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BEQUEST CHOICES UNDER UNCERTAINTY

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SUMÁRIO

RESUMO

Este trabalho desenvolve uma modelagem para o comportamento dos pais em relação à transmissão de herança de propriedades agrícolas, abordando explicitamente estratégias de consumo e investimento em capital humano especializado em atividades rurais. Podemos distinguir dois tipos de modelagem incorporando ou não um comportamento estratégico dos filhos para com os pais. No primeiro modelo, assumimos que os filhos não se comportam estrategicamente e neste caso o pai escolhe o quanto transferir dependendo do retorno do capital humano de cada filho. Conclui-se sob algumas condições que o filho com habilidades agrícolas produzindo retorno suficientemente grande comparado ao retorno de seus irmãos é mais propenso a receber uma parcela maior da terra. Este resultado aponta para um comportamento não altruísta. O segundo modelo contempla o caso em que cada filho influencia a escolha da herança transmitida prestando serviços aos pais. Exibimos uma simulação numérica em que a estratégia do filho em prestar determinado nível de serviços é suficiente para garantir uma parte maior da herança em um equilíbrio de Nash, independentemente das discrepâncias nos retornos em capital humano.

ABSTRACT

This paper develops a theoretical model for parental behavior regarding land inheritance, explicitly accounting for consumption and land-specific human capital savings strategies. We distinguish two types of modeling: one with and another without strategic behavior. In the first model, we assume that children do not act strategically towards their parent; in this case, the parent chooses how much to bequeath contingent upon each child's return to human capital. We find that the child with the highest return to human capital is more likely to receive a larger share of the land if difference in offspring's returns is large enough. This result points to a non-altruistic behavior. In the second model, we allow for each child to influence parent's optimal choice of bequest by providing services to the latter. We show a numerical example in which the child's strategy for service provision is sufficient to assure that the one providing more service will receive a larger share of the bequest in a Nash equilibrium. This holds, regardless of differences in offspring returns to land-specific human capital.

JEL classification: K11; P51

Keywords: Land inheritance; Altruism; Exchange motive; Nash equilibrium;

1 Introduction

The study of income transfers within a family is of great interest for the economic theory. Private transfers act as a way to improve the well-being of individuals, either smoothing or exacerbating inequality among family members (Becker and Tomes, 1979; Tomes, 1981). It can also affect the effectiveness of government redistribution and intergenerational transmission of wealth depending on agent's motives (Barro, 1974; Cox and Jakubson, 1995; Altonji et al., 1997).

In this paper we model bequests from parents to children by considering a two-period model where parents take into account offspring's well-being by making unilateral transfers of bequests. In addition, we assume that children have different relative skills, yielding different returns to their investment in human capital. Parents then value this random return in their decision on how much to bequeath. Our theoretical model also contemplates the situation where children provide services for their parents. We find theoretical results for the amount bequeathed based on assumptions on preferences and the marginal propensity to save.

The literature on transfers usually contemplates two type of motives: altruistic (Becker, 1974; Becker and Tomes, 1979; Altonji et al., 1997; McGarry, 1999; Stark and Zhang, 2002) and exchange (Bernheim et al., 1985). The first type of approach focuses on how parents use transfers to smooth consumption of needed family members, while the second considers transfers as a way to induce behavior of beneficiaries who provide services of interest. Baker and Miceli (2005) propose a model of land inheritance based on strategic behavior. Their model considers a production function including children's random relative skill and stock of human capital. Differing from typical altruistic models of transfers, parents do not take into account children's well-being. The model considers a simultaneous move game yielding results under a Nash equilibrium and does not include wealth transfers over consecutive periods. The authors find that children overinvest in human capital when compared to equal sharing.

In this work we focus on how parents transfer wealth to their children, mainly in the form of bequests. We develop a theoretical model where agents live for two periods in an environment where wealth transfers between two consecutive periods are available. In the first period, children save to invest in human capital[1] characterized by specific skills, such as training on how to grow perennials or how to raise cattle. In the second period, income from job specialization generates returns to the previous investment in human capital as in Baker and Miceli (2005). Uncertainty in the first period is a consequence of differences among children's relative ability. To model this uncertainty, we use the concept of subjective probability as in Savage (1964), which consists of assigning a subjective probability distribution for the returns in the second period. These probability distributions can differ among children and are of common knowledge within the family.

Parents also transfer wealth between consecutive periods by means of a free risk asset with known return. For simplicity, we assume that family members live for two periods and the parent passes on part of the total wealth[2], consuming the remaining wealth to her death. Without loss of generality, we assume that all bequeatable wealth is represented by *intended bequests*. At last, we assume that children do not invest in the second period, all surviving to be eligible as heirs.

In particular, we consider two different approaches. In one approach, the parent makes transfers in the second period and no strategic behavior from the children arises. The lack of strategic behavior holds because children don't provide any good or service valued by the parent. As an important result, we provide conditions to show that the child with the highest subjective return to human capital receives the largest share of bequest. For this situation to hold, we assume a certain boundary condition on the magnitude of the highest return child's propensity to save. As shown by Altonji et al. (1997), this result contradicts altruistic behavior. In the other approach, each child may provide a good or service for the parent as a way to alter her optimal choice for bequests. As in Baker and Miceli (2005), all family members are in a simultaneous move game in which each child strategically chooses to invest in human capital. In this case, the sharing rule is exogenous. Furthermore, the parent strategy consists in establishing a sharing rule for bequest. In contrast to Baker and Miceli (2005), we consider that children's well-being directly affects the parent's preference. In addition, the sharing rule for the bequeatable wealth is endogenously defined. We exhibit a numerical simulation showing that the lowest return child supply additional levels of services to compensate parent's predisposition to transfer a higher share to the most productive child.

[1]Measured in units of the consumption good and represented by the opportunity cost of alternative paid activities.
[2]Consisting of the total savings from period one, the income from period two, and the bequest

2 Motives for private transfer

The effects of private transfers on individuals' well-being and their relations to public transfers are directly linked to the reasons for why individuals engage in passing on wealth to others within the family. In general, there are two main motives for familial transfers: altruism (Becker, 1974; Altonji et al., 1997; McGarry, 1999) and exchange motive (Bernheim et al., 1985; Cox, 1987). We describe briefly each one of these models and discuss their strengths and limitations.

The original altruism model (Becker, 1974) assumes that an unselfish individual has her utility function directly affected by her own consumption level and by the well-being of others she cares about. The altruistic individual can adjust her behavior if one of her close relatives suffers any change in well-being. The model predicts that altruistic parents try to equalize utility levels across their offspring, making the largest transfer to the least well-off child. Moreover, if altruism is operative, it should lead to the Ricardian equivalence, crowding out the effect of public transfers (Barro, 1974).

Under the traditional altruistic models, an increase in a child's pre-transfer income reduces the parents' marginal utility of making a transfer to that child and thus we expect a negative relationship between the incidence of transfers and the recipient's pre transfer income under the altruism hypothesis. However, Altonji et al. (1997) and McGarry (1999) argue that this relation depends on the kind of transfer. Inter vivos transfers should be negatively associated to child's current income because low-income children are considered to be liquidity constrained. Bequests, on the other hand, hold weak (positive in families who make inter-vivos transfers) or no (families making no inter-vivos transfers) relationship with a child's current income. Stark and Zhang (2002) emphasize that the available empirical evidence is not in agreement with theoretical predictions. They present a model for parents who are equally altruistic towards their children. Their model, however, predicts that parents would actually make the largest transfers to the child whose current income or potential earnings is higher. The key point is that altruistic parents believe that their children are equally altruistic, and as family members are interlinked, indirect transfers between siblings would smooth out direct transfers from parents to children. An optimal choice would be to transfer more to the child with the largest earning potential, because this child would transfer part of the augmented wealth to the less talented sibling. This result holds only if the beneficiary child has a marginal propensity to save sufficiently large relative to her siblings. The authors, however, do not take this fact into account in their theoretical model of transfers.

The exchange motive (Bernheim et al., 1985; Cox, 1987) assumes that parents bequeath as re-payments for services provided by their children at some point. They do so by assigning an ex-ante sharing rule for bequests, directly caring about some actions taken by their children. This direct care establishes the motive for strategic as opposed to nonstrategic influence, as in Becker's rotten kid theorem (Becker, 1974). The model predicts, for instance, that the child who helped the parents more often (with household chores, companionship, or agricultural tasks) would receive the largest transfer or bequest. Transfers profile will be the result of a second best Nash equilibrium among credible beneficiaries in situations where there are at least two of them. Bernheim et al. (1986) model generalized this game problem to more than one period.

Baker and Miceli (2005) develop a theoretical model of land inheritance showing that the knowledge about parents' sharing rules for bequest affects children's investment in human capital under certain circumstances. They argue that in the absence of a formal land market, fixed inheritance rules prevent wasteful competition among potential heirs because they know what their future wealth will be and can maximize their investment in human capital. For example, if the family decides to leave the largest amount of land to the eldest son (primogeniture), he can invest in agricultural training boosting his productivity.

The existence of a formal land market undermines the fixed inheritance rule because the potential heir could sell the land if this would yield a greater return than keeping it (Baker and Miceli, 2005). This behavior is also affected by ecological conditions. More productive land can induce agents to behave as when a land market is absent; in this situation, land selling is not an optimal choice (Hrdy and Judge, 1993).

In our study area, there is evidence of a formal land market. VanWey et al. (2012) suggest a level of change in land ownership due to selling and buying practices, which can affect parent's preferences. Furthermore, land quality varies significantly across our study area. Older farmers own land of better quality and closer to a main road whereas new farmers are pushed to worse quality land, which implies differences in land accessibility and soil quality. Empirical evidence by Carneiro (2001) corroborates the importance of land quality on parents' behavior towards land division. According to the author, farmers in less productive areas follow equal division of land, whereas those in more productive areas follow a fixed division rule. In the next section we present our theoretical model of land inheritance based on the framework of choice under uncertainty.

3 The model

We propose a theoretical model for bequests based on the notion of uncertainty as in Savage (Savage, 1964). We assume that parents assign subjective probabilities for the children's returns, anticipating an exogenous prior for each child's expected return. We develop two different models. In the first (*unilateral transfers*), the child derives utility by her own consumption and the transfer received. In the second (*bilateral transfers*), a service is provided by the child to the parent; this service enters both the parent's and the child's objective function.

3.1 Unilateral transfers

Suppose that a parent has two children and lives for two periods. In the first period she chooses her optimal consumption and investment. In the second period she decides about the amount of wealth to bequeath, consuming the remaining wealth until her death[3]. A subjective prior defined by the parent characterizes the return to each child's human capital. These priors are of common knowledge. This is a sufficient condition to characterize the expected returns of offspring's investment in human capital. Consider[4] $y = (y_p, y_j, y_k)$ and $a = (a_p, a_j, a_k)$ the income and optimal investment profile of parent p and the two children, j and k, respectively. All agents have the same utility function $u : \mathbb{R}_+ \to \mathbb{R}$, representing the benefit for each consumption level. For the sake of simplicity, we assume that there is a single good, with the amount of consumption and savings given in units of this good.

Write t_i the optimal bequest from the parent to the child $i = j, k$. The value function of child i's problem is given by

$$v_i(t_i) = \max\left\{u(c_i) + \beta \int_{\mathbb{R}_+} u((1+r)a_i + t_i)f_i(r)dr : c + a \leq y_i\right\} \tag{1}$$

and the corresponding choices given by

$$(\hat{c}_i(t_i), \hat{a}_i(t)) = \text{argmax}\left\{u(c_i) + \beta \int_{\mathbb{R}_+} u((1+r)a_i + t_i)f_i(r)dr : c_i + a_i \leq y_i\right\} \tag{2}$$

where f_i is the density function inducing the subjective probability of returns and β the intertemporal discount rate characterizing agent's impatience. The parameter β is also related to the marginal saving rate[5]. We assume that f_j has a finite support, that is, there exists $\bar{r}_j > 0$ such that $f_j(r) = 0$ for all $r \geq \bar{r}_j$.

Consider μ the parameter representing parent's utility discount relative to the child's utility. A risk-free asset with returns r_p is available for the parent. The problem of the parent p is given by

$$v_p = \max\{u(c_p) + \beta u((1+r_p)a_p - t_j - t_k) + \mu v_j(t_j) + \mu v_k(t_k)\} \tag{3}$$

where the max is evaluated over all $(c_p, a_p, t_j, t_k) \in \mathbb{R}^4_+$ such that $c_p + a_p \leq y_p$ and $t_j + t_k \leq (1 + r_p)a_p$. In Eq. (3), t_j and t_k represent the transfers for child j and k respectively.

This model predicts larger bequests to the child with the highest subjective expected return to human capital. This prediction contradicts the altruistic behavior. In order to show this, we provide certain conditions on the child's optimal investment strategies and on the densities f_j, f_k such that the child with the highest return must receive the largest transfer.

[3]For the sake of simplicity we use a two-period instead of a three-period model. Ideally, parents should pass on the land after their death. In our model, however, they bequeath in the second period. This assumption does not alter our predictions and numerical solutions.

[4]The model described in Altonji et al. (1997) contemplates uncertainty on income only, not on children's return, as assumed in our model. Our assumption allows us to isolate the direct effect of human capital's return on bequests. This was not done by any of the models revised in the literature.

[5]This concept is not considered in the theoretical literature on private transfers.

To assure this result we establish two assumptions. Assumption (3.1) below is based on two main insights. First, the child with the lowest return always invests a strictly positive amount on human capital under the optimal choice.[6] Last, the minimal present value of the unit good must be higher than the negative saving rate in relation to the transfer. This means that the more productive child must have a propensity to save even when her return is at its highest.

Assumption 3.1. Let a_j, a_k be given by (5). There exits a constant $\bar{a}_k > 0$ such that $a_k(t) \geq \bar{a}_k$ for all $t \in \mathbb{R}_+$. Moreover, we assume that a_j is differentiable and[7] $|a_j'(t)| < 1/(1+\bar{r}_j)$ for all $t \in \mathbb{R}_+$.

Assumption (3.2) states that the expected value of the marginal benefit for the more productive child when she consumes at her maximum in the second period is still higher than the expected value of the marginal benefit for the less productive child when she consumes at her minimum in the second period. Thus, even when consumption is at its maximum for the more productive child, she still has a propensity to save, rendering an expected benefit higher than the one for her sibling.

Assumption 3.2. Consider $\bar{a}_k$ the constant given in Assumption 3.1 and write $y = \max\{y_j, y_k\}$. The subjective densities f_j, f_k satisfy

$$\int_{\mathbb{R}_+} u'((1+r)y + t) f_j(r)dr > \int_{\mathbb{R}_+} u'((1+r)\bar{a}_k + t) f_k(r)dr \text{ for all } t \in \mathbb{R}_+.$$

The following result exhibits certain conditions favoring a non-altruistic behavior. Intuitively, Assumption (3.2) assures that child j has higher expected returns.

Theorem 3.3. *Suppose Assumptions 3.2 and 3.1. Then, the optimal transfers t_j, t_k must satisfy $t_j > t_k$.*

Proof: See appendix. □

3.2 Bilateral transfers

Suppose now that the children provide a service to the parent as in Bernheim et al. (1985). Consider $y = (y_p, y_j, y_k)$ and $a = (a_p, a_j, a_k)$ the income and optimal investment profile of parent p and the two children, j and k, respectively. The amount of consumption and saving is given in units of the single good. Write t_i the bequest from the parent to the child $i = j, k$. This variable may depend on the amount of services provided by the child. The variable s_i represents the amount of service supplied by the child i to the parent measured as an opportunity cost. Children have the same utility function $u_i : \mathbb{R}_+^2 \to \mathbb{R}$, with $u_i(c_i, s_i)$ representing the benefit of a consumption level c_i and supplied services s_i for $i = j, k$. A representative parent has a utility function $u_p : \mathbb{R}_+^3 \to \mathbb{R}$, with $u_p(c_p, s_j, s_k)$ yielding the benefit of a consumption level c_p and a profile (s_k, s_j) of services received from the children. We assume that services generate disutility for the children and utility for the parent, that is, $\partial_{s_i} u(c_i, s_i) < 0$ and $\partial_{s_i} u(c_p, s_j, s_k) > 0$ for $i = j, k$. We consider utility functions separable on consumption and services, that is, $u_i(c_i, s_i) = u(c_i) + \bar{u}_i(s_i)$ for $i = j, k$ and $u_p(c_p, s_j, s_k) = u(c_p) + \bar{u}_p(s_j, s_k)$ for some concave differentiable $u : \mathbb{R}_+ \to \mathbb{R}_+$, $\bar{u}_i : \mathbb{R}_+ \to \mathbb{R}_+$ for $i = j, k$ and $\bar{u}_p : \mathbb{R}_+^2 \to \mathbb{R}_+$ satisfying the INADA conditions. A risk-free asset with return r_p is available for the parent.

The value function of child i's problem is then given by

$$v_i(t_i) = \max\left\{u_i(c_i, s_i) + \beta \int_{\mathbb{R}_+} u((1+r)a_i + s_i t_i) f_i(r)dr : c_i + a_i + s_i \leq y_i\right\} \tag{4}$$

and the corresponding choices given by

$$(\hat{c}_i(t_i), \hat{a}_i(t_i), \hat{s}_i(t_i)) = \text{argmax}\left\{u_i(c_i, s_i) + \beta \int_{\mathbb{R}_+} u((1+r)a_i + s_i t_i) f_i(r)dr\right\} \tag{5}$$

where the max is chosen over all (c_i, a_i, s_i) such that $c_i + a_i + s_i \leq y_i$.

[6] Under this assumption, the more productive child always invest because she has a higher return than her sibling.

[7] Recall that $\bar{r}_j$ satisfies $f_j(r) = 0$ for all $r \leq \bar{r}_j$.

Consider μ the parameter representing parent's utility discount relative to the child's utility. Write

$$\tilde{u}_p(c_p, a_p, s_j, s_k, t_j, t_k) = u_p(c_p, s_j, s_k) + \beta u((1 + r_p)a_p - s_j t_j - s_k t_k) + \mu v_j(t_j) + \mu v_k(t_k)$$

the parents' utility function. The parent's problem is given by

$$v_p(s_j, s_k) = \max\{\tilde{u}_p(c_p, a_p, s_j, s_k, t_j, t_k) : (c_p, a_p, t_j, t_k) \in \mathbb{R}^4_+\} \tag{6}$$

and such that $c_p + a_p \le y_p$ and $s_j t_j + s_k t_k \le (1 + r_p)a_p$. The corresponding optimal choice

$$(\hat{c}_p(s_j, s_k), \hat{a}_p(s_j, s_k), \hat{t}_j(s_j, s_k), \hat{t}_k(s_j, s_k))$$

written as $\hat{x}_p(s_j, s_k)$ is given by

$$\hat{x}_p(s_j, s_k) = \text{argmax}\left\{\tilde{u}_p(c_p, a_p, s_j, s_k, t_j, t_k) : (c_p, a_p, t_j, t_k) \in \mathbb{R}^4_+\right\}$$

and such that $c_p + a_p \le y_p$ and $s_j t_j + s_k t_k \le (1 + r_p)a_p$.

The next theorem characterizes the solution for this case. It will be given as a Nash Equilibrium. Its proof is similar to that found in Nash (1951).

Theorem 3.4. *There exists a profile of bilateral transfers* $(t_j^*, t_k^*, s_j^*, s_k^*)$ *such that:*

$$s_j^* = \hat{s}_j(t_j^*),\ \ s_k^* = \hat{s}_k(t_k^*),\ \ t_j^* = \hat{t}_j(s_j^*, s_k^*),\ \ t_k^* = \hat{t}_k(s_j^*, s_k^*)$$

3.3 Examples

To illustrate our theoretical model of unequal transfers under uncertainty, we show three numerical examples. Examples 3.5 and 3.6 illustrate the model in Section 3.1, while example 3.7 elucidates the model in Section 3.2. In the first example, returns are quite close and the parent will smooth the aggregate benefit transferring more to the child with the lowest return. In the second example, the difference in children's returns to human capital is very large. In this case, the numerical solution shows that the parent transfers to the more productive child, increasing the aggregate benefit of the family. We conclude that parents' altruistic behavior depends on the variance of capital returns among their offspring.

In the next example, we show that when the variability of children's human capital returns is low, parent behaves altruistically. Intuitively, when children have high contrast in returns, the smoothing effect prevails since the additional benefit obtained with the consumption smoothing is greater than the gain obtained by the additional investment in the more productive child.

Example 3.5. Suppose that $\beta = 1/2$, $y_p = y_j = y_k := y$, and $\mu = 1/2$. The probability distributions of returns are induced by the densities $f^j \sim U[0, 0.6]$ and $f^k \sim U[0, 0.5]$. Under these distributions the expected values of human capital returns are given by 0.3 and 0.25, respectively. In the case of identical preferences with utility function given by $u(c) = yc/2 - c^2$, the optimal choices and bequests, conditional to some specific income level y, are[8]

$$\begin{aligned} c_j(t_j) &= 0.3495t_j + 0.5094y & \qquad c_k(t_k) &= 0.3488t_k + 0.4942y \\ a_j(t_j) &= -0.3495t_j + 0.4906y & \qquad a_k(t_k) &= -0.3488t_k + 0.5058y \\ t_j &= 0.2364y & \qquad t_k &= 0.2385y \end{aligned}$$

Observe that child k has a lower expected return on her investment and receives the largest bequest.

In the next example, the variability between the returns on human capital is larger enough so that the transfers do not favor altruistic behavior of parents.

[8] All the following examples were calculated using the software *Mathematica*.

Example 3.6. Suppose that $\beta = 1/2$, $y_p = y_j = y_k := y$, and $\mu = 1/2$. The probability distributions of re-turns are induced by the densities $f^j \sim U[0,1]$ and $f^k \sim U[0,1/2]$. Under these distributions the expected values of human capital returns are given by 0.5 and 0.25, respectively. In the case of identical preferences with utility function given by $u(c) = yc/2 - c^2$, the optimal choices and bequests, conditional to some specific income level y, are

$$\begin{aligned} c_j(t_j) &= 0.3461t_j + 0.5673y & c_k(t_k) &= 0.3488t_k + 0.4941y \\ a_j(t_j) &= -0.3461t_j + 0.4326y & a_k(t_k) &= -0.3488t_k + 0.5058y \\ t_j &= 0.2415y & t_k &= 0.2356y \end{aligned}$$

Observe that child j has a greater expected return on her investment and receives the largest bequest.

In the following example, children behave strategically providing services to the parent.

Example 3.7. Suppose that $\beta = 1/2$, $y_p = y_j = y_k := 2$, and $\mu = 9/10$. The utility function of the children is identical and given by $u(c_i) = 8c_i - c_i^2$ and their utility for services is given by $\bar{u}_i(s_i) = -s_i$ for $i = j, k$. The utility function of the parent is given by $u(c_p) = 8c_p - c_p^2$ and his utility for services given by $\bar{u}(s_j, s_k) = s_j + s_k$. A risk-free asset with return $r_p = 3/2$ is available for the parent. The probability distributions of returns are induced by the densities $f^j \sim U[0,1]$ and $f^k \sim U[0,0.75]$. Under these distributions the expected values of returns are given by $E[r_j] = 0.5$ and $E[r_k] = 0.325$, respectively. In this case, the optimal choices and bequests are

$$\begin{aligned} c_j(t_j) &= 1.8142 & c_k(t_k) &= 1.9895 & a_j(t_j) &= 0.1857 & a_k(t_k) &= 0.0104 \\ s_j &= 2.3301 & s_k &= 3.1030 & t_j &= 0.3419 & t_k &= 0.3419 \end{aligned}$$

Observe that child k has the lowest expected return on her land-specific human capital investment and receives the largest bequest since $t_j s_j < t_k s_k$.

4 Conclusion

In this paper we find that the child with the highest return to human capital is more likely to receive a larger share of the land if the contrast in offspring's returns is large enough. For this situation to hold, we assume a certain boundary condition on the magnitude of the highest return child's propensity to save. This result points to a non-altruistic behavior. In the case which each child influences parent's optimal choice of bequest by providing services to the latter we conclude that the child's strategy for service provision is sufficient to assure that the one providing more service will receive a larger share of the bequest in a Nash equilibrium. This holds, regardless of differences in offspring returns to human capital.

5 Appendix

Theorem 5.1. *Suppose Assumptions 3.2 and 3.1. Then, the optimal transfers t_j, t_k must satisfy $t_j > t_k$.*

Proof: Assuming interior solution,[9] differentiating with respect to t and applying the Envelop theorem to Equation (1) we get for each $t \in \mathbb{R}_+$

$$v_i'(t) = \beta \int_{\mathbb{R}_+} u'((1+r)a_i(t) + t) f_i(r) dr \text{ for } i = j, k. \tag{7}$$

Differentiating with respect to t again we get

$$v_i''(t) = \beta \int_{\mathbb{R}_+} ((1+r)a_i'(t) + 1) u_i''((1+r)a_i(t) + t) f_i(r) dr \text{ for } i = j, k. \tag{8}$$

[9]The condition $\lim_c \to 0^+ u'(c) = \infty$ assures the interior optimum.

Moreover, the F.O.C. of problem 3 with respect to t_j, t_k for a interior solution gives

$$\beta u'((1+r_p)a_p - t_j - t_k) = \mu v_i'(t_i) \text{ for } i = j, k$$

where (a_p, t_j, t_k) is the optimum value.

Assumption 3.2 and Equation (7) imply that[10]

$$\begin{aligned}
v_j'(t_k) &= \beta \int_{\mathbb{R}_+} u'((1+r)a_j(t_k) + t_k) f_j(r) dr \\
&\geq \beta \int_{\mathbb{R}_+} u'((1+r)y + t_k) f_j(r) dr \\
&> \beta \int_{\mathbb{R}_+} u'((1+r)\bar{a}_k + t_k) f_k(r) dr \\
&> \beta \int_{\mathbb{R}_+} u'((1+r)a_k(t_k) + t_k) f_k(r) dr \\
&= v_k'(t_k) = v_j'(t_j).
\end{aligned}$$

Therefore we conclude that $v_j'(t_j) > v_j'(t_k)$. Assumption 3.1 assures that $(1+\bar{r}_j)|a_j'(\cdot)| < 1$ and hence

$$(1+r)a_j'(\cdot) \geq -(1+r)|a_j'(\cdot)| \geq -(1+\bar{r}_j)|a_j'(\cdot)| > -1 \text{ for all } r \leq \bar{r}_j.$$

Thus $v_j''(\cdot) < 0$ by (8). This implies that $t_j > t_k$ since $v_j'(t_k) > v_j'(t_j)$. □

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[10]Observe that in the second and fourth inequality we use that u' is decreasing, $a_j \leq y$ and $a_k \leq \bar{a}_k$.

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