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notes on countertendencies to the fall of the rate of profit, simulation models and metamorphoses of capitalism

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MARX, PROFITS AND FRACTAL PROPERTIES:
notes on countertendencies to the fall of the rate of profit, simulation models and metamorphoses of capitalism [*]

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SUMÁRIO

ABSTRACT

What really matters to understand capitalist dynamics in the long run are the countertendencies to the tendential fall of the rate of profit. For researchers in 2015, with all historical and statistical information on capitalist dynamics (not available to Marx, Schumpeter or Bain), capitalism can be seen as an engine for the creation of countertendencies to the fall of the rate of profit. Since classical political economy (Smith, Ricardo, Mill) and Marx the behavior of the rate of profit is a key subject of investigation, that has been also investigated by Schumpeter, evolutionary economist and modern industrial economics. Contemporary debates on the rate of profit would have three advantages vis-à-vis previous rounds of this long-lasting discussion: 1) the MEGA Project has provided more information Marx's works; 2) there are data on the long-term behavior of the rate of profit; 3) there are new tools to investigate the logic of capitalism as a complex system - a dialogue with physics is useful for this analysis. This paper combines different approaches and methods: a short review of the history of economic thought, lessons from economic history, data analysis of the movements of the rate of profit and a simulation model to test our understanding of those movements - a model based on two very simple rules, inspired on an interpretation of Marx's insights about the contradictory interaction between the tendency and the countertendencies to the fall of the rate of profit. These different approaches and methods organize this paper.

Key words: the rate of profit, Marx, MEGA2 Project, complex systems, metamorphoses of capitalism

RESUMO

Debates contemporâneos sobre os movimentos da taxa de lucro tem três vantagens em relação às duas rodadas anteriores: 1) o Projeto MEGA2 que disponibiliza novas informações sobre as obras de Marx; 2) dados sobre o comportamento a longo prazo da taxa de lucro; 3) novas ferramentas para investigar a lógica do capitalismo como um sistema complexo - um diálogo com a física. Diferentes abordagens e métodos organizam este texto: a primeira seção revisa a literatura sobre esses debates; a segunda avalia lições da história econômica do capitalismo; a terceira apresenta uma avaliação empírica do comportamento da taxa de lucro; a quarta apresenta um modelo de simulação baseado em duas regras simples, inspirado em uma interpretação de ideias de Marx. A conclusão discute esses resultados e sugere uma agenda de pesquisas.

PALAVRAS-CHAVE: queda da taxa de lucro, Marx, Projeto MEGA2, propriedades fractais, metamorfoses do capitalismo

JEL Classification: P16, O33, B510

INTRODUCTION: THE RATE OF PROFIT AND ECONOMIC DYNAMICS

Since classical economists, the behavior of the rate of profit is a key subject of academic investigations. A consensus might be found among them: over time, the profit rate falls. Investigating the movements of profits, Adam Smith (1776, I. 9.1) puts forward that "[t]he increase of stock, which raises wages, tends to lower profit" (I.9.2). A tendency to lower profits is identified, for different reasons, by Ricardo (1821) and John Stuart Mill (1848). Marx did try a synthesis of these debates about what he at least once thought of as the "most important law of political economy" (Rosdolsky, 1968, p. 319).

Classical economists also identified factors that raised them, however. Adam Smith might be the first to indicate such counteracting factors: "The acquisition of new territory, or of new branches of trade, may sometimes raise the profits of stock, and with them the interest of money, even in a country which is fast advancing in the acquisition of riches" (I.9.12).

Ricardo (1821) mentions both the "tendency" of profits to fall and the role of "improvements in machinery" to "check" "this tendency, this gravitation" – "[t]he natural tendency of profits then is to fall; for in the progress of society and wealth, the additional quantity of food required is obtained by the sacrifice of more and more labor. This tendency, this gravitation as it were of profits, is happily checked at repeated intervals by the improvements in machinery, connected with the production of necessaries, as well as by discoveries in the science of agriculture which enable us to relinquish a portion of labor before required, and therefore to lower the price of the prime necessary of the laborer" (6.29).

John Stuart Mill (1848), in turn, is original in his explicit introduction of a special role for "counteracting circumstances" (IV.4.16), "counter-agencies" (IV.4.19) or "counter-forces which check the downward tendency of profits" (IV.4. 25). Those "counteracting circumstances" are "waste of capital in periods of over-trading and rash speculation" (IV.4.17); "improvements in production" (IV.4.19); "cheap commodities from foreign countries" (IV.4.21); and "the perpetual overflow of capital into colonies or foreign countries" (IV.4. 25).

Marx, in the *Grundrisse*, discusses the "Tendenz der Profitrate zu fallen" (Mega II.1, p. 435, 532, 621-625, 627, 629, 635 and 636), but also factors that block that fall – reductions in existing taxes (the state is here), a decrease in ground rents, new branches of production (p. 624) and monopolies. In the *Theories of Surplus-Value*, Marx reviews classical economists' elaboration of the tendency of the rate of profit to fall, with a special focus on Ricardo's theory (Mega II.3.3, pp. 1063-1093).

In the manuscripts for *Capital* Volume III (Mega II.4.2, pp. 285-340), Marx presents factors that define the tendency of the rate of profit to fall. He later contrasts those with a list of six counteracting factors (the numbers are from Marx): "1) *Erhöhung des Exploitationsgrad der* Arbeit" (Mega II.4.2, p. 302) (title introduced by Engels: "more intense exploitation of labor"); "2) *Herunterdrücken des Arbeitslohns unter seinen Werth*" (Mega II.4.2, p. 305) (Engels's title: "reduction of wages below their value"); Marx's topic 3 has no title, the main subject might have been summarized in the second paragraph, which connects "der *Entwicklung der Industrie*" with "*Depreciation des vorhandnen Capitals*" (Mega II.4.2, lines 32-33, p. 305) (Engels's title: "cheapening of the elements of constant capital"); "4) Die *relative Surpluspopulation*" (Mega II.4.2, p. 305) (Engels's title: "the relative surplus population"); Marx's topic 5 also has no title, but begins as follows: "5) *Soweit der auswärtige Handel*" (Mega II.4.2, p. 306) (Engels's title: "foreign trade"); finally, in topic 6, with no title from Marx (Mega II.4.2, p. 309), Marx deals with "share capital", without using this word (Engels's title: "the increase in share capital").[1]

Marx distinguishes his approach from Ricardo's, taking the tendency and countertendency as a starting point: "these two aspects involved in the accumulation process cannot just be considered as existing quietly side by side, which is how Ricardo treats them; they contain a contradiction, and this is announced by the appearance of contradictory tendencies and phenomena" (Marx's words, without editorial interference from Engels: Mega II.4.2, p. 323, translation Marx, 1894, p. 357). This dialectics is also a step forward vis-à-vis Mill's counteracting factors – remember that the latter's final analysis leads to a stationary state (Mill, 1848, IV.6.3). Marx's explorations, with a dialectics between the tendency and the countertendency of the rate of profit to fall, are his major contribution to this debate.

The decline of the profit rate is a problem not only for classical economists and Marx. Schumpeter (1911) suggests a dynamics wherein profits are created by the introduction of a successful innovation and are eroded by competition from imitators. Schumpeter is an author that connects classical economists, Marx and contemporary theories, especially the evolutionary approach (Nelson and Winter, 1982) and modern industrial economics (Bain, 1951; Caves and Porter, 1977; Caves, 1998; Chandler, 1992). The microeconomics of large firms and market structures show how leading firms struggle to preserve profits from existing and potential competitors.

Contemporary transnational corporations have grown using the two routes discussed by Adam Smith in 1776 (Chandler, 1992, p. 83). For Chandler, organizational capabilities "accounted for the long-term persistence of profits by the same players over the decades. Such capabilities and the resulting retained earnings became the basis for their continued growth" (1992, p. 83). In microeconomic terms, it is a huge problem for firms to avoid falling profits; "the long-term persistence of profits" is not an easy task, and not all firms are able to keep them or even survive: the forces of "creative destruction" are restless (Schumpeter, 1942). Capitalist macro-dynamics have microfoundations on the behavior of the rate of profit.

This paper combines different approaches and methods. The first section reviews debates on the fall of the profit rate, including what we may learn with the findings of the MEGA2 Project. The second section searches for lessons from the economic history of capitalism during the last 150 years. Those two sections prepare the third, which presents an empirical evaluation of the long-term behavior of the profit rate – an introductory investigation about its properties as a complex system. Those findings support an interpretation of capitalism as a complex system and suggest specific tools for modeling its long-term dynamics. The fourth section thus presents a simulation model that helps interpret the nature of the long-term movements of the profit rate.

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[1] Engels inserted the expression "so-called dividends" in topic 6. However, Marx mentions "dividends" (line 14, p. 502) in the topic on "*Die Rolle des Kredits in der kapitalischen Produktion*." (MEGA II.4.2, pp. 501-505). In a subtopic – "*III- Bildung der Aktiengesellschaften*" – of this part, Marx explicitly identifies their role as a countertendency (Mega II.4.2, p. 502). This is one connection between the movements of the profit rate and the credit system (Callinicos, 2014, pp. 277-278).

I. A LONG AND UNFINISHED DEBATE

A chapter of the history of economic thought may be written about the long debate on the Marxian elaboration about the tendential fall of the rate of profit. At least three rounds of this debate may be identified, two of which came before MEGA2.

I.1. Pre-MEGA2 deabtes during the 20th century[2]

The first round was triggered by Sweezy's (1942) critique. The author argues that the very process behind the falling rate of profit (the rising organic composition of capital), predicated upon labor-saving technical change and hence growing unemployment, is coterminous with a rising rate of exploitation. The latter would thus not be a countertendency, but rather an integral element of the law. With no grounds to abstractly ascertain the prevalence of either force, only in particular conjunctures could the final effect be known. In light of this, Sweezy proposes a theory that, integrating the state, allows for crises to spring from the rising value composition of capital, decreasing rates of exploitation, disproportionality between sectors or, as he would later highlight, underconsumption.[3]

The second round starts with Okishio (1961), who seeks to break the connection between the introduction of innovations profitable for individual capitalists and a declining average rate of profit. In other words, under the conditions proposed it is suggested that a rising organic composition of capital cannot lead to a declining profit rate, a pillar of the law. Although the best-known proof is in the Okishio theorem (1961), other authors had obtained similar results (e.g., Bortkiewicz, 1907; Moszkowska, 1935; Shibata, 1939). Some dispute the theorem over interpretations of value-theory (e.g., Carchedi, 2009), and the validity of the hypotheses is controversial, as the author himself admits (Okishio, 2000). Its impact on the debate has, nevertheless, been tremendous – even motivating obituaries of the law (Parijs, 1980).

I.2. The status of the "falling rate of profit" after MEGA2

The findings of the MEGA2 Project help promote a better understanding of Marx (Cerqueira, 2014), casting new light on Volume III of *Das Kapital*. Engels had given numerous clues about the state of the manuscripts for Volume III and how difficult his editorial work was for publishing this volume in 1894. Volumes II.4.2 and II.14 of MEGA2 shows the problems Engels confronted in his edition. Further investigation on other unpublished Notebooks about the crisis of 1866, expected in volume

[2] For overviews of the twentieth century debate before MEGA2, see Groll and Orzech (1989), Howard and King (1992) and Parijs (1980).

[3] A later interpretation along these lines is found in Fine and Harris (1979, p. 64), who suggest it would be better called 'the law of the tendency of the rate of profit to fall and its counteracting influences'. Reuten (2004) suggests this leans towards a cyclical view of the rate of profit.

IV.19 of MEGA2, will also be very helpful. The end result might be a much more nuanced, problematic, open-ended Marx: *human, all- too- human*.

The third round of this debate (Carchedi and Roberts, 2013; Kliman et al, 2013; Heinrich, 2013b; Harvey, 2015), triggered by Michel Heinrich's (2013a) researches on Volume III, repositions Marx's elaboration. Heinrich stresses that Marx, after his 1863-1865 Manuscripts (Mega II.4.2), did not return to the topic of the fall of the rate of profit and did let clues about his own doubts about such "law".[4]

This round of the debate suggests, first, that Marx had only general ideas, truly insights, for eventual later developments – Heinrich evaluates that Engels's edition delivered a more elaborated version of those insights. Second, given the overall stage of elaboration of Volume III – a draft, as Heinrich did suggest –, today's researchers may use those insights according to their agenda, but with a clear understanding that it is an unfinished, unedited topic of his elaboration. Nobody can be sure, furthermore, of what Marx would have done if he had prepared a final version of Volume III.

The post-MEGA2 literature puts in perspective Marx's elaboration, stressing how unfinished, how incomplete it was left for Engels – and he had no instructions from Marx on how to proceed. Probably, the post-MEGA2 literature suggests how much was left for later generations of researchers to develop. An indication of the complexity of capitalism, of the difficulties to understand a system always under change – metamorphoses of capitalism do mean changes in the nature of crises.

Heinrich's revision of the Volume III manuscripts and later notes by Marx should be taken into account and incorporated by contemporary researchers that would like to use Marx's insights on the behavior of the rate of profit. Assuming that there is not a "law", there is no reason to discard Marx's insights.

I.3. Marx's insights and the emphasis on countertendencies

Marx's manuscripts of Volume III present another very important insight for the understanding of capitalist dynamics – the role of surplus profit or extra-surplus-value (1894, p. 279) and the profits capitalists gain by using the most modern production techniques. This decisive element of Marx's elaboration on competition was well understood by Schumpeter (1911), who elaborated a complete theory of capitalist dynamics based on a straight relationship between innovation and profits – a translation of Marx's concepts to his own framework.

The struggle between the factors that push the rate of profit down and the counter-acting factors that push it up suggests that the rate of profit is a resultant. A "synthesis of multiple determinants," a variable with multiple causes – a multidimensional variable. Being the result of those multiple causes, the profit rate impacts the dynamics of the whole system, in its ups and downs. Probably a variable that may be under the "reciprocal effects" dynamics: both cause and effect – *Wechselwirkung*.

[4] Without access to the MEGA2 edition, Grolll and Orzech (1989) also advance this argument, based on a note Marx wrote on his copy of *Capital* vol. 1, published in the third German edition.

II. TWO CENTURIES OF CAPITALISM AND THE KEY ROLE OF COUNTERTENDENCIES

Limits to, or metamorphoses of, capitalism? The history of capitalism in the last 150 years show its flexibility, it shows that limits may be overcome and that capitalist dynamics include a strong capacity to introduce changes in the system as a whole. Therefore, metamorphoses of capitalism are part of this dynamics, and today we are in a position to see that they should be taken into account (Furtado, 2002). The huge literature on what is the best periodization of capitalism (Albritton et al, 2001) discusses the role of crises as moments of transition between phases – they have heretofore been associated with the reshaping of capitalism, not with its collapse (Duménil and Lévy, 2002).

Three approaches can be presented as an attempt to understand this complex system called capitalism (Marques, 2014). First, technology is a driving force of change. This feature is illustrated by the approach focused on the technological dimension of capitalist dynamics: the long waves of capitalist development research agenda (Freeman and Louçã, 2001). Second, the leading centers of capital accumulation have changed over time. Those geographical changes are grasped by an approach that incorporates geopolitical struggles, deeply connected to the nature of world money: the systemic cycles of accumulation agenda (Arrighi, 1994). Third, the nature of the locus of capital accumulation – the firm – has been transformed over time, with a systematic expansion towards new sectors and new regions. This feature is identified by the approach focused on the internationalization of capital (Dunning and Lundan, 2008).

What might connect those different and certainly complementary approaches? The movements of the rate of profit.

Technological innovation pushes the profit rate upwards as it opens new sectors for capital accumulation, improves existing ones and, at a microeconomic level, preserves the profit rate of leading firms in oligopolistic structures. Geopolitical changes, transitions of hegemony, mean the flow of capital towards new regions where the profits are higher. The internationalization of capital is a combination of the first two dimensions, as leading firms, supported by strong innovative capabilities, are able to expand productive and innovative capabilities towards new geographic regions and sectors. Multinational firms can be seen as machines to explore the international division of labor in their favor, to increase profits globally – multinationality is a new source of profits. And, as they do this, they change the face of global capitalism (see Ribeiro et al, 2015) – they are agents of the metamorphoses of capitalism.

This investigation has in Grossmann (1929) a relevant contribution, which may be read as a book on the countertendencies to the fall of the rate of profit. This may be seen as a paradox, given that the collapse of capitalism is in the book's title. Nevertheless, Grossmann actually devotes 132 pages to the factors that contribute to the fall of the rate of profit and 166 pages to the countertendencies.[5]

[5] The discussion of Grossmann's elaboration is beyond the subject of this paper. Pannekoek (1934) presents a very interesting critique, focused on politics, labor organization and action, against an automatic and economically determined end to capitalism. He concludes his analysis indicating that "self-emancipation of the proletariat is the collapse of capitalism."

The creation of new countertendencies is very clear after the crisis of 1929: the long list of Grossmann's countertendencies need to be even further extended, with a new role of the state as a strong countertendency to the fall of the rate of profit. This was left for Poulantzas: "state interventions" should be investigated as "countertendencies" to the tendential fall of the rate of profit (1978, p. 199). This opens room for integrating changes in the state (quantitative and qualitative) and metamorphoses of capitalism.

What did we see in the last 150 years of capitalism? First, technological revolutions generating new sectors (chemistry, electricity, combustion engine, electronics, computers, health-related technologies, microelectronics, internet) and transforming health and information into commodities – and, therefore, into new spaces for capital accumulation. Second, the growth of the role of the state in capitalist economies (both quantitatively and qualitatively). Third, the exponential growth of share capital. Fourth, the geographical dislocation of capital, including a change in the leading country (and several small peaks of capital accumulation in important, but not leading, countries – Germany, Japan, China). Fifth, a tremendous geographical expansion of capital, now directed to an invented new region: the Internet and the World Wide Web. The latter were created inside scientific institutions (CERN, Switzerland, at its core), as part of strong innovation systems,[6] and were later appropriated by capital – they constitute a strong countertendency, brought forth by non-market institutions. This list supports the conjecture that capitalism has been an endless creator of countertendencies to the fall of the rate of profit.

III. PROFIT RATE DATA ANALYSIS FOR THE UNITED STATES

Data on the profit rate are not easy to find, process and evaluate; there are, however, important works organizing and discussing it: Duménil and Lévy (1993), Maito (nd), and Basu and Vasudevan (2013).

Maito's paper stresses the falling rate of profit in the long-term, as his graphs with global averages show (see his Figure 8). Nevertheless, there is another phenomenon visible in Maito's graphs – over time, the profit rate sequentially peaks in different key capitalist economies. Maito's (nd) Figure 2 shows how between 1860 and 1920 the UK's profit rate was higher than the USA's, and how Germany's rate was, between 1870 and 1885, higher than that of the UK. Figure 3 in turn shows how Japan's rate of profit between 1950 and 1975 was the highest of all developed countries. Finally, Maito's Figure 7 shows that the rate of profit in China between 1978 and 2007 was far above the "world average".

Maito's data hint at broad international movements that indicate the operation of countertendencies to the fall of the rate of profit – the geographical movements of capital towards new regions. They also show that a strictly national framework is incapable of grasping all movements of the profit rate. They might display an Arrighian sequence: a juxtaposition of data for the UK, the USA and China might suggest a pattern of international movements of the rate of profit. In sum: Maito's (nd)

[6] WWW was proposed in 1989 by Tim Berners-Lee at CERN, Switzerland.

paper may be read as a snapshot of international capital mobility in the long-run. Therefore, the conclusions of the analysis Basu and Manolakos (2013) present for the US economy between 1948 and 2007 may be put in a broader perspective: "the rate of profit declines at a rate of approximately 0.2 percent per annum after controlling for the counter-tendencies" (p. 93). The long-term decline of the rate of profit in the US is confronted by its rise elsewhere.

With this general picture as a background, this section focuses on the data for the USA (Duménil and Lévy, 2015). The USA reached the summit of technological application of science, and of financial organization, together with the largest public sector (in absolute terms) of the capitalist world. Therefore, it is the country in global capitalism where the countertendencies operated more fully.

Figure 1 shows the rate of profit for the USA between 1869 and 2011.[7]

FIGURE 1
Rate of profit in the United States (1869-2011)

Source: Duménil and Levy (2015).

During this period capital accumulation expanded, as measured by a huge increase in USA's GDP: from US$ 98 billion in 1870, it reached US$ 7,394 billion in 1998 – an important change in the scale (size) of that profit producing economy (in 1990 international US$, according to Maddison, 2001, p. 184).[8] The ups and downs of the rate of profit are, of course, related to broad institutional changes that took place in the history of USA, involving both technological revolutions, the New Deal reforms, new roles in the world economy, World Wars, the growth of the state intervention etc.

[7] Duménil and Lévy (2015) define the rate of profit as follows: Profit Rate = (NDP-W*L)/KN; where, NDP is Net Domestic Product (current US$); W is Hourly Wage (current US$), L is Number of Hours Worked, and KN is Net Stock of Fixed Capital (current US$).

[8] Those data have implication for later evaluation. First, because size matters – different scales might be related to properties linked to fractal phenomena. Second, because although the rate of profit may be lower, the mass of profits could be larger.

Given our conjecture – capitalism as a complex system, where the rate of profit is a key variable –, this section processes those data with a decomposition of the cyclical movements of the rate of profit for USA.

The turbulent (in the economic sense) behavior of the rate of profit in Figure 1 looks like a "fractal curve"[9] – compare, for instance, with Ribeiro's (2004, p. 24) Figure 2.4.a. We employ a commonly used tool to analyze this kind of curve: the Fourrier transform (FT). The FT decomposes a function into is constituent frequencies. In more details, the transformation writes the original function as an infinite sum of other periodic functions (cos(...) + *i* sen(...)), each which a different period/frequency, until it sweeps all possible periods/frequency. This kind of decomposition might have implications for debates on long waves research, as a possible quantitative methodology (mathematical tool) to measure their duration and periodicity.

Figure 2 shows the result of applying the FT to the data on the profit rate of the USA.[10] The horizontal axis is the frequency of the periodic function and the vertical one is the coefficient that multiplies this function, which means the weight of the frequency in the original function.

The resulting "power law regression" (y=0.0004*x^-1.0001) in Figure 2 confirms that the profit rate curve (Figure 1) is a fractal, and the exponent near -1 (-1.0001) indicates a complex system behavior (see section IV for a more detailed discussion).

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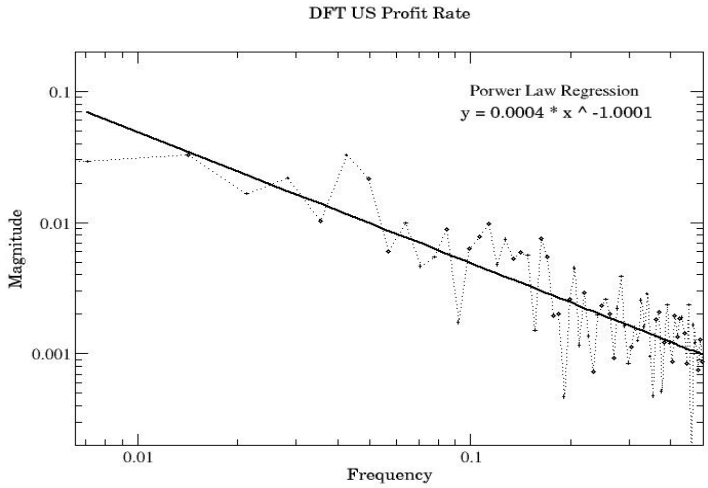
[9] Under this average rate there is a deep process of intersectoral differentiation. Ribeiro et al (2015) disaggregate data for the USA between 2005 and 2014, using the ORBIS database, and show that in 2014, there were 14 NACE main sectors with profit margins below the average, and 4 NACE main sectors above that average. The same differentiation repeats itself in intra-manufacturing sectors (in 2013, 42 out of 160 NACE subsectors were above the NACE C average) and in intra-information-and-communication sectors (in 2014, 6 out of 17 NACE subsectors were above the NACE J average). Firms such as Pfizer, Apple, Google, Amgen and IBM have profit margins above the general average, above their respective main NACE sector average and above their NACE four digit subsector - firms that get super profits based on innovative capabilities.

[10] A Fast Fourrier Transform (FFT) is applied here.

FIGURE 2
USA profit rate data: Magnitude and frequency of the FT decomposition



Source: authors' elaboration.

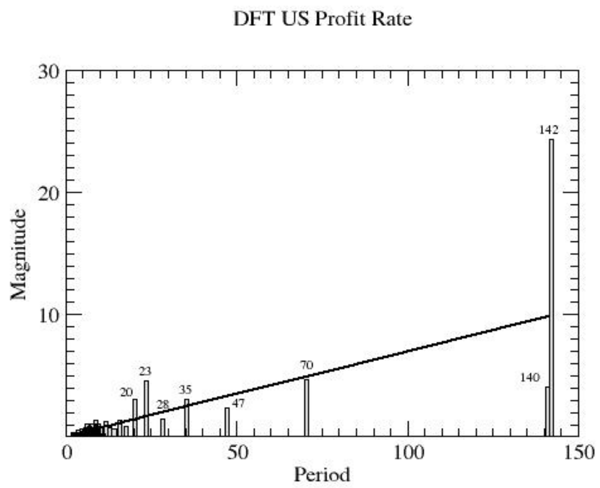
Figure 3 reorganizes the FT results (Figure 2), showing at the horizontal axis the period (the inverse of frequency, $T=1/f$ where T is the period and f is the frequency). This facilitates the visualization of those cycles with a greater weight in the behavior of the profit rate.

Figure 3 shows that, in the decomposition of the data for the USA profit rate, the most important cycles are the 23-, the 20- and the 35-year long ones.[11] The decomposition also shows diverse and overlapping cycles of different temporalities, revealing the combination and superposition of different dynamics. Again, an identification of properties related to complex systems and fractal properties (Ribeiro, 2004, pp. 20-28).

Mandel (1981) and Shaikh (1992) discuss long waves focusing on the rate of profit. On the one hand, Figure 3 shows a 47-year cycle, very close to the Kondratiev cycle – approximately 50 years long, according to Schumpeter (1939). On the other hand, there are four other cycles more important than the 47-year one: in order, the 23-, the 20- cycle, the 35-, and the 70-year ones. This evaluation is not the subject of this paper, but this analysis might contribute to discussions on this subject.

[11] The period of 142 years is disregarded here. This period is in the graph for strictly technical reasons related to the algorithm used to perform the Fourier transform (the FFT) and to the range of the analyzed data.

FIGURE 3
USA profit rate data: Magnitude and frequency of the FT decomposition



Source: authors' elaboration.

Those features (complexity, fractal properties) are indications of what methods may be used for the modeling of such dynamics. This is the topic of the next section.

IV. MODELING COUNTER-TENDENCIES AND LONG-RUN DYNAMICS OF CAPITALISM

Gravitation as a reference for the rate of profit might probably be an approach more related to the Ricardian interpretation (see Ricardo, 1821, 6.29). Our suggestion is that capital might be evaluated by methods that analyze fractal phenomena (Ribeiro, 2004, chapter 2). This suggestion is a starting point for a simulation model of long-term capitalist dynamics, based on the struggle between the factors that push the rate of profit down and the counter-tendencies that push it up.

IV.1 Capital as a nonequilibrium and complex system

Nelson and Winter (1982) pioneered the use of simulation models in economics. Ribeiro et al (2010, section 2) review the literature on the use of simulation models to help investigate capitalist dynamics. Using those tools, Ribeiro et al (2006 and 2010) present preliminary attempts to model world wealth distribution fueled by science and technology. Those models deal with a system that grows over time – GDP per capita increases. This growth follows key institutional changes – the growth of scientific production, a basic element for the implementation of that important Marx's insight on capitalism as a system with "systematic technological application of science". In those models growth is related to changes in the nature of the firm – the driver of technological change, in permanent interaction with the scientific dimension.

Although not directly related to the matter at hand, those works lead us to a search for tools that may be useful to investigate capitalism as a dynamic and complex system that includes a turbulent (in an economic sense, *à la* Arrighi) growth trajectory – that is the question. Rephrasing it: why does our discussion in section II and data in section III lead to a very specific set of simulation models?

In physics, statistical mechanics explain the thermodynamic behavior of large systems (i.e., formed by a large number of particles). That is, it provides exact methods to connect macroscopic thermodynamic quantities (such as temperature, pressure, heat, entropy) to the microscopic behavior (such as velocity, momentum) of the particles that comprise the system.

An equilibrium system (i.e. a system in thermodynamic equilibrium) is the subject of a line of inquiry of statistical mechanics that deals with systems isolated from the rest of the universe. This isolation may be achieved by improvements (thicker walls, adequate material etc.) until the environment has a negligible effect on the system.

This line of statistical mechanics also deals with systems in contact with a large reservoir with constant temperature, so that there is no energy unbalance between them, and/or with constant chemical potential, so that no chemical (number of particles) unbalance obtains between them. It is expected that it will take some time for transient effects to cease. If the system reaches such a state, it is in thermodynamic equilibrium – this is an equilibrium system.

Having defined a system in (thermodynamic) equilibrium, a nonequilibrium system is anything else. Therefore, a system is nonequilibrium when it is neither isolated from the rest of the universe nor in contact with a constant temperature and/or bath of constant chemical potential. A system that is under transient effects before reaching equilibrium is also nonequilibrium. External fields (e.g., electric or magnetic) or perturbations can also induce a nonequilibrium state, as the system is not isolated. A very particular nonequilibrium system's behavior obtains when there is no time dependence (i.e. a steady state situation). In such this state, there is a balance between the injection/subtraction of energy, particles or other characteristics that the system might exchange with the environment, and its properties are thus time-invariant.

How might those concepts be applied to an economic system, since it is not strictly a thermodynamic system as understood in physics?

In general, the economic system is formed by a large number of individuals, firms, banks, institutions, and countries. Those components interact with each other through their consumer preferences, production of goods, trade rules, sales and credit operations, monetary wage relationships, monetary flows, knowledge flows etc.

Individuals enter and leave the system, determining different levels of demand for products and different levels of employment. Firms are created and may survive or die, producing different levels of activity. New technologies lead to new firms, new products, and lower production costs. Taking individuals, firms, banks and institutions as the "particles" forming this system, the economic system can be interpreted as a non-fixed-particles thermodynamic system.

Furthermore, state intervention leads to different rules for the domestic and international production and trade of goods. Wars may lead to different rules of marketing and supply of goods. This dynamic of definition of rules can be compared to the influence of external fields in a thermodynamic system.

Therefore, using these analogies, the economic system can be seen as a nonequilibrium system.

Although there is no concise definition of a complex system on which all scientists agree, available definitions converge on some key points and on a core set of features widely associated to them. Some convergent points are that a complex system is formed by a large number of interacting components and exhibits different organizations at different scales of observation, which lead to different behaviors at different scales.

Bringing the economic system into the frame again, the smallest possible level of disaggregation are the individuals. To have their needs met, they interact with other individuals (and other components of the system) through their consumption preferences. Additionally, some groups of individuals are organized, along specific rules, creating firms. These firms, although an aggregation of individuals, interact with others firms and individuals through rules different from those presiding over the interaction between individuals. The growth of firms and resulting market structures are defined by complex interactions between production costs, the reinvestment of profits and competitive strategies. These complex processes lead (at this aggregation level) to the logic of the market and of capital accumulation. Other forms of organization take place with the formation of governments, institutions and universities. These in turn interact with individuals, firms and between themselves, through specific rules that differ from those of other patterns of interaction. The result (at this higher aggregation) is new institutions, such as laws, labor qualification, job training, public finance, the stock market, national innovation systems etc.

Those "aggregations" are in turn organized and generate countries that interact with each other and with the other (lower level) "aggregations" through particular rules. This leads to the logic of exchange, international trade, and global innovation systems (international flow of knowledge). There are also intermediate aggregations such as local clusters, sectoral innovation systems and so on.

These different organizations at myriad aggregation levels (individuals, firms, institutions, governments, countries, innovation systems...) produce distinct behaviors/features at each scale, all of which influence or contribute to the dynamics of the system. Regarding a capitalist economic system, the rate of profit is determined with the contribution of all those organizations, at different scales (of different orders of magnitude): as a synthesis of multiple determinants, the complex operation and interactions between the factors and counter-acting factors to the fall of the rate of profit is represented by the movements of the rate of profit.

The decomposition of the frequencies that comprise the behavior of a large variety of evolutionary systems at low frequencies exhibits a power law $1/f$ relation (where f equals frequency). This $1/f$ behavior has been observed in a wide range of systems, such as condensed matter system, river discharge, DNA base sequence structure, cellular automata, traffic flow, financial markets and other complex systems with self-organizing elements (Gontis et al, 2004; Gilden et al, 2005; Wong et, 2003; Kaulakys et al, 1998; Yamamoto et al, 1995; Maxim et al, 2005). It is common for parts of such systems to be fractals, and their statistics exhibit scaling (i.e., their behavior is the same in any scale of measurement). The universality[12] of 1/*f* behavior suggests that it does not arise as a consequence of particular forms of interaction, but that it is rather a characteristic signature of complexity and self-organization (due the contribution of different scales of organization to the global behavior of the system).

There are multiple self-organized systems. One of the most popular is the sand pile model (Maslov et al, 1999). This nonlinear model was introduced to explain 1/*f* noise as a result of self-organized criticality.[13] The great interest of this line of research regards the complex behavior that mimics a noise with fractal characteristics. In this model, however, noise originates from nonlinear deterministic interactions.

It is also possible to define a stochastic model exhibiting fractal statistics and 1/*f* noise. Explaining the evolution of complexity into a chaotic regime, the stochastic processes may be used to describe phenomena that occur as random sequences of events, exhibiting scaling of several statistics (Thurner, 1997). A considerable part of real stochastic sequences of events in physics, biomedicine, geophysics and economics are fractal (Kaulakys et al, 1998; Kaulakys et al, 1999; Gontis, 2001; Gontis, 2002).

From this point of view, and in light of the behavior obtained from the empirical analysis, we propose a model based on a stochastic process, representing the struggle between the tendency and the countertendencies to the fall of the rate of profit.

IV.2 A simulation model and its results

We propose a stochastic model that summarizes all mechanisms leading to a higher profit rate, regardless of their specific form and operation, in their final result: the increase (I) of the profit rate. The model likewise summarizes the countertendencies in their final result: the decrease (D) of the profit rate.

The dynamics and interaction the between tendencies and countertendencies to the fall of the rate of profit are thus expressed by two equations. The first (1) conveys the factors that pull the profit rate down, and might be seen as a formalization of Marx's insights in Mega II.4.2 (pp. 285-301), which Engels edited as chapter 13 of *Das Kapital* Volume III. The second (2) expresses the factors that push the profit rate up, and might be regarded as a formalization of Marx's insights in Mega II.4.2 (pp. 301-309), edited by Engels as chapter 14.

(1) $D(t) = \alpha_D * \xi + \beta_D * I(t\text{-}1) * \xi$;
(2) $I(t) = \alpha_I * \xi + \beta_I * D(t\text{-}1) * \xi$;

[12] This means a large group of systems display similar characteristics of interest (e.g., the relation between the frequency of events and their magnitude) regardless of the mechanisms that generate them.

[13] Self-organized criticality suggests that certain systems *inherently* tend towards the critical state wherein fractal characteristics and 1/*f* noise are observed.

$D(t)$ is the intensity of the factors that pull the profit rate down at time (t) (the tendency of the rate of profit to fall). $I(t)$ is the intensity of the factors that push the rate of profit up, at that same moment t (the countertendency). ξ is a random number uniformly distributed between 0 and 1 - and generated as many times as it is used in those equations.

Firstly, the terms $\alpha_{D,I}*\xi$ of equations 1 and 2 vary within the range between 0 and $\alpha_{D,I}$. That is, $\alpha_{D,I}$ is the maximum for those terms. These terms ($\alpha_{D,I}*\xi$) do not explicitly depend upon neither the tendency nor the countertendency for the rate of profit to fall. They are just random terms resulting from all the actions of the system's components and their interactions. The latter might result, in the case of equation 1, in a tendency for the profit rate to fall – obversely, in the case of equation 2, it might result in an upwards countertendency.

Secondly, the term $\beta_D*I(t\text{-}1)*\xi$ (and its correlate $\beta_I*D(t\text{-}1)*\xi$ in equation 2) varies between 0 and $\beta_D*I(t\text{-}1)$. This term is then proportional to the intensity of the factors that pull the rate of profit up, the constant of proportionality being β_D.

Therefore, this term ($\beta_D*I(t\text{-}1)*\xi$) and its analogue in equation 2 integrate, through a crossing, the behavior of the tendencies and the countertendencies for the rate of profit to fall. This crossing means that an increase in the forces that pull the rate of profit down triggers an increase in the upward trend of profit rate. Conversely, when the forces that push the rate of profit up increase, this triggers counteracting forces that reduce the rate of profit. This crossover behavior, called "coupling" in Physics, does not allow the collapse of the system (the rate of profit going to zero or infinity).

This cross interaction might be thought of as a formalization of Marx's insights in Mega II.4.2 (pp. 309-340), edited by Engels as chapter 15, where he explores the interactions between the tendency and its countertendency. In this draft, there are five paragraphs (Mega II.4.2, p. 323), not rewriten by which Engels (see Marx, 1894, p. 357).[14] Marx, after critiquing Ricardo, argues that "the contending agencies function simultaneously in opposition to one another", and then presents three ways in which those contending agencies function "simultaneously" (*Gleichzeitig mit*...). He concludes that they may "exhibit themselves" sometimes "side by side, spatially, at other times one after another, temporally".[15] This adverb (simultaneously) is another support for including those two terms, ($\beta_D*I(t\text{-}1)*\xi$) in equation 1 and its analogue in equation 2. As presented in the introduction, this contradictory interaction between the tendency and countertendency is a major insight from Marx.

Finally, equation 3 expresses the change in the profit rate ($T(t)$ - $T(t\text{-}1)$) as the difference between the intensity of the factors that push the rate of profit up ($I(t)$) and the counteracting factors that pull it down ($D(t)$).

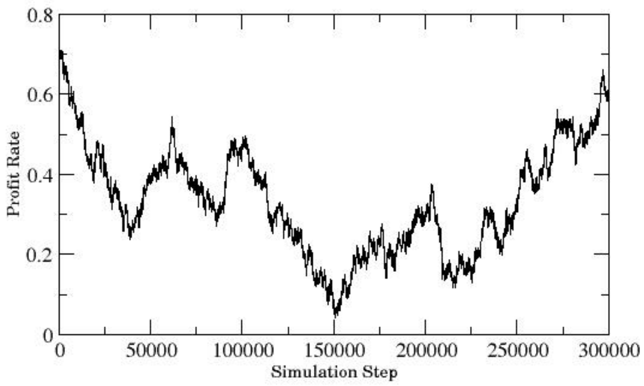
[14] Engels only included Marx's second paragraph (Mega II.4.2, line 10, p. 323) in the first of those five paragraphs in the English edition (1894, p. 357). Therefore, there were 6 paragraphs in Marx's manuscripts and 5 paragraphs in Engels's edition.

[15] An example of Marx's articulation between the tendency and countertendency: "Simultaneously with the fall in the profit rate, the mass of capital grows, and this is associated with a devaluation of the existing capital, which puts a stop to this fall and gives an accelerating impulse to the accumulation of capital value" (Marx, 1894, p. 357; Mega II.4.2, lines 15-18, p. 323).

(3) $T(t) - T(t-1) = I(t) - D(t)$

Figure 4 shows the profile of the movements in the rate of profit generated by a simulation of this model, starting at t=0 with a profit rate equal 0.7, parameters αD=αI=βD=βI=0.2 and running 300,000 iterations. We run simulations varying each parameter (α_D, α_I, β_D and β_I) in range 0.01 at 1.00 with steps of 0.01. The model exhibits no collapse, it is indicative of its robustness and its controlled state due to coupling between trends and countertendency of profit rate falling.

FIGURE 4
Model 1: movements of the rate of profit in a simulation with 300,000 iterations

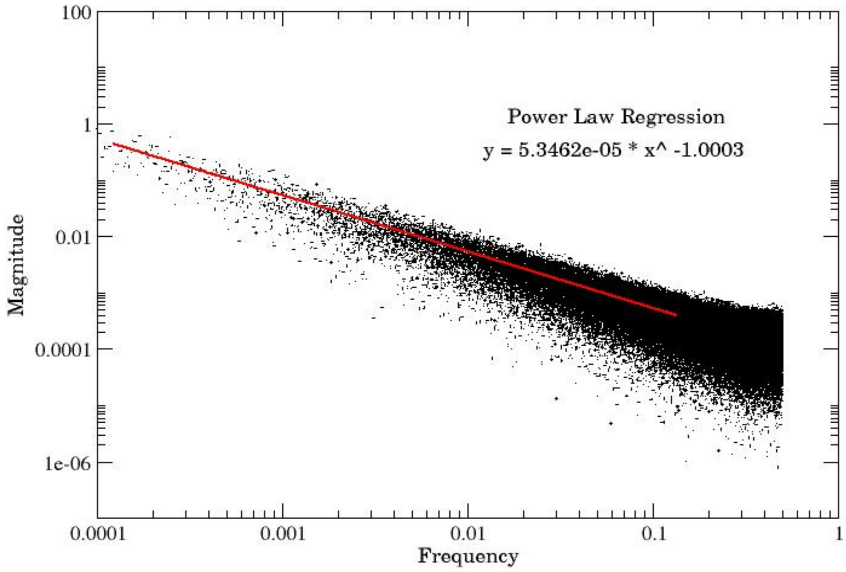


Source: authors' elaboration

The profile of the movements of the profit rate once again looks like a fractal, with widely varying peaks replicated in different scales of the graph. As with the profit rate of the USA (Figure 1), we also apply a FT to analyze the data our simulation generated (Figure 4). The FT results are presented in Figure 5.

The power law regression (y=5.3462e-05*x^-1.0003) presents an exponent near -1 (-1.0003), similar to the case of the USA (Figure 2). The exponent of the power law, frequently called the signature of the relation, is a particularly important result – roughly speaking, it describes the temporal structure of the phenomenon. It identifies, in mathematical terms, the relationship between the frequency of the signals that comprise the result and their intensity. In this case, the intensity is inversely related to the frequency, meaning short-term cycles have a smaller impact on the profit rate. Furthermore, the exponent close to -1 is characteristic of processes with fractal characteristics, a feature observed in the data for the USA. The importance of this power law with -1 exponent is that correspond to the 1/f behavior discussed in section (IV.1). The universality of this result suggests that it does not arise as consequence of particular interaction but it is a characteristic signature of complexity and self-organization. Therefore, it indicates, empirically, that the capital is at a self-organized state, which would be responsible for control of the system not allowing it to collapse or convergence to a fixed profit rate. In other words, this result confirms that the profile of the profit rate movements in our simulation is also a fractal, indicating a complex system behavior.

FIGURE 5
Model 1 data: Magnitude and frequency



Source: authors' elaboration

Figures 4, 5 and our interpretation of their results suggest that the simulation model proposed in this section replicates key features of the data presented in section IV for the USA. Namely, the long-run dynamics of the profit rate displays features related to complexity, self-organization and out-of-equilibrium behavior. In sum: fractals, and not gravity, might be the best analogy from physics to understand capital's long-term dynamics. Capital is fractal.

V. PRELIMINARY CONCLUSIONS

An empirical analysis of data for the USA between 1869 and 2011 has uncovered clues (physically speaking: signatures) of a complex system with self-organizing properties and out-of-equilibrium. Those clues (signatures) were revealed by applying a Fourier transform and identifying that the behavior of the profit rate in the USA can be described by a "power law".

These findings underpin the suggestion of a three-equation simulation model based on two very simple rules, inspired by an interpretation of Marx's insights. The FT analysis of the results of this simulation model also showed a "power law" behavior. Importantly, both "power law regressions"

(Figures 1 and 4) delivered very similar exponents, near -1, which suggests that the simulation model replicates the real long-run dynamics and the temporal structure of the profit rate in the USA. This should not be taken in the sense of the model being able to predict the evolution of the profit rate, or that it is a precisely calibrated model for such rate in the USA. The result rather indicates that this approach to modeling captures essential characteristics of the system, and thus some of its fundamental dynamic properties – namely, out-of-equilibrium behavior, complexity, self-organization and fractal properties.

Those results stimulate two broader reflections.

First, it supports this paper's conjecture that Marx's insights on the behavior of the profit rate as a result of simultaneous and contradictory interactions between a tendency and a countertendency to its fall can help understand the long-term dynamics of capitalism.

Second, this real capitalist dynamics shows fractal properties. It is revealing that an investigation of such a synthetic variable (rate of profit) may unveil features related to complexity, self-organization and out-of-equilibrium behavior. The results of this paper suggest that the dialogue between political economy and physics is fertile, and that it is possible to build a theoretical and empirical dialogue between those two disciplines based on a common understanding of capitalist economies as complex systems.

An agenda for further research might have at least five related lines:

1) a discussion of the theoretical implications of empirical findings of this paper (the nature of Marx's insights, the need for a dialogue with other approaches, a critical evaluation of long waves of capitalist development, the role of crises and their aftermath in reshaping capitalism);

2) a search for other databases and an effort to systematize the available data, including other key countries in the empirical analysis of the profit rate;

3) a fine-tuning of the suggested simulation model, to include other factors in the contradictory interaction between the tendencies and countertendencies of the rate of profit to fall – advancing, when feasible, towards a more global simulation model;

4) an investigation using available data to track the ups and downs of profits before and after the last crisis (2007-2008), with a focus on intersectoral and inter-firm differentiation – an investigation of the turbulent underpinnings of the average rate of profit.

5) further steps in the dialogue between political economy and physics, after the understanding of capitalist economies as evolving complex systems. Subjects such as long waves and systemic (in)stability might be explored in this dialogue.

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