral analysis, are traced back to Fourier and key figures working on economic applications of harmonic analysis in the first decades of the 20th century. Revolving around its ability to both decompose and build cycles, this tool still presents an untapped potential for contemporary analyses of long-term economic dynamics. Hence, it is worthwhile to determine Fourier's role in the history of economic thought, what necessarily leads to the long-lasting contributions of the creative institute headed by Nikolai Kondratiev.

Keywords: harmonic analysis; business cycles; Fourier transform; spectral analysis; Eugen Slutsky.

RESUMO

O artigo traz uma perspectiva histórica da análise harmônica de ciclos de negócios. Assim, apresentam-se a teoria do matemático Jean-Baptiste Fourier e sua recepção no Instituto de Conjuntura de Moscow, mediada por Henry L. Moore e Albert L. Vainshtein, o que culminou com o famoso artigo de 1927 de Eugen Slutsky sobre processos cíclicos a partir de causas randômicas. Adicionalmente, abordagens mais recentes à análise de ciclos econômicos, como a teoria dos ciclos reais de negócios e a análise espectral, são contextualizadas à luz das contribuições de Fourier e economistas que trabalharam em aplicações econômicas da análise harmônica durante as primeiras décadas do século XX. Em que se pese sua capacidade de tanto decompor quanto construir ciclos, essa ferramenta ainda apresenta um potencial não totalmente explorado em análises contemporâneas da dinâmica econômica de longo prazo. Portanto, justifica-se o resgate da figura de Fourier na história do pensamento econômico, o que necessariamente leva às duradouras contribuições do criativo instituto liderado por Nikolai Kondratiev.

Palavras-chave: análise harmônica; ciclos de negócios; transformada de Fourier; análise espectral; Eugen Slutsky.

Jel Classification: B16, B23, C02, E32

I. INTRODUCTION

Research and elaboration on cycles or waves is a common element in the works of Nikolai Dmitrievich Kondratiev, Evgenii "Eugen" Evgenevich Slutsky and Jean-Baptiste Joseph Fourier. The importance of their theoretical legacies on this subject is hard to overestimate, as shown by Kondratiev's influential studies on long waves in the economy (Schumpeter, 1939; Freeman and Louçã, 2001; Aghion et al., 2014, p. 551), Slutsky's contribution to real business cycle theories (Kydland and Prescott, 1990; Plosser, 1989), and Fourier's outstanding achievements in pure and applied mathematics (Hawkings, 2007, pp. 519-590).

Different approaches, different points of view, different subjects: Kondratiev ([1922] 2004) looks for broad dynamic forces shaping capitalism and triggering long cycles; Slutsky (1927) shows how random causes may combine into undulatory patterns; Fourier (1822) builds mathematical tools to understand wave-like natural phenomena. In a very peculiar and rich historical moment, there was an interplay of such theories and methods at the Moscow Conjuncture Institute (MCI) (Kondratiev, 1925; [1927] 1998; Barnett, 1998, pp. 8-13). The MCI was created by Kondratiev in 1920 and led by him until 1928; it was closed in 1929 under the yoke of Stalin.[1]

There are several accounts on Kondratiev's intellectual legacy (e.g., Makasheva et al., 1998; Barnett, 1998; Louçã, 1999). His elaboration on cycles and waves is based on a broad and careful literature review, surveying the most important works available at the time.[2] One of Kondratiev's sources was Henry L. Moore, a professor at Columbia University, and his "use of Fourier's theorem" (Moore, 1914, pp. 6-13). The curiosity about Moore's works amongst the members of the MCI seems to have been so great that one of its senior researchers, Albert Lvovich Vainshtein, prepared at least two detailed reviews of his works. In fact, this is an indication of their level of interest in Fourier's harmonic analysis. In the end of 1925, Slutsky was invited by Kondratiev to work at the MCI, joining in January 1926 (Barnett, 2011, pp. 81-82). In 1927, Slutsky would use harmonic analysis to support his own theory on random causes of cyclical patterns (Slutsky, 1927).

The subject connecting Kondratiev, Slutsky and Fourier would also appear in Joseph A. Schumpeter's *Business Cycles*.[3] He discusses both sides of harmonic analysis: wave composition and

[1] Schumpeter (1954, pp. 1157-1158) mentions the existence of an institute "for business or cycle research, which for the time being enjoyed some freedom not only in collecting but also interpreting economic data". Schumpeter also mentions other "competent economists" (Pervushin, Oparin, Sokolnikov, and others), a "proof that serious economics survived until the rigors of the Stalinist regime fully asserted themselves" (p. 1158). In a footnote Schumpeter writes that "Kondratieff was exiled in Siberia" (p. 1158).

[2] "Problems of economic dynamics increasingly attract theoreticians. Cournot, Jevons, Walras, Pareto, Clark, Marshall, Schumpeter and some others... The works of Juglar, Tugan-Baranovsky, Spiethoff, Pohle, Eilenburg, Lescure, Aftalion, Bunyatan, Mitchell, and others, using empirical data, approached closely to the problem of economic cycles and elucidated much here" (Kondratiev, [1922] 2004, pp. 23-24). While some would acknowledge that long cycles do exist, others would think that conjunctural movements do not answer "the question of the cyclic nature of the transition between those periods"; a "third group" would reject that such movements "are cyclic, viewing them as the result of random factors and movements in economic life" (Kondratiev, [1926] 1998, p. 27). More specifically, Kondratiev ([1928] 1992, pp. 423-424) describes six different lines of research on business cycles: (1) identification of cyclical price movements without attempts to explain them; (2) cycles as consequences of random causes, like wars, revolutions, or fluctuations in gold production; (3) gold production, monetary circulation, and credit; (4) socio-economic conditions of capital accumulation; (5) relationship between supply and demand; and (6) relationship between supply and demand and the volume of gold production. Certainly, one important question would be how to combine all those causes (on this issue, see our concluding remarks).

[3] Mitchell (1927) also cited those three authors in the same context, although he did not provide a more integrated perspective on their works.

decomposition. On the one hand, Schumpeter (1939, p. 180) mentions the "Slutsky effect"; on the other hand, he reviews authors who apply the so-called periodograms – which were developed by Arthur Schuster (1898) – to investigate periodicities in economic life (Crum, 1923; Moore 1923; Wilson, 1934). Unfortunately, neither these references nor Schumpeter's assessment stimulated at that moment a broader use of harmonic analysis in economics.

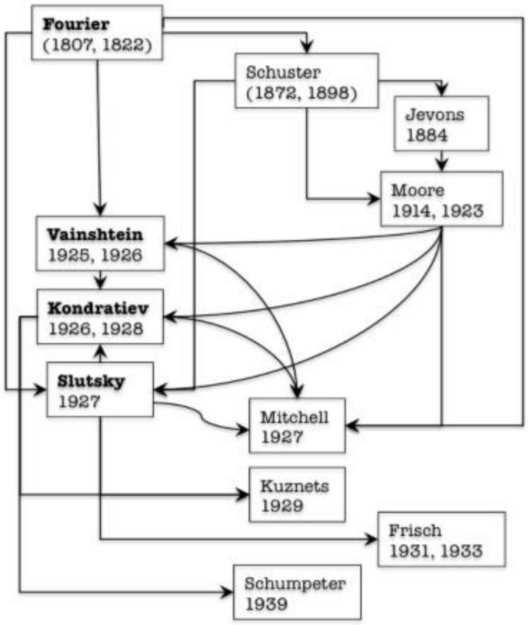
Schumpeter's views on this subject are somewhat ambiguous. In *Business Cycles*, he attempted to integrate different cycles into one resulting wave, and had glimpses of how combination, superposition and other articulations between different dynamic forces shaping waves could operate. On the periodogram. Schumpeter (1939, fn. 2, pp. 165-166) presents a long footnote to attest that it was "gaining ground in economics". He refers to Moore as a pioneer, and also to William H. Beveridge, W. Leonard Crum and Edwin B. Wilson. He believes periodograms would work for applications in meteorology and other natural sciences, while for others such as economics "[i]rregularities and interferences between components need not to be very great to upset everything" (p. 166). Schumpeter fears that "no amount of closeness of fit proves in itself that the individual components have any meaning in the sense that distinct phenomena correspond to them" (p. 168). After his choice and justification of his "three-cycle schema" – Kondratiev, Juglar and Kitchin cycles – once again he mentions Fourier with some reservations (pp. 199, 212, 215).

Although it is not the aim of the present inquiry, we hypothesize that two issues might have limited Schumpeter's ability to grasp the full potential of harmonic analysis for economic cycles. Firstly, his theoretical view was still within the equilibrium framework, with innovation acting as a disequilibrium force that triggers a process that would, in the end, return to a new equilibrium position. Secondly, his three-cycle schema would impose a high pressure on the regularity of such cycles. These two theoretical constraints might have blocked his vision of a more turbulent and chaotic economic dynamics, which he would later suggest along with his elaboration of the process of creative destruction (Schumpeter, 1942, ch. 7).

Schumpeter performed a great service in terms of the acknowledgment and dissemination of the contributions of Kondratiev, Slutsky and Fourier to research on business cycles. However, important changes have taken place since 1939 in relation to data availability, new phases of capitalism (three more Kondratiev cycles), and, after the breakdown of Stalinism, contextual information on Kondratiev and the MCI. Therefore, this article proposes a historical assessment of harmonic analysis of business cycles. It addresses Fourier's reception at the MCI and how it was mediated by Moore and Vainshtein (Figure 1 summarizes direct and indirect links between Fourier, Slutsky and Kondratiev in terms of citations. In the figure, Schumpeter is assessed as a business cycle theoretician and not as a historian of thought).

Accordingly, this article is organized as follows. The next section presents Fourier's main idea leading to harmonic analysis, its origins, and the potential two-sided contributions to investigations on cycles and waves. Section III focuses on Moore as a pioneer in the use of harmonic analysis in economics, and on his inspiration in Schuster's periodograms. The following section travels to the MCI and assesses the role of Vainshtein as part of the knowledge flow from the French mathematician to Kondratiev and Slutsky. Section V finally reaches Slutsky and Kondratiev during the final years of the MCI, establishing how they managed to thrust harmonic analysis into business cycle debates then and now. Section VI sheds light on early interpretations of Slutsky's work and elucidates how more recent approaches to economic cycles, such as real business cycles and spectral analysis, can be traced back to a long line of thinkers. Section VII brings concluding remarks.

FIGURE 1
A citation-based tentative genealogy of the links between Fourier and the Moscow Conjuncture Institute



Source: Authors.

II. THE FOURIER TRANSFORM AND THE ANALYSIS OF CYCLES AND WAVES

Isaac Newton proposed the law of cooling, which states that the rate of heat transfer is proportional to the temperature difference between two bodies (Bergles, 1988). Since then, there would have been few theoretical advances on the subject of heat transfer until 1807, when Fourier conceived the differential equation which describes heat transfer phenomena and developed a non-trivial technique to solve this complicated equation (Narasimhan, 1999). Fourier's partial differential equation mathematically describes the rate at which temperature is changing at any location in the interior of a solid body as a function of time. Fourier's equation has the following form:

$$\frac{\partial u_{(x,y,z,t)}}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$u_{(x,y,z,t)}$ denotes the temperature at points *x*, *y*, and *z* at time *t*. The equation mixes time and space, having a tough solution, which becomes straightforward if Fourier's technique is used.

Fourier realized that, if the spatial behavior of the temperature inside the body follows a unique sinusoidal function, i.e., in the form

$$u_{(x,t)} = f(t)\sin\omega x$$

(for simplicity we will solve for a one-dimensional body, like a wire, although the result can be extended to a three-dimensional body), the spatial and temporal variables at his equation can be separated and solved independently. Inserting this last equation into Fourier's equation and calculating the second derivative of a sinusoidal function yield the following result:

$$\frac{\partial u_{(x,t)}}{\partial t} = -\alpha\omega^2 u$$

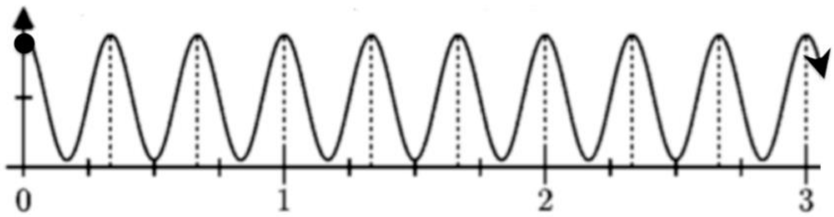
This partial derivative equation for the temporal part is quite simple and has as a solution an exponential function. Solving for the spatial and temporal parts, we have a spatial sinusoidal behavior as well as a temporal exponential decay:

$$u_{(x,t)} = \sin(\omega x)e^{-\alpha\omega^2 t}$$

However, this solution only works for a specific spatial temperature behavior, i.e., a sinusoidal function with angular frequency ω, and not for a generic temperature distribution inside the body. What if we could write a generic temperature distribution as a sum of sinusoidal functions? In this case, we could just add up Fourier's equation solution for each angular frequency which composes the initial temperature distribution and obtain the generic solution over time and space.

With this in mind, Fourier developed a technique to write a generic function $f(x)$ as a sum of oscillatory functions (sines and cosines), known as the Fourier transform. To understand how that is possible, think of a sinusoidal wave, as shown in Figure 2.

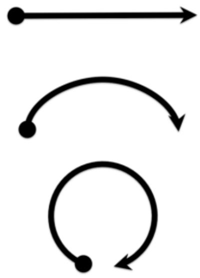
FIGURE 2
A sinusoidal wave



Source: Authors.

Now, imagine that the horizontal axis is curled in the way shown in Figure 3.

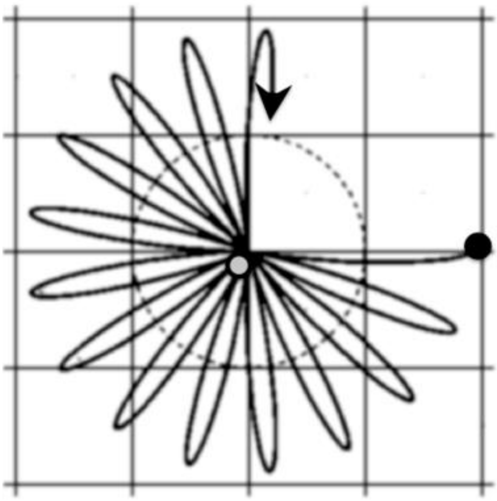
FIGURE 3
Curling the horizontal axis



Source: Authors.

This transformation to the horizontal axis is also applied to the original wave, as shown in Figure 4 (notice where the circle and arrow marks which identified the initial and ending points of the original curve in Figure 1 were, and where they are in Figure 3 after the curling process).

FIGURE 4
Curling the original wave



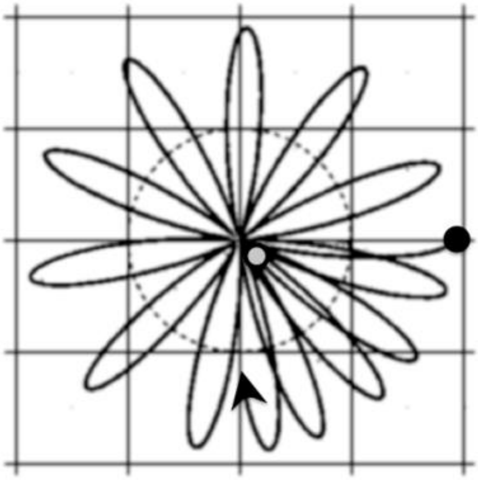
Source: Authors.

Nevertheless, the speed that is used to curl the original wave is arbitrary and the transformation can be done using different speeds. To clarify this issue, Figure 5 shows the transformation using a slightly higher curling frequency (again, observe where the circle and arrow marks are).

Curling the wave using an even higher frequency might lead to beautiful patterns, as shown in Figure 6.

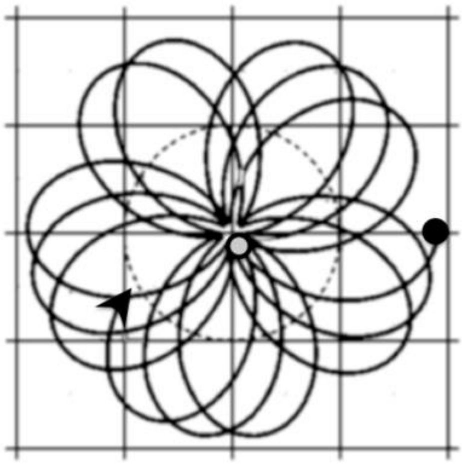
However, something different happens when the curling frequency is exactly the same as the frequency of the original wave. If the latter is fixed and the former continuously altered, visiting all possible values, at some point they will coincide. In this case, Figure 7 shows what happens. Most of the curve is now on the right-hand side of the grid. Hence, the position of the center of mass of this curve (shown as a gray circle in Figure 7) differs from the center of the grid, as opposed to the cases in Figures 4, 5 and 6.

FIGURE 5
he curled wave in a slightly higher frequency



Source: Authors.

FIGURE 6
Patterns arising from higher curling frequencies

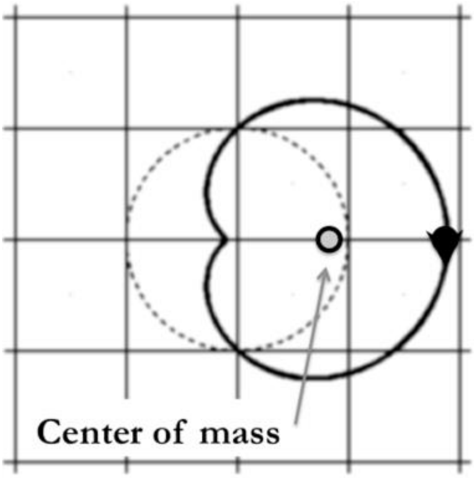


Source: Authors.

If the position of the center of mass of the curled wave is plotted as a function of the curling frequency, the following chart is obtained (Figure 8). The center of mass remains near the origin for almost all curling frequencies and a peak appears – the center of mass moves away from the origin – when the curling frequency is equal to the frequency of the original wave.

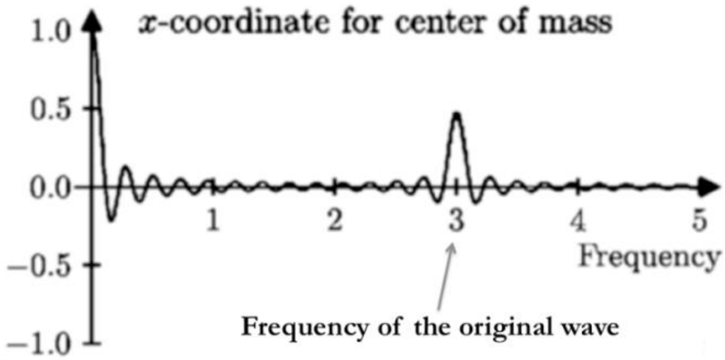
The example above is applicable to original waves with a single frequency. If we think of an original wave resulting from the overlap of two waves with different frequencies, perform the curling transformation, and plot the centers of mass for all possible curling frequencies, then two different peaks would appear on the chart of Figure 8. Therefore, the different frequencies which compose the original curve can be identified. The peak height corresponding to a given frequency can be interpreted as the weight of that frequency on the composition of the original curve. The same is valid for an original curve composed of many different frequencies.

FIGURE 7
The curled wave when its curling frequency matches that of the original wave



Source: Authors.

FIGURE 8
The position of the center of mass as a function of the curling frequency



Source: Authors.

Mathematically speaking, the curling transformation is equivalent to multiplying the original curve by an exponential of an imaginary number, i.e., $e^{-i\omega t}$, whereas ω is the angular frequency of the curling transformation and equal to $f/2\pi$ (f being the curling frequency). Calculating the position of the center of mass is equivalent to integrating the curve over its domain for a unitary total mass. Thus, decomposing a function $F(x)$ into the different waves which constitute it is equal to calculating

$$F(f) = \int_{-\infty}^{\infty} F(x) e^{-\frac{2\pi i t}{f}} dx$$

Using this technique, it is possible to decompose an arbitrary initial temperature distribution inside a solid body into each of its constituent frequencies and, in turn, to solve the heat transfer equation to each one of them, before finally adding up the temporal and spatial solutions from each frequency in order to get the general solution.

Wave Composition

In an analogous manner, the Fourier transform can also be used to build an arbitrary function from periodic waves by means of the inverse transform. Due to the properties of the exponential of an imaginary number, the inverse Fourier transform is obtained by a simple sign change of the exponent in the previous equation:

$$F(x) = \int_{-\infty}^{\infty} F(f) e^{\frac{2\pi i t}{f}} df$$

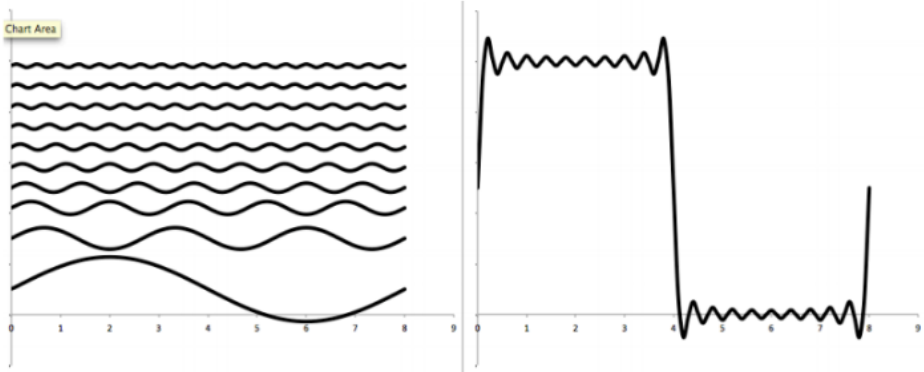
As the exponential of an imaginary number can also be written as

$$e^{i\omega t} = \sin \omega t + i \cos \omega t,$$

(for simplicity, we deal here only with the real part), the exponential term represents an oscillatory wave with angular frequency ω and frequency *f*.

In the inverse Fourier transform, we basically multiply an oscillatory wave (the exponential term) by its weight in the original function, $F(f)$,which is the result of the Fourier transform to frequency *f*, and then add up all the waves with different frequencies (the integration over df). Therefore, once the weight and the frequency of each wave are known, it is possible to build the original function by multiplying the weight along the wave of frequency *f* and adding up all present frequencies.

FIGURE 9
The inverse Fourier transform for a square function



Source: Authors.

An illustration of this process is shown in Figure 9. At the left-hand side, there is a sequence of sinusoidal waves with different frequencies and weights, which are expressed by their amplitudes. When the waves are added up, the curve shown at the right-hand side of the figure emerges. It is similar to the square function and gets closer and closer to it whenever more waves are added up. It becomes a perfect square function when all sinusoidal waves which compose it are included.

Context and Implications of Fourier's Investigation on Wave-Like Phenomena

Why did Fourier research heat and its diffusion? From a point of view of history of science, a starting point to answer this question could be the scientific environment of Fourier's time, in which thermodynamics emerged as a new branch of science (Taton, [1958] 1995, pp. 533-545; [1961] 1995, pp. 261-298). Heat conductivity entailed a theoretical problem in the 18th century (James Watt's first patent would only materialize in 1794). Narasimhan (1999, p. 119) lists "significant contributions to diffusion", starting with Daniel G. Fahrenheit's "mercury thermometer and standardized temperature scale" in the first quarter of that century. Maurey (2019, p. 14) corroborates that "[t]he study of heat was a serious subject around 1800, especially with the rise of the steam engine."

From the point of view of Rosenberg (1982) on "how exogenous science is", Fourier can be an excellent case study. Firstly, in relation to the role of research tools for the progress of basic science. Fourier's investigations included experimentation which was dependent on the progress of instruments such as the thermometer for measuring heat and its diffusion (Fourier, 1828). Secondly, Fourier's breakthroughs are a classical example of how trying to solve practical problems – how heat diffuses – unintentionally leads to a key contribution to basic science. To Hawkings (2007, p. 527), Fourier "revolved around his solution of the problem of heat diffusion in solid bodies. (…) In the course of this work, he developed a new mathematical technique that would not likely have arisen otherwise".

Once developed, Fourier's achievement would spread through different branches of science and engineering, with countless applications (Narasimhan, 1999, pp. 122-135). In economics, its ability to compose and decompose waves has been particularly applied to the study of cyclical and undulatory phenomena. In this case, however, Fourier's contribution made a relatively late arrival.

III. MOORE AS A PIONEER: A MISTAKEN GUESS CONNECTING FOURIER AND ECONOMICS

Moore seems to be the first economist to deal with Fourier in an explicit and well-developed way.[4] His 1914 book brings a complete section on "The Use of Fourier's Theorem" (pp. 6-13); in a second book, Moore (1923, pp. 42-46) includes a topic on "data and method" that again uses Fourier.

The structure of the first book, *Economic cycles: their law and cause*, reveals his main point: a chapter on "cycles of rainfall"; a second one on "rainfall and crops"; an intermediate chapter on "the

4 Bachelier (1900) is mentioned by Narasimhan (1999, p. 133) as a first use of Fourier in economics, namely for an investigation on stock prices. This reference would lead to another branch in our genealogy shown in Figure 1, as Mandelbrot (1963, 1987) mentions Bachelier and links Fourier to a modern conception of complexity.

law of demand"; and the final chapter, "the mechanism of cycles". In the latter, Moore (1914, p. 127) is very clear: "[t]he fundamental, persistent cause of the cycles in the activity of industry and of the cycles of general prices is the cyclical movement in the yield of crops". And the causes of such movements in the yield of crops could be explained by meteorological causes. Therefore, Moore relies on one key link: rainfall and its cycles as a "persistent cause of the cycles of the crops" (p. 57).

Moore's line of investigation explains why Schumpeter (1939, p. 176; 1954, p. 1133) classifies him under the heading "harvest theories". Schumpeter establishes a connection between Moore and William Stanley Jevons, who is mentioned by Moore (1923, p. 110) as having "attempted to work out [John F.W.] Herschel theory of the synchronism of sunspots and the yield of crops". This reference is important because it might hint toward how Moore first learned about Fourier. Jevons's elaborations on sunspots, crops and cycles has for a long time been known amongst economists. He published two articles in *Nature* in 1878 and 1879 on the relationship between commercial cycles and sunspots which, given the prestige of the author and the journal, might have been a relevant source for Moore.

Furthermore, Moore might have known about Schuster's works either through Jevons's articles or his subsequent book (Jevons, 1884). In the 1878 text, Jevons states that "Dr. Schuster [Schuster, 1872] has pointed out a coincidence between good vintages and minima of sun-spots which can hardly be due to accident" (p. 36).[5] In the same passage, Jevons reveals how important the subject was at that time: "...and the whole controversy about the connection of Indian famines with the sun-spot period is of course familiar to all readers of *Nature*". This is true: browsing the 1877 issues (vol. 46) of *Nature*, it is possible to verify the relevance of the discussion on the relationship between sunspots, rainfall and agriculture. This might explain Jevons's (and Moore's) interest in this subject.

Vivid discussions on sunspots and meteorological issues would continue into the 20th century, with great efforts looking for regularities and periodicities in these phenomena, including the development of new research tools. For these investigations, Schuster developed the periodogram, a technique which uses Fourier series to uncover hidden periodicities. Moore (1914; 1923) applied this technique to business cycles, reason why Schumpeter (1939, p. 166) sees him as a pioneer in this subject.

Schuster's (1898) periodograms facilitate the use of harmonic analysis to investigate periodicities.[6] Basically, it organizes the data being processed in a Fourier transform and delivers a graphic presentation of $f(t)$. Periodograms identify the existing cycles and show the duration of its constituent periods. For example, "[t]he periodogram of sunspots would show a 'band' in the neighborhood of a period of eleven years, while the periodogram of tides would have a line coincident with the lunar month" (Schuster, 1898, p. 25)."[7]

Moore (1914; 1923) uses periodograms to define the duration of rainfall cycles (Figure 10). Once the periodicity of rainfall is determined, the next step would be to investigate its correlation with

---

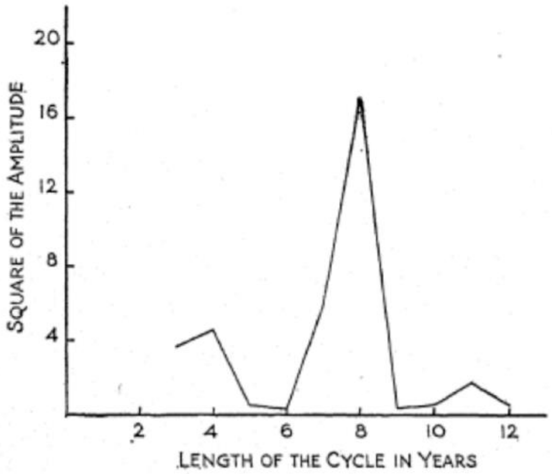
[5] H. S. Foxley, editor of Jevons's *Investigations of Currency and Finance*, includes a letter from Schuster in an Appendix (Jevons, 1884, pp. 332-333), in which Schuster criticizes other findings on the periodicities of sunspots.

[6] According to Brillinger ([1981] 2001, p.9), "the taking of the Fourier transform of an empirical function was proposed as a means for searching hidden periodicities in [George G.] Stokes (1879). Schuster (…), in order to avoid the annoyance of considering relative phases, proposed the consideration of the modulus-squared of the finite Fourier. He called this statistic the periodogram. His motivation was also the search for hidden periodicities".

[7] Shumway and Stoffer (2011) mention Schuster and hint toward the usefulness of his periodograms to modern time series analysis.

economic data. Moore (1914, pp. 46-48) then correlates cycles of rainfall with cycles of crop yields.[8] However, it is worth noticing that his use of harmonic analysis and periodograms did not lead to an investigation on wave composition, despite the fact that periodograms allude to the presence of different cycles as part of the same phenomenon.

FIGURE 10
Moore's periodogram applied to rainfall cycles



Source: Moore (1923).

One important finding for Moore was his ascertainment of eight-year cycles (1923, pp. 70-102). While Jevons (1884) credits his eleven-year economic cycles to sunspot periodicities, Moore attributes his eight-year cycles to "the planet Venus in its eight-year periodic motion with respect to the Earth and the Sun" (1923, p. 102).

Moore's work on the relationship between the Sun, planets, and economic cycles, mediated by rainfall and crop yields, followed a research line of the late 19th century which, as will be discussed in the next section, would soon face fierce criticism and be retrospectively seen as a mistaken guess.[9] However, this mistaken guess made at least two important developments possible. The first one is the search for new uses of harmonic analysis and for new tools such as the periodogram. The second one is the development of a strong opposition to correlations derived from the simple juxtaposition of two coincident time series, what would lead to new research questions in statistics and econometrics (e.g., Yule, 1926; Slutsky, 1927).

---

[8] Beveridge (1921; 1922) presented a similar elaboration.

[9] Michael Polanyi (1958, p. 144) differentiates between "scientific guesses which have turned mistaken" and "unscientific guesses which are not only false but incompetent"; Moore's falls into the former category.

IV. VAINSHTEIN ON MOORE: FOURIER ARRIVES AT THE MOSCOW CONJUNCTURE INSTITUTE

Vainshtein was a physicist and mathematician interested mainly in statistical economics, a subject in which he would be trained at the Moscow Commercial Institute, later renamed as Plekhanov Institute. He worked on a variety of issues, such as agricultural economics, systems of national accounts, linear programming, history of economic thought, and translations of academic works. He became well-known for his contributions on national income and wealth (Vainshtein, 1969), although his legacy as an economist of the 1920s working for the development of the new socialist economy has not gone unnoticed (Campbell, 2012).

Vainshtein served as a senior consultant at the MCI, alongside Slutsky and Nikolay Nikolaevich Shaposhnikov, a figure better known for his role in the Russian synthesis between classical political economy and marginalism (Allisson, 2015). Vainshtein had previously worked at the Office for Agricultural Economics of the People's Commissariat of Agriculture (*Narkomzem*) and was associated with the Timiryazev Agricultural Academy, what might have been a steppingstone leading to Kondratiev's invitation to work at the MCI.[10] Vainshtein remained at the MCI between 1923 and 1928, being "strongly involved in all the work of the Conjuncture Institute" (Campbell, 2012, p. 425), for which he would suffer long jail and exile sentences after the Institute became a target of Stalinist repression.

Overshadowed by his prolific scientific production, Vainshtein's reviews of Moore's work are usually not mentioned amongst his main contributions. However, they sparked an interest in harmonic analysis within the ranks of the MCI, given its inclusion in formal research projects on economic cycles led by Slutsky and others. The first of them was *Novaia Teoriia Ekonomicheskikh Tsiklov* ("New Theory on Economic Cycles"), an article published in the first issue of *Voprosy Koniunktury* ("Questions of Conjuncture"), an irregular series of the MCI dedicated to methodological and theoretical expositions (Vainshtein, 1925).[11] In the following year, Vainshtein would publish an extended appraisal of Moore's work in a collection organized by the Research Institute for Agricultural Economics of the Timiryazev Agricultural Academy (Vainshtein, [1926] 2010).

Firstly, Vainshtein presents Moore's main proposition of eight-year cycles in rainfall fluctuations and their strong relationship with fluctuations in harvest yields, as well as raw material cycles for the same period, which would constitute key economic cycles caused by non-economic factors and, hence, intensely affect general price cycles:

> Hence, we get the following coherent reasoning by Prof. Moore: cyclic and strictly periodic fluctuations of meteorological factors cause fluctuations primarily in the production of goods, depending on climatic and environmental conditions, and in general cause similar fluctuations in the production of raw materials going into industrial processing. These are the main (*generating*) economic cycles. Fluctuations in the production of raw materials are

[10] The MCI itself was created by the Council of the Professors of the Timiryazev Agricultural Academy (Kondratiev, 1925).

[11] The other publication of the MCI was *Ekonomicheskii Biulleten Koniunkturnogo Instituta* ("Economic Bulletin of the Conjuncture Institute"), focused on the production of economic data and indicators.

> correspondingly reflected in the prices of these goods, transmitted according to the law of production costs, and also in the prices of final products. Cyclical price fluctuations are, therefore, already derived from the main economic cycles; they are strictly periodic and can be explained by one reason (Vainshtein, 1925, pp. 165-166, translated by the authors).

Vainshtein then makes a thorough exposition of how empirical series can have their cyclicity investigated by means of what he calls the "Fourier method", according to which "any periodic motion, however complex it may be, can be decomposed into a series of simple harmonic oscillations" (p. 167). Moreover, Schuster's periodogram is presented as a criterion to probabilistically distinguish between "true" and "random" cycles.

Moore's search for mathematically strict periodicities (as opposed to Jevons's average values derived from different periods) is systematically questioned by Vainshtein. Although one cannot deny its possibility, he highlights the lack of evidence from different perspectives, such as the shortcomings in the empirical data provided by Moore, especially in the face of diverging results obtained in other studies, and the legitimacy of statistical conclusions based on the use of periodograms (e.g., the inobservance of other likely periods or Moore's controversial removal of the secular trend prior to the application of the harmonic analysis). Similar criticism is given for Moore's results on raw material production and price cycles.

Furthermore, Vainshtein dismisses Moore's hypothesis for the cause of the eight-year cycles, for whom periodic changes in the position of Venus in relation to Earth and the Sun – either due to its influence on solar radiation on its way to Earth or to its direct effect as a source of magnetic fields – would account for changes in meteorological and, consequently, economic conditions. There would have no available knowledge in theoretical physics to support such claims. In addition, while the period of Venus's movement is strictly eight years long, those of rainfall and crop yields are only approximate figures. Vainshtein also weighs that, even if we accept such a strict relationship, there are non-meteorological phenomena which influence the cyclical fluctuations of agricultural productivity: improvements in land cultivation, soil conditions, crop composition, etc. Moore's method would be of little help in the consideration of these elements.

Overall, Vainshtein acknowledges Moore's arguments in favor of the relationship between rainfall, harvest yields, and prices of agricultural products, although the remains skeptical on Moore's elucidation of the mechanism through which raw materials cycles influence prices and the production of final goods. His main reproach to Moore's works relates to the "absence of an economic explanation for the mechanism of influence in the interconnections he establishes, an explanation which could not be given using his research method" (1925, p. 179). The formal statistical analysis, to Vainshtein, must be aided by an economic appraisal of the issue, requiring a detailed analysis of the dynamics of the development of an economy. As an attempt to find the root cause of economic phenomena, Moore's approach is unsuccessful. However, his theory is deemed as "elegant in form, lean in content, and rich in statistical material" (p. 179). With these statements, Vainshtein recognizes the merits of harmonic analysis for the study of economic cycles, despite the overstretched, naïve, and sometimes illogical inferences Moore seems to make based on his own results. While Vainshtein seems to understand the potential and limitations of the "Fourier method" in terms of explanatory power, Moore's ill-conceived use of harmonic analysis in order to develop theories such as those correlating planetary movements and economic cycles is easy prey for his sharp mathematical understanding of the method.

In his second and expanded review of Moore's books, Vainshtein ([1926] 2010, p. 265, translated by the authors) warns about an intrinsic limitation of the harmonic analysis:

> But there is a very significant consideration of a mathematical nature which forces the economist to be very careful when using the method of harmonic analysis to study economic phenomena. (…) [T]he values of the components of a static series are *completely independent* from each other. In reality, the complete independence of individual members of the series from each other rarely takes place. One of the founders of this method, Prof. Schuster, on whose work Moore has relied, in his first article on harmonic analysis indicates that, even in the case of meteorological data, we are not dealing with independent values.

Economic phenomena (e.g., prices and production output) would entail even stronger "inertial" regularities over time. This observance implies that the elements of the Fourier series do not present the same probabilities, and the likely effect is usually a decrease in the amplitude of shorter periods. Unlike Beveridge and G. Udny Yule, Moore would have completely ignored how this condition affected his results.

Vainshtein's approach to harmonic analysis raises some questions in the context of the MCI. Why was he interested in this subject matter? In intellectual terms, it could be seen as a continuation of his earlier works on agricultural prices. Alternatively, the originality and peculiarity of harmonic analysis within economic science, as well as the then international outlook of the MCI, might point to Kondratiev and other senior colleagues as active supporters of this initiative.[12] According to the latter, the MCI provided a fruitful atmosphere for the exchange of ideas: "the Institute carries on another kind of research work requiring a greater period of time. These are the researches of individual members, although the results and the conclusions are also discussed collectively" (Kondratiev, 1925, p. 323). To Barnett (1998, p. 11), "Kondratiev worked very closely with his consultants on specific issues". In addition, Kondratiev, who had first conceived his long waves hypothesis a few years earlier (Kondratiev, [1926] 1935), and Vainshtein, the second-in-command at the MCI[13], shared the view in which an economic explanation for the mechanisms of influence in the generation of cycles was absolutely necessary.[14]

Another question relates to Vainshtein's influence over Slutsky's well-known 1927 article. Despite the different approaches which would be adopted, on the one hand, by Vainshtein and Kondratiev, and, on the other hand, by Slutsky, the fact that the application of harmonic analysis to

---

[12] The international outlook of the MCI resembled that of Kondratiev himself, who sought to establish an exchange with leading research centers in different countries. An example of such interactions is Kondratiev's visit to the newly founded National Bureau of Economic Research in the end of 1924. On the other hand, in the following year Vainshtein would, for unknown reasons, not be given permission to go on a similar business trip to Germany, France, and Italy (Barnett, 1998, p. 95).

[13] According to Barnett (1998, p.214, fn.35), "Vainshtein occupied a position above the other two consultants, as he earned an extra 75 rubles per month in the form of a bonus."

[14] Vainshtein ([1926] 2010, p. 279) also posed this question for the purpose of forecasting: "we must point out that, even if one assumes that one of the components of the general dynamic process is the true harmonic oscillation of certain elements or the influence of cosmic factors, this is far from enough for forecasting purposes. The perturbing forces in economic and social events are so significant that any attempt to make an economic forecast based on only periodicity or due to cosmic factors is doomed to failure in advance. To obtain a true picture of the world around us for the near future, it is necessary to combine various types and methods of forecasting, using, along with the formal mathematical and statistical method, an economic analysis of causal relationships and dependencies."

business cycles was a given at the MCI might have caught the latter's attention upon his arrival in 1926. This possibility was also raised by Barnett (2006, p.425), although we believe, based on a careful assessment of Vainshtein's reviews of Moore's works, that he might have been less critical of Slutsky's article than suspected. To that effect, we would give more credit to Vainshtein's legacy on this specific matter than Barnett (2005, p. 112) does, as a detailed appraisal of harmonic analysis applied to business cycles could thitherto be found in none of the other published documents of the MCI.

V. SLUTSKY, KONDRATIEV, AND THE END OF THE MCI

Slutsky arrived at the MCI in January 1926, after 13 years working at the Kiev Institute of Commerce. By that time, he was already reluctant to engage in economic research, although his intellectual interest in such issues would never completely disappear.[15] In his autobiography, Slutsky affirms that the new planned socialist economy of the Soviet Union offered problems for which he would neither care nor be equipped to tackle (Sheynin et al., 2010). In this sense, it seems strange that he would choose (not without some hesitation) to relocate to the MCI, even though he rightly believed that his mathematical and statistical skills would be deservingly acknowledged in his new position. In a letter to Ladislaus Bortkiewicz dated May 16th, 1926, he explained that his move to Moscow had been due to problems with the Ukrainian language (a compulsory decree came into effect stating that the Ukrainian language was to be used in all lectures held at academic institutions of that republic, and Slutsky was not a fluent speaker) (Sheynin et al., 2010, letter n. 7). Thus, the MCI would rather be a solution at the personal level than an opportunity to make the transition toward non-economic applications of mathematics and statistics. Yet, while his first three articles as a member of the MCI (all published in 1927) deal with economic subjects, his subsequent contributions, a total of eight pieces dated from 1927 to 1930, focus on mathematics and statistics.[16]

Furthermore, as shown above, Slutsky was far from being the only economist at the MCI who was familiar with harmonic analysis and its use in the study of business cycles. According to Chetverikov ([1959] 2010, p. 254), "soon upon his arrival there, he was attracted by some scientific investigations (the study of cycles in the economy of capitalist countries). (…) E.E. [Slutsky] became an active participant of this research, and, as usual, surrendered himself to it with all his passion". A letter to his wife written in March 1926 reads as follows:

> *I am head over heels in the new work, am carried away by it. I am almost definitively sure about being lucky to arrive at a rather considerable finding, to discover the secret of how are wavy oscillations originating by a source that, as it seems, had not been until now even suspected. Waves, known in physics, are engendered by forces of elasticity and rotary movements, but this does not yet explain those wavy movements that are observed in social phenomena. I obtained waves by issuing from random oscillations independent one from another and having no periodicities when combining them in some definite way* (p. 255, italics in the original).

[15] Slutsky's efforts to add new materials for the 1937 translation of his 1927 article is an example of his lasting academic interest in economics (Barnett, 2011, p. 89).

[16] The complete list of Slutsky's articles while serving as a consultant at the MCI can be found in Barnett (2011, p. 82).

Hence, Barnett (2006, p. 425) correctly raised the possibility that "Slutsky had first realized that such statistical techniques might have been relevant to economic matters through exposure to such discussion in the Conjuncture Institute" (e.g., Kondratiev's long waves, Timofei Ivanovich Rainov's association between economic cycles and scientific breakthroughs, Vainshtein's review of Moore's books, Nikolai Sergeevich Chetverikov's connection between prices and harvests etc.). Conversely, Slutsky's background and brilliancy would stand out at the MCI. His 1927 article draws from a wide variety of sources, which go way beyond the usual references in Soviet economics (e.g., Tugan-Baranovsky). To an extent, "the impetus for the new approach of the 1927 article had originated from Slutsky himself" (Barnett, 2006, p. 417).[17] In any case, it remained little known until its English translation and publication in *Econometrica* in 1937. A few years later, it would be already deemed as a "classical contribution" (Allen, 1950).

Slutsky's 1927 article successfully applies harmonic analysis to support his claim on the random causes of cyclical patterns. He showed that an oscillatory series resembling actual business cycles could be generated from a random series by taking a moving sum or difference, regardless of the presence of weights or repetition in the process. Thus, instead of looking for ways to decompose such cycles, Slutsky sets forth the possibility to build similar cycles using random series, and by doing so he brings into economics this other side of Fourier's contribution (the inverse transform for the purposes of wave composition). The implications of Slutsky's results are at least two-fold: on the one hand, actual business cycles might be explained as "statistical artefacts" and, hence, can do without explanations in terms of economic mechanisms (the "spurious" or "statistical" interpretation); on the other hand, the inability to distinguish between randomly generated cycles and those with underlying periodic causes, given the fact that both might exist as manifestations of economic life, needs to be taken into account when studying cyclical economic phenomena (the "real" or genuine" interpretation) (Barnett, 2006, pp. 412-413).[18] Slutsky summarizes his 1927 article as follows:

> *The summation of random causes generates a cyclical series which tends to imitate for a number of cycles a harmonic series of a relatively small number of sine curves. After a more or less considerable number of periods every regime becomes disarranged, the transition to another regime occurring sometimes rather gradually, sometimes more or less abruptly, around certain critical points* ([1927], 1937, p. 123, italics in the original).

Although Slutsky acknowledges Schuster's periodograms as a means to discover hidden periodicities, he deems that the lack of independence between the components of empirical series (as noted by Vainshtein in his review of Moore's book) renders the use of Schuster's method unreliable, as "casual" waves could be interpreted as bearing strict regularity (also given the fact that empirical series are not long enough to provide proof of such strict regularities).[19] From these premises, Slutsky – inspired by Yule (1926) in relation to what would be dubbed as the Slutsky-Yule effect – formulates the

[17] The use of harmonic analysis is also present in Slutsky (1929; 1934).

[18] If one could gather data for a sufficient amount of time, then, according to Slutsky, it would be possible to distinguish between randomly generated cycles and those with periodic causes, as the former go through "regime changes" – structural breaks leading to new regularities with different parameters – while that was not the case for the latter. However, Slutsky does not say how long must the series be for that to be the case.

[19] On a footnote, Slutsky mentions how further developments in Schuster's method can overcome this difficulty.

question: "is it possible that a definite structure of a connection between random fluctuations could form them into a system of more or less regular waves?" ([1927] 1937, p. 106). Instead of searching for the source of regularity in "the superposition of regular waves complicated only by purely random components" (p. 107), why not seek an explanation in random causes? This would have been the main course of action in promising fields within physics and biology, in which order (regularity) arises from chance or randomness precisely due to its arbitrary nature.

After 1927, Slutsky would not carry on with economic applications of his sophisticated analytical toolkit, preferring to work on mathematical and statistical theory or its implications to the natural sciences. Neither would his colleagues take up the challenge. It seems the technical staff of the MCI was becoming increasingly aware of the dangers of being associated either with economic issues of political relevance or, more precisely, with the figure of Kondratiev. According to Barnett (1995, p. 417), "members of the Conjuncture Institute must have realised that their personal positions within the Soviet economic bureaucracy were inextricably connected to the fate of the actual markets which they studied and analysed in such an enthusiastic and original manner." That was certainly so for Slutsky, according to his letters to his wife discussing the personal risks associated with his work (Chetverikov, [1959] 2010).

Another reason for the interruption of this line of research at the MCI could be the overly advanced mathematical content of Slutsky's work. Barnett (2011, pp. 91-92) raises the question whether most researchers at the MCI would possess the necessary mathematical skills to contribute to Slutsky's study on random causes of periodic waves. After all, Slutsky was not a clear political target; in fact, he was not arrested after the end of the MCI, to the contrary of other highly ranked scholars. Whatever the case, a look into Kondratiev's writings in the context of his role as head of the MCI is quite insightful regarding the level of interest in harmonic analysis of business cycles among the members of the MCI.

Kondratiev (1989, pp. 416-450) wrote a report about the activities of the MCI ("Report on works of the Conjuncture Institute in 1926-1927 and prospects for 1927-1928"), including a topic on "Work on the methodology of the study of conjuncture conditions and dynamics" (p. 430). Kondratiev explained that "[w]ork of this kind was carried out partly in connection with the formulation of problems on the study of patterns in the development of conjuncture conditions and dynamics, partly in connection with the development of issues related to the analysis of the current conjuncture".[20] He mentions the leaders of the group working on these issues:

> "[W]orks on techniques for studying cyclic waves: in this area, the mathematical part of Slutsky's study "Summation of random causes as the source of cyclic processes" (printed in *Voprosy Koniunktury*, v. 3) has started the work on the economic application of the methods he found for explaining cyclic waves. This also includes the study of the question "On the application of harmonic analysis to conjunctural cycles" – a work coming under the direction of N.S. Chetverikov and E.E. Slutsky (p. 430).

[20] The first topic on this subject – "Work on the methodology of the study of conjuncture conditions and dynamics" – referred to "research on levelling techniques to find out the main trends within their dynamics. It includes work on the application of the least squares method and the method of mechanical levelling. They have been ongoing for several years under the leadership of N.S. Chetverikov" (Kondratiev, 1989, p. 430). Kondratiev also mentions Chetverikov's assistance after explaining the need to use "more complex tools to process statistical series" (Kondratiev, [1926] 1998, p. 29). These comments show how Chetverikov's work, as described in Kondratiev's report (1989, p. 430), was integrated with investigations on long waves.

At this stage, the "Fourier method" discussed and applied at the MCI seems to have fulfilled two different but related roles. Firstly, following Moore's use of periodograms, harmonic analysis led to the development of tools for investigating the duration and amplitude of existing cycles. Secondly, with Slutsky (1927), harmonic analysis would be a tool for theoretically building a cycle. Thus, the MCI pioneered this two-sided use of Fourier's elaboration on cycles. This is additional evidence that harmonic analysis had found at the MCI a friendly host environment, and that a very fruitful interaction between those two uses might have been in motion at that specific time and place.

Toward the end of the 1920s, freedom of thought would quickly vanish in Stalin's Soviet Union, especially in fields and subjects associated with public policy. Kondratiev's defense of pro-market industrialization, his disagreements with Gosplan officials and their views on economic development, and his involvement in agrarian issues – based on which he was accused of being one of the leaders of the "Working Peasants Party" – would cost him his position at the MCI in 1928, his personal liberty in 1930, and his life in 1938.[21] After his dismissal, the Institute would be transferred from the auspices of the People's Commissariat of Finance to the Central Statistical Administration, and many of its senior scholars would also be sacked. The MCI was officially closed at the end of 1929; however, in its short existence it managed to become one of the worldwide leading research centers in the study of market conjuncture (Barnett, 1995; 2011).

VI. KUZNETS, FRISCH, AND THE REVIVAL OF HARMONIC ANALYSIS IN THE 1960S

The implications of Slutsky's results mentioned above, the "real" or genuine" interpretation and the "spurious" or "statistical" interpretation, would be respectively formulated by two of Slutsky's well-known contemporaries, Simon S. Kuznets and Ragnar A. K. Frisch. Their interpretations would contribute to the development of different lines of research throughout the 20th century, enduring to this day.

Kuznets was a member of Wesley C. Mitchell's National Bureau of Economic Research (NBER). He could read Russian, so his views on Slutsky's article were already published in 1929. His corroboration of Slutsky's main thesis is accompanied by a discussion on a so-called "inverted inference", i.e., whether the fact that a summation of random causes yields a periodic wave also means that such waves are primarily a result of this type of causes and can be explained by their characteristics. Kuznets (1929, p. 274) is skeptic: "[s]uch inference, of course, cannot carry with it any certainty, since we are never certain of all the contingencies of other hypotheses to which the formations of cycles in economic data may be reduced." He acknowledges the significance of the influence of random factors, albeit it is not an exclusive one. Other conditions would need to be assessed (e.g., how economic mechanisms act upon probability distributions), a hint toward the desirability of an integrated approach hitherto rarely to be seen in the literature.

---

[21] Klein (1999) discusses how, within the debates on business cycles, Slutsky's formal mathematics of stationary stochastic processes might be seen as a bigger threat to orthodox Marxism and, in turn, to the Soviet establishment than Kondratiev's empirical assessment of the material base and political superstructure of the economy. However, it is more likely that Kondratiev's political involvement in pressing issues, as opposed to Slutsky's complex mathematical theories, would be deemed as an imminent threat to Stalin. The international outlook and exchanges which were so esteemed by members of the MCI might also have played a role in its downfall.

For Kuznets, treating cycles as the mere summation of random events leads to the conclusion that they do not require hypotheses on underlying, recurrent economic reasons; all that is necessary is to explain how a determined cumulative process acts within the economy in order to produce a determined cyclical oscillation. In this respect, Kuznets alludes to how

> institutional explanations of business cycles deal mainly with the economic forces that make for cumulation, with forces that explain why a given random event is not immediately cancelled by an opposite reaction but allowed to exert its influence for some time to come, an economic counterpart of the statistical mechanism of a moving average (1929, p. 275).

This is what Barnett (2006, p. 417) calls the "Kuznets elucidation" of Slutsky's 1927 article, which would be in favor of a "real cyclical process occurring in an economy modelled as a moving average procedure itself". It is to be contrasted with the "Frisch elucidation" (p. 419), focused on "spurious cycles generated by some types of statistical manipulation of data series". Frisch (1931) pointed to the econometrically relevant aspects of Slutsky's work, mainly how to use his results to eliminate spurious cycles, instead of considering the potential role of the summation of random causes in the formation of business cycles.[22]

Klein (1999, p. 156) attributes Slutsky's "subsequent reputation for discovering spurious qualities of random functions" to Yule (1926) and "Frisch's (1931) spin on Slutsky's work", i.e., to the "Frisch elucidation".[23] As a result, "Slutsky's name is now more closely associated with the generation of spurious cycles than with his advocacy of the summation of random disturbances as an explanation for actual cyclical phenomena" (p. 157).

While both elucidations can be somehow traced back to Slutsky ([1927] 1937), the "Frisch elucidation" seems to be a misinterpretation or at least an unintended consequence from his point-of-view. His belief in the actual manifestation of economic cycles as a result of the effects of linear combinations of random elements is clear in the text. For example, one of his premises is "[s]uppose we are inclined to believe in the reality of the strict periodicity of a business cycle (…)" ([1927] 1937, p.106). Whereas his mention to Yule's work on nonsense correlations might be seen as legitimization of the "Frisch elucidation" (Barnett, 2006), it is unlikely that Slutsky would agree that his findings only imply that there is a problem with the manipulation of time series.

Slutsky's employment of harmonic analysis to the study of business cycles would have a long-lasting impact. After the initial responses of his contemporaries, this part of his legacy would be revived in the second half of the 20th century by theorists associated with real business cycles. At roughly the same time, modern spectral analysis would redeem the contributions of Beveridge, Moore and others in relation to the decomposition of cycles by means of the Fourier transform.

---

[22] Two years later, however, Frisch (1933) would also contribute to the real interpretation of Slutsky's work, while attempting to explain how the summation of random causes can take place in an economic system (Barnett, 2006). Moreover, by means of his correspondence with Slutsky, he might have played a role in the revisions done by Slutsky for the 1937 translation article in *Econometrica*, given the adoption of elements of Frisch's model, such as periodic vibrations and random disturbances (Barnett, 2011). For a more recent account bearing on Frisch's contribution, see Louçã (1997).

[23] To Klein (1999), the works of Slutsky, Yule and Frisch would also have caught the attention of statisticians to the possibility that random components might be important elements of the data-generating process, and not only errors of observation.

In the 1980s, the real business cycle analysis of Edward C. Prescott, Finn E. Kydland and Robert E. Lucas Jr. (Kydland and Prescott, 1990; Prescott, [1986] 1998) would draw directly from Slutsky. According to the Royal Swedish Academy of Sciences (2004, p. 26), "[m]ore generally, Kydland and Prescott's approach to business cycle analysis is linked to early articles by Frisch (1933) and Slutsky ([1927] 1937), which showed how an economy's adjustment to a sequence of random shocks can give rise to cyclical fluctuations reminiscent of business cycles." Apart from the terminological change from "causes" to "shocks", what Barnett (2006) attributes to Frisch's (1933) use of "erratic shocks" as the energy which maintains oscillatory waves, they would have sought to determine how a specific type of random cause (e.g., productivity changes) leads to cycles.

Alternatively, economic applications of the so-called modern spectral analysis have appeared in the 1960s, including analyses of business cycles (e.g., Hatanaka, 1963; Nerlove, 1964; Granger, 1966; Harvey, 1975). They are based on wave decomposition in the frequency domain using the Fourier transform. According to Granger and Engle (1983, p. 93), the "forerunners of modern spectral analysis were Fourier series fitting techniques, which assumed a series contained important deterministic cycles of known period, and the periodograms, which assumed the same model but the components had periods that needed to be determined". These authors mention Moore (1914) and Beveridge (1921; 1922) as the first applications of the method, and Davis (1941) as a forgotten link between Fourier series, periodograms, and modern approaches. However, Granger and Engle deem that spectral analysis would have been "too successful", in the sense that an "unlikely multiplicity of cycles" were yielded by the basic model (p. 93), and, hence, for some time neglected in favor of Yule's autoregressive moving average models. Time-domain and frequency-domain models would then coexist since the 1960s, as the problems with the latter were surpassed by means of smoothing techniques to estimate spectra.[24] Cargill (1974) offers an overview of the early stages of development of spectral analysis in economics.[25]

The application of spectral analysis to business cycles would be further developed, for instance, in the works of Klotz and Neal (1973) and Van Ewijk (1982), which attempt to assess a myriad of proposed cycles (e.g., Kondratiev cycles, Kuznets swings, Juglar and Kitchin cycles). More recent examples include A'Hearn and Woitek (2001), Korotayev and Tsirel (2010), and Tsirel (2012), not to mention contributions to the methodology itself (e.g., Baxter and King, 1995).

These recent developments show how far harmonic analysis has come within economic science. Even so, the lines of research and working groups on modern spectral analysis do not exhaust the list of economists who can be portrayed as Fourier's intellectual heirs. Other scholarly branches exist, as, for example, that of Bachelier (1900) leading to Mandelbrot (1963; 1987) and his approach to the concept of complexity.[26] Therefore, harmonic analysis of economic cycles presents itself as a research agenda which overcomes Schumpeter's skeptical views.

---

[24] A large research project on the economic applications of spectral analysis was initiated at Princeton in 1959 by Oskar Morgenstern, with the support of John von Neumann. The supervisor was John W. Tukey, and staff included Michio Hatanaka, Clive Granger, and Michael Godfrey, among others (Granger and Engle, 1983; Cargill, 1974).

[25] Cargill treats as synonyms the terms "harmonic analysis", "Fourier analysis", "periodogram analysis", and "spectral analysis", all meaning frequency-domain analysis. Spectral analysis would be the most common term since the 1960s. Here, the term "harmonic analysis" is used in a broader sense, including wave composition and decomposition as they evolved based on Fourier transforms.

[26] See fn. 4.

VII. CONCLUDING REMARKS

Fourier's development of harmonic analysis has a history in economics, especially in the analysis of business cycles, which evinces how useful his contributions can be in different circumstances. The account of such a history revolves around harmonic analysis as a tool to decompose cycles as well as to build them. Moreover, it can be assumed that these uses entail an untapped potential for further analyses of long-term economic dynamics.

The Schumpeterian tradition has improved the investigation on cycles from a theoretical thread which goes back to Kondratiev. Currently, a combination between views on general purpose technologies (GPT) (Bresnahan and Trajtenberg, 1995; Rosenberg, 1998; Bresnahan, 2010) and long cycles may lead to a fruitful research field aiming to understand business cycles as a mixture and superposition of different GPTs which define the long-term dynamics of capitalism. This combination between GPTs and Kondratiev waves may suggest a more accurate image of economic dynamics than Schumpeter's elaboration on the three-cycle dynamics based on Kitchin, Juglar and Kondratiev cycles (Schumpeter, 1939). The ability of harmonic analysis to decompose a wave into its constitutive parts or cycles may be a starting point in this regard.

Alternatively, the multifarious determinations of capitalist economic dynamics calls for an approach which includes and integrates variables beyond Schumpeterian elements, i.e., beyond strictly technological driving forces. Kondratiev ([1928] 1992, pp. 423-424) listed different interpretations of business cycles, which might be revisited to help define a course of action combining of those various potential causes. The relationship between technology and monetary and financial elements was noticed by Schumpeter (1911) and more recently by Perez (2002). Geopolitical considerations and the role of power, both in political and in military terms, were summarized by Arrighi (1994). The diversity of factors which can shape different phases of capitalism might be exemplified by the different approaches to the periodization of capitalism organized by Albritton et al. (2001). These different causes of perturbations in the economy can be assessed by means of the techniques developed by Fourier.

Our suggestion on the importance of harmonic analysis for contemporary investigations on capitalist dynamics, which include a broader analysis of the economy as a complex system, as hinted by Mandelbrot (1987), brings with itself the relevance of the MCI in relation to its pioneering application of Fourier's mathematical conceptions in economic dynamics. The role played by Fourier in such contemporary issues also reveals the long-lasting contributions of the creative institute headed by Kondratiev.

REFERENCES

AGHION, P.; AKCIGIT, U.; HOWITT, P. (2014). What Do We Learn from Schumpeterian Growth Theory? In: AGHION, P.; DURLAUF, S. (eds.) *Handbook of Economic Growth*. Amsterdam: Elsevier, pp. 515-563.

A'HEARN, B.; WOITEK, U. (2001). More International Evidence on the Historical Properties of Business Cycles. *Journal of Monetary Economics*, v. 47, pp. 321-346.

ALBRITTON, R.; ITOH, M.; WESTRA, R. ZUEGE, A. (eds.) (2001). *Phases of Capitalist Development: Booms, Crises and Globalization*. New York: Palgrave.

ALLEN, R.G.D. (1950). The Work of Eugen Slutsky. *Econometrica*, v. 18, n. 3, pp. 209-216.

ALLISSON, F. (2015). *Value and Prices in Russian Economic Thought: A Journey Inside the Russian Synthesis, 1890–1920*. New York: Routledge.

ARRIGHI, G. ([1994] 2009). *The Long Twentieth Century*. London: Verso.

BACHELIER, L. (1900). Théorie de la Spéculation. *Annales Scientifiques de l'École Normale Supérieure*, 3e série, tome 17, pp. 21-86.

BARNETT, V. (1995). A Long Wave Goodbye: Kondrat'ev and the Conjuncture Institute, 1920–28. *Europe-Asia Studies*, v. 47, n. 3, pp. 413-441.

BARNETT, V. (1998). *Kondratiev and the Dynamics of Economic Development: Long Cycles and Industrial Growth in Historical Context*. Hampshire: Macmillan Press.

BARNETT, V. (2005). *A History of Russian Economic Thought*. New York: Routledge.

BARNETT, V. (2006). Chancing an Interpretation: Slutsky's Random Cycles Revisited. *European Journal of the History of Economic Thought*, v. 13, n. 3, pp. 411-432.

BARNETT, V. (2011). *E.E. Slutsky as Economist and Mathematician: Crossing the Limits of Knowledge*. London/New York: Routledge.

BAXTER, M.; KING, R.G. (1995). *Measuring Business Cycles Approximate Band-Pass Filters for Economic Time Series*. NBER Working Paper Series, n. 5022.

BERGLES, A.E. (1988). Enhancement of Convective Heat Transfer: Newton's Legacy Pursued. In: LAYTON, E.T., Jr; LIENHARD, J.H. (eds.) *History of Heat Transfer: Essays in Honor of the 50th Anniversary of the ASME Heat Transfer Division.* New York: The American Society of Mechanical Engineers, pp. 53-63.

BEVERIDGE, W.H. (1921). Weather and Harvest Cycles. *The Economic Journal*, v. 31, n. 124, pp. 429-452.

BEVERIDGE, W.H. (1922). Wheat Prices and Rainfall in Western Europe. *Journal of the Royal Statistical Society*, v. 85, n. 3, pp. 412-475.

BRESNAHAN, T. (2010). General Purpose Technologies. In: HALL, B.; ROSENBERG, N. (eds.) *Handbook of the Economics of Innovation* (vol. 2). Amsterdam: North Holland, pp. 761-791.

BRESNAHAN, T.; TRAJTENBERG, M. (1995). General Purpose Technologies: 'Engines of Growth'? *Journal of Econometrics*, v. 65, n. 1, pp. 83-108.

BRILLINGER, D.R. ([1981] 2001). *Time Series: Data Analysis and Theory* (Classics in Applied Mathematics, vol. 36). Philadelphia: Society for Industrial and Applied Mathematics.

CAMPBELL, R.W. (2012). *A Biobibliographical Dictionary of Russian and Soviet Economics*. New York: Routledge.

CARGILL, T.F. (1974). Early Applications of Spectral Methods to Economic Time Series. *History of Political Economy*, v. 6, 1-16.

CHETVERIKOV, N.S. ([1959] 2010). The Life and Scientific Work of Slutsky. In: SHEYNIN, O.; RAUSCHER, G.; WITTICH, C. (eds.) *Evgeny Slutsky Collected Statistical Papers*. Berlin: Oscar Sheynin. Available at https://sheynin.de.

CRUM, W. (1923). Cycles of Rates on Commercial Paper. *The Review of Economics and Statistics*, v. 5, n. 1, pp. 17-29.

DAVIS, H.T. (1941). *The Analysis of Economic Time Series*. Bloomington, IN: Principle Press.

FOURIER, J.B. (1822). *Théorie Analytique de la Chaleur*. Paris: Firmin Didot.

FOURIER, J.B. (1828). Recherches Expérimentals sur la Faculté Conductrice des Corps Minces Soumis à l'Action de la Chaleur; et Description d'Un Nouveau Thermomètre de Contact. *Annales de la Chimie et de Physique*, pp. 291-315.

FREEMAN, C.; LOUÇÃ, F. (2001). *As Time Goes By: From the Industrial Revolutions to the Information Revolution*. Oxford: Oxford University Press.

FRISCH, R. (1931). A Method of Decomposing an Empirical Series into Its Cyclical and Progressive Components. *Journal of the American Statistical Association* [Suppl.: Proceedings of the American Statistical Association], v. 26, n. 173, pp. 73-78.

FRISCH, R. (1933). *Propagation Problems and Impulse Problems in Dynamic Economics* (Publikasjon n.3). Oslo: Universitetets Økonomiske Institutt.

GRANGER, C.W.J. (1966). The Typical Spectral Shape of an Economic Variable. *Econometrica*, v. 34, n.1, pp. 150-161.

GRANGER, C.W.J.; ENGLE, R. (1983). Applications of Spectral Analysis in Econometrics. In: BRILLINGER, D.R.; KRISHNAIAH, P.R. (eds.) *Handbook of Statistics* (vol. 3). Amsterdam: Elsevier, pp. 93-109.

HARVEY, A.C. (1975). Spectral Analysis in Economics. *Journal of the Royal Statistical Society*. *Series D* (*The Statistician*), v. 24, n. 1, pp. 1-36.

HATANAKA, M. (1963). *A Spectral Analysis of Business Cycle Indicators: Lead-Lag in Terms of All Time Points*. Econometric Research Program, Princeton Research Memorandum n. 53.

HAWKINGS, S. (2007). *God Created the Integers: The Mathematical Breakthroughs that Changed History*. Philadelphia/London: Running Press.

JEVONS, W.S. (1878). Commercial Crises and Sun-spots. *Nature*, v. 19, pp. 33-37.

JEVONS, W.S. (1879). Sun-spots and Commercial Crises. *Nature*, v. 19, pp. 588-590.

JEVONS, W.S. (1884). *Investigation on Currency and Finance*. London: Macmillan.

KLEIN, J. (1999). The Rise of "Non-October" Econometrics: Kondratiev and Slutsky at the Moscow Conjuncture Institute. *History of Political Economy*, v. 31, n. 1, pp. 137-168.

KLOTZ, B.P.; NEAL, L. (1973). Spectral and Cross-Spectral Analysis of the Long-Swing Hypothesis. *The Review of Economics and Statistics*, v. 55, pp. 291-298.

KONDRATIEV, N.D. ([1922] 2004). *The World Economy and Its Conjunctures During and After the War*. Moscow: International Kondratiev Foundation.

KONDRATIEV, N.D. (1925). The Conjuncture Institute at Moscow. *The Quaterly Journal of Economics*, v. 39, n. 2, pp. 320-324.

KONDRATIEV, N.D. ([1926] 1935). The Long Waves in Economic Life. *Review of Economic Statistics*, v. 17, n. 35, pp. 105-115.

KONDRATIEV, N.D. ([1926] 1998). Long Cycles of Economic Conjuncture. In: MAKASHEVA, N.; SAMUELS, W.; BARNETT, V. (eds.) *The Works of Nikolai D. Kondratiev* (vol. 1). London: Pickering and Chato, pp. 25-60.

KONDRATIEV, N.D. ([1927] 1998). The Moscow Institute for Economic Research and its Work. In: MAKASHEVA, N.; SAMUELS, W.; BARNETT, V. (eds.) *The Works of Nikolai D. Kondratiev* (vol. 4). London: Pickering and Chato, pp. 272-281.

KONDRATIEV, N.D. ([1928] 1992). La Dynamique des Prix des Produits Industriels et Agricoles. In: FONTVIELLE, L. (ed.) *Les Grands Cycles de la Conjoncture*. Paris: Economica, pp. 377-492.

KONDRATIEF, N.D. (1989). *Problemy Ekonomicheskoi Dinamiki* [Problems of Economic Dynamics]. Moscow: Economica.

KOROTAYEV, A.V.; TSIREL, S.V. (2010). A Spectral Analysis of World GDP Dynamics: Kondratieff Waves, Kuznets Swings, Juglar and Kitchin Cycles in Global Economic Development, and the 2008–2009 Economic Crisis. *Structure and Dynamics*, v. 4, n.1. Available at https://escholarship.org/uc/item/9jv108xp.

KUZNETS, S. (1929). Random Events and Cyclical Oscillations. *Journal of the American Statistical Association*, v. 24, n. 167, pp. 258-275.

KYDLAND, F.; PRESCOTT, E. (1990). Business Cycles: Real Facts and a Monetary Myth. *Quaterly Review Federal Reserve Bank of Minneapolis*, v. 14, n. 2, pp. 3-18.

LOUÇÃ, F. (1997). *Turbulence in Economics: An Evolutionary Appraisal of Cycles and Complexity in Historical Processes*. Cheltenham: Edward Elgar.

LOUÇÃ, F. (1999). Nikolai Kondratiev and the Early Consensus and Discussions About History and Statistics. *History of Political Economy*, v. 31, n. 1, pp. 169-205.

MAKASHEVA, N.; SAMUELS, W.; BARNETT, V. (eds.) (1998). *The Works of Nikolai D. Kondratiev*. London: Pickering and Chato.

MANDELBROT, B. (1963). The Variation of Certain Speculative Prices. *The Journal of Business*, v. 36, n. 4, pp. 394-419.

MANDELBROT, B. (1987). Towards a Second Stage of Indeterminism in Science. *Interdisciplinary Science Reviews*, v. 12, n. 2, pp. 117-127.

MAUREY, B. (2019). Fourier, One Man, Several Lives. *European Mathematical Society Newsletter*, September, pp. 8-20.

MITCHELL, W.C. (1927). *Business Cycles: The Problem and its Setting*. Chicago: NBER.

MOORE, H.L. (1914). *Economic Cycles: Their Law and Cause*. New York: Macmillan.

MOORE, H.L. (1923). *Generating Economic Cycles*. New York: Macmillan.

NARASIMHAN, T.N. (1999). Fourier's Heat Conduction Equation: History, Influences and Connections. *Proceedings of the Indian Academy of Sciences - Earth and Planetary Sciences*, v. 108, n. 3, pp. 117-148.

NERLOVE, M. (1964). Spectral Analysis of Seasonal Adjustment Procedures. *Econometrica*, v. 32, n. 3, pp. 241-286.

PEREZ, C. (2002). *Technological Revolutions and Financial Capital*. Cheltenham: Edward Elgar.

PLOSSER, C. (1989). Understanding Real Business Cycles. *Journal of Economic Perspectives*, v. 3, n, 2, pp. 51-77.

POLANYI, M. (1958). *Personal Knowledge: Towards a Post-Critical Philosophy*. London: Routledge.

PRESCOTT, E.C. ([1986] 1998). Theory Ahead of Business Cycle Measurement. In HARTLEY, J.E.; HOOVER, K.D.; SALYER, K.D. (eds.) *Real Business Cycles: a Reader*. London: Routledge.

ROSENBERG, N. (1982). How Exogenous Is Science? In: ROSENBERG, N. *Inside the Black Box: Technology and Economics*. Cambridge: Cambridge University Press, pp. 141-160.

ROSENBERG, N. (1998). Chemical Engineering as a General Purpose Technology. In: HELPMAN, E. *General Purpose Technologies and economic growth*. Cambridge, MA/London: The MIT Press, pp. 167-192.

ROYAL SWEDISH ACADEMY OF SCIENCES (2004). *Finn Kydland and Edward Prescott's Contribution to Dynamic Macroeconomics: The Time Consistency of Economic Policy and the Driving Forces Behind Business Cycles*. Stockholm: Royal Swedish Academy of Sciences. Available at https://www.nobelprize.org/uploads/2018/06/advanced-economicsciences2004.pdf

SCHUMPETER, J. ([1911] 1983). *The Theory of Economic Development*. London: Routledge.

SCHUMPETER, J. (1939). *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process* (2 vols.). New York/London: McGraw-Hill.

SCHUMPETER, J. (1942). *Capitalism, Socialism, and Democracy*. New York: Harper and Brothers.

SCHUMPETER, J. (1954). *History of Economic Analysis*. London: Allen & Unwin.

SCHUSTER, A. (1872). Sun-spots and the Vine Crop. *Nature*, v. 5, p. 501.

SCHUSTER, A. (1898). On the Investigation of Hidden Periodicities with Application to a Supposed Twenty-Six-Day Period of Meteorological Phenomena. *Terrestrial Magnestism*, v. 3, n. 1, pp. 13-41.

SHEYNIN, O.; RAUSCHER, G.; WITTICH, C. (eds.) (2010). *Evgeny Slutsky Collected Statistical Papers*. Berlin: Oscar Sheynin. Available at https://sheynin.de.

SHUMWAY, R.; STOFFER, D. (2011). *Time Series Analysis and Its Applications – With R Examples.* New York: Springer.

SLUTSKY, E.E. (1927). Slozhenie Sluchainykh Prichin, kak Istochnik Tsiklicheskikh Protsessov [The Summation of Random Causes as the Source of Cyclic Processes]. *Voprosy koniunktury*, v. 3, pp. 34-64.

SLUTSKY, E.E. (1929). Sur l'Extension de la Theorie de Periodogrammes aux Suites des Quantités Dependentes. *Comptes Rendus de l'Académie des Sciences*, v. 189, pp. 722-734.

SLUTSKY, E.E. (1934). Alcuni Applicazioni di Coefficienti di Fourier al Analizo di Sequenze Eventuali Coherenti Stazionarii. *Giornale dell'Istituto Italiano degli Attuari*, v. 5, n. 4.

SLUTSKY, E.E. ([1927] 1937). The Summation of Random Causes as the Source of Cyclic Processes. *Econometrica*, v. 5, n. 2, pp. 105-146.

STOKES, G.G. (1879). Note on Searching for Hidden Periodicities. *Proceedings of the Royal Society*, v. 29, pp. 122–125.

TATON, R. ([1958] 1995). *La Science Moderne*. De 1450 a 1850. Paris: Quadrige/PUF.

TATON, R. ([1961] 1995). *La Science Contemporaine*. 1/Le XIXe Siècle. Paris: Quadrige/PUF.

TSIREL, S.V. (2012). Kondratevskie Volny v Rossii i Drugikh Stranakh [Kondratiev Cycles in Russia and Other Countries]. In: AKAEV, A.A.; GRINBERG, R.S.; GRININ, L.E.; KOROTAEV, A.V.; MALKOV, S.Iu. (eds.) *Kondratevskie Volny: Aspekty i Perspektivy* [Kondratiev Cycles: Aspects and Perspectives]. Volgograd: Uchitel, pp. 361-372.

VAN EWIJK, C. (1982). A Spectral Analysis of the Kondratieff Cycle. *Kyklos*, v. 35, n. 3, pp. 468-499.

VAINSHTEIN, A.L. (1925). Novaya Teoriya Ekonomicheskikh Tsiklov [New Theory on Economic Cycles]. *Voprosy konyunktury, 1*, pp. 163-179.

VAINSHTEIN, A.L. (1969). *Narodnyi Dokhod Rossii i SSSR: Istoriia, Metodologiia Ischisleniia, Dinamika* [National Income in Russia and USSR: History, Calculation Methodology and Dynamics). Moscow: Nauka.

VAINSHTEIN, A.L. ([1926] 2010). Urozhainost, Meteorologicheskie i Ekonomicheskie Tsikly, Problema Prognoza (po Nekotorym Noveishim Rabotam) [Crop Yields, Meteorological and Economic cycles, Forecasting Problems (According to Some New Works)]. In KLIUKIN, P.N. (ed.) *Izbrannye Trudy Kondratevskogo Koniunkturnogo Instituta*. Moscow: Ekonomika. pp. 242-279.

WILSON, E. (1934). The Periodogram of American Business Activity. *The Quaterly Journal of Economics*, v. 48, n. 3, pp. 375-417.

YULE, G.U. (1926). Why Do We Sometimes Get Nonsense-Correlations Between Time-Series? – a Study in Sampling and the Nature of Time-Series. *Journal of the Royal Statistical Society*, v. 89, n. 1, pp. 1-63.