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PRODUCTION PRICES AND THE WAGE-PROFIT CURVE IN BRAZIL: AN INPUT-OUTPUT ANALYSIS, 1975

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C E D E P L A R

PRODUCTION PRICES AND THE WAGE-PROFIT CURVE IN BRAZIL: AN INPUT-OUTPUT ANALYSIS, 1975

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SUMÁRIO

PRODUCTION PRICES AND THE WAGE-PROFIT CURVE IN BRAZIL: AN INPUT/OUTPUT ANALYSIS, 1975*

I. INTRODUCTION

The input/output approach has been instrumental in the analysis of basic value-theoretic problems dating back to Marx (see Brody, 1970). An advantage of this approach is that we can examine the trade-off between the wage rate and the profit rate, and the effect of this trade-off on production prices. But the empirical analysis of basic Marxian economic categories, using the input/output approach, is still not extensive (Wolff, 1975; Shaikh, 1984; Ozol, 1984; Okishio and Nakatani, 1985; Ochoa, 1986; Alberro and Nieto, 1986; Da Silva, 1987).

The principal objective of this study is to examine the effects of changes in income distribution on Marxian production prices, as the profit margin is allowed to vary from zero to its maximum level. This study is applied to the most recently available Brazilian input/output accounts for 1975. Other things equal, we suggest that changes in income distribution will cause non-uniform effects on Marxian prices. In the concluding section we draw a preliminary sketch of some major policy implications of the empirical results.

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II. THEORETICAL FRAMEWORK

In the absence of capital stocks, Marxian prices are proportional to cost-prices:[1]

$$p = (1 + r)\, p\, A + (1 + r)\, w\, L \qquad (1)$$

$$p\, y = 1 \qquad (2)$$

where p A + w L denotes cost-prices.

Equation (2) represents Marx's (1971, pp. 159-60) postulate that the sum of prices equals to the sum of labor values; it is the numeraire of prices and wages.[2] The other symbols can be defined as follows:

p (1,n) is a vector of Marxian prices.

A (n,n) is a matrix of input/output coefficients.

L (1,n) is a vector of direct labor coefficients.

y (n,1) is a vector of net output per total employment.

r (1,1) and w (1,1) are scalars denoting the profit margin and the money wage rate per worker.

This system contains (n + 1) equations and (n + 2) unknown variables to be determined: the prices p_i (i = 1, 2, ..., n), profit margin, and money wage rate. Its solution is obtained by making some standard assumptions, which are regarded as constituting the "core" of the surplus approach to the analysis of value and income distribution between wages and profits (see Garegnani, 1984, pp. 293-94). This means that we take as given the following magnitudes:

(i) the technology in use, [A, L];

(ii) the level and composition of net output, Y (n,1); and

(iii) the value of net output is assumed to be completely divided between wages and profits.

From these assumptions, we can solve (1) for p as function of the profit margin, by taking the following steps:

$$p\,[I - (1 + r)\,A] = (1 + r)\,w\,L$$

$$p = (1 + r)\,w\,L\,[I - (1 + r)\,A]^{-1} \qquad (3)$$

where I (n,n) is an identity matrix.

Obviously, we also assume that the input/output matrix is productive, such that the matrix $[I - (1 + r)\,A]$ is invertible (Brody, 1970, pp. 175-76). Equation (3) permits the computation of p for any given pair of the distribution variables (r, w).

Likewise, for a given vector y (n,1), the wage share can be solved as function of the profit margin. We postmultiply (2) by y, and then solve for w as the share of wages in net output:

$$p\,y = 1 = (1 + r)\,w\,L\,[I - (1 + r)\,A]^{-1}\,y$$

$$w = 1 / (1 + r)\,L\,[I - (1 + r)\,A]^{-1}\,y, \qquad (4)$$

where r is an independent variable.

As shown, this wage-profit relation (4) exhibits a basic property of capitalism that a higher wage share is necessarily associated with a lower profit margin, and vice versa. In reality, equation (4) represents Marx's (1969, Part II, p. 419) suggestion that "wages have to be reckoned according to the **relative share** of the value of the total product ... The position of the classes to one another depends more on **relative wages** than on the absolute amount of wages."

Equations (3) and (4) do not constitute a self-contained system because the profit margin is treated as an exogenous

variable. But although we do not determine a specific level of r, we can certainly calculate its range, $0 \leq r < R$ = maximum profit margin. The computation of R simply involves setting $w = 0$ in equation (1). This implies, by hypothesis, that the whole net output is appropriated by the capitalist class. Then, by using (4), we can interpolate values for the wage share as the profit margin is allowed to increase from zero to R. Thus, setting $w = 0$, the price equation (1) is reduced to the following system:

$$p = (1 + R)\, p\, A$$

$$p\,[I - (1 + R)\, A] = 0$$

$$p\,[u\, I - A] = 0, \qquad (5)$$

where $u = 1/(1 + R) < 1$ is the dominant Frobenius root of the input/output matrix A (n,n).

From some well-known theorems of Frobenius and Perron (see Brody, 1970, pp. 24, 171-72), a positive value for the maximum profit margin can be found if the determinant of the homogeneous equation (5) is set to zero:

$$\det\,[u\, I - A] = 0 \qquad (6)$$

From this n-th order polynomial u_i ($i = 1, 2, \ldots, n$), such that for each u_i there corresponds a vector p_i, we choose the maximum Frobenius root (u), and calculate $R = (1/u) - 1 > 0$. The associated price vector is obtained by substituting u into equation (5). Now we have the equation system needed to compute the wage-profit curve as well as Marxian production prices, both as function of the profit margin.

III. EMPIRICAL RESULTS

For the empirical analysis we use the input/output accounts of the Brazilian economy aggregated into 9-sectors (see Table 1). All the relevant data come from IBGE (1987). Accordingly, we calculated the maximum Frobenius root for Brazil in 1975, as $u = 0.423$; the corresponding maximum profit margin is $R = (1/u) - 1 = 1.364$. Using equation (4), we interpolated values for the wage share, in the range $0 \leqslant r < R = 1.364$. This generated a convex wage-profit curve.[3] See the first two columns of Table 2 and Figure 1.

After substituting (4) into (3), we computed Marxian prices as function of the profit margin, again in the range $0 \leqslant r < R = 1.364$. See Table 2 and Figures 2(a-c). The computed prices are **relative** to the value of net output; their **dimension** is employment - that is, the number of workers directly engaged in production - **per** million of 1975 **cruzeiros.**

Figures 2(a-c) - which group the relative prices of the different sectors of the Brazilian economy according to comparable scales - show that as the profit margin increases towards its maximum level,

a) p_1, p_8, and p_9 decrease linearly;

b) p_6 increases linearly;

c) p_2, p_3, and p_7 increase at an increasing rate;

d) p_4 increases at a decreasing rate; and

e) p_5 has a gentle quadratic shape.

The empirical results imply that the policy posture that increases in the wage share - or decreases in the profit margin or in the profit share - always leads to increases in relative prices does not hold generally. Instead, this position seems to be restricted to those cases in which our numeraire is violated because prices are better explained by the markup equation, that is, when prices are only flexible upwards but not downwards.

IV. SOME CONCLUSIONS

The absence of capital stocks poses a major restriction on the interpretation of the empirical results. Accordingly, this study should be interpreted as a search to understand structural relationships in the Brazilian economy, using the Marxian framework; the empirical results are rough approximations rather than exact measurements.

As a conclusion, this study suggests that a policy which limits the profit margin or the profit share is necessary if workers are to increase their share in net output. In particular, this study shows that a policy-induced increase in the wage share - or a decrease in the profit margin or in the profit share - is quite compatible with decreases in the relative prices of important sectors of the Brazilian economy. In this way, the anti-labor price stabilization programs which have been implemented in Brazil in the 1970s are shown not to have been an economic necessity.

NOTES

[1] Marx established an inverse relationship between the wage share and the profit rate. But since we do not have a capital stock matrix for the Brazilian economy, we work with a circulating capital model. Thus, we estimate the profit margin instead of the profit rate and prices proportional to cost-prices instead of production prices. See Brody (1970, pp. 43-44) for the price equation with fixed capital.

[2] Marx's invariance postulate is perhaps better recognized if we rewrite (2) as follows:

$$p\,Y = v\,Y = N \qquad (2')$$

$$\text{or} \quad p\,y = v\,y = 1, \qquad (2)$$

where $v = L\,[I - A]^{-1}$ is a row vector of labor values, Y (n,1) is net output, N (1,1) is total employment, and y (n,1) is net output scaled by total employment.

[3] The computation of the average wage share is fraught with problems. As an example, we calculated the (weighted by value-added) average wage share as $w = 0.346$, paired with $r = 0.611$. But this ratio includes the income of workers directly engaged in production, salaried workers, and management personnel. The available data are not sufficient to isolate the wage share of workers directly engaged in production.

Fig. 1 - Brazil, 1975: Wage-Profit Curve

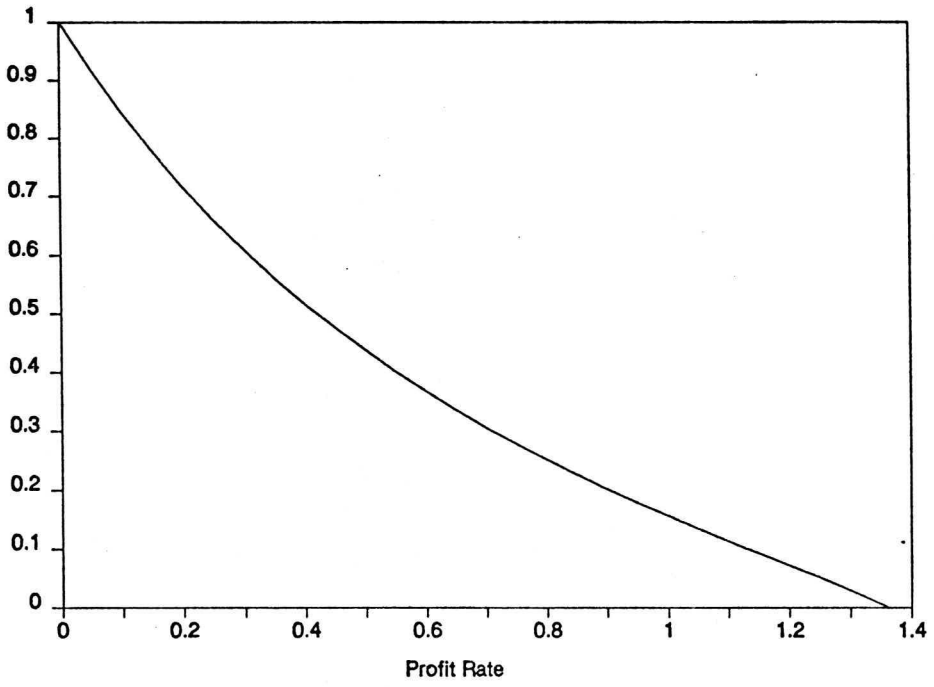

Fig. 2a - Brazil, 1975: Relative Prices as f(r)

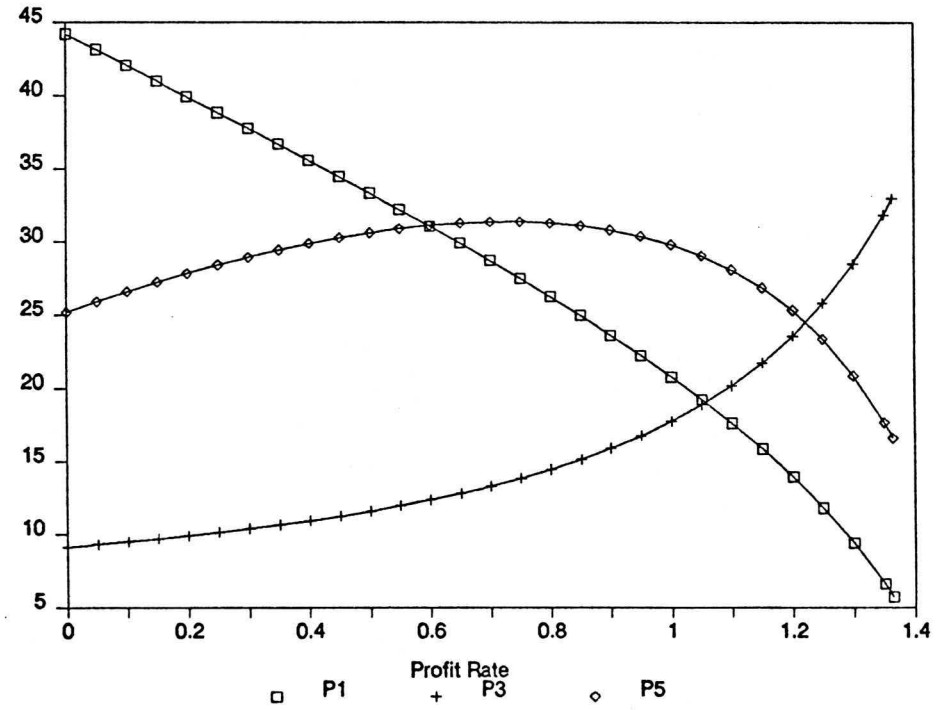

Fig. 2b - Brazil, 1975: Relative Prices as f(r)

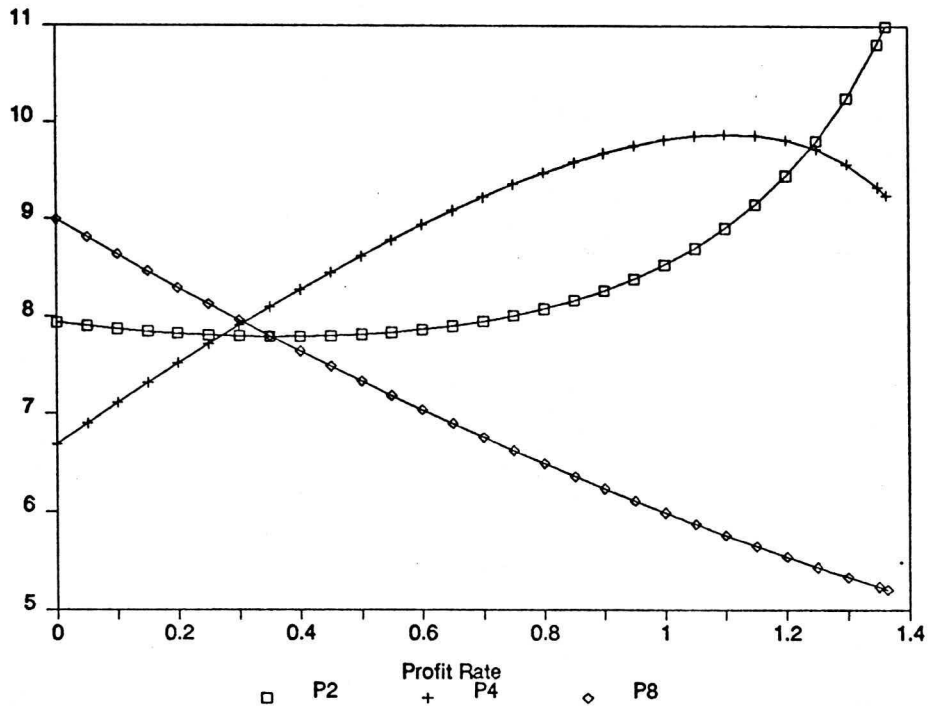

Fig. 2c - Brazil, 1975: Relative Prices as f(r)

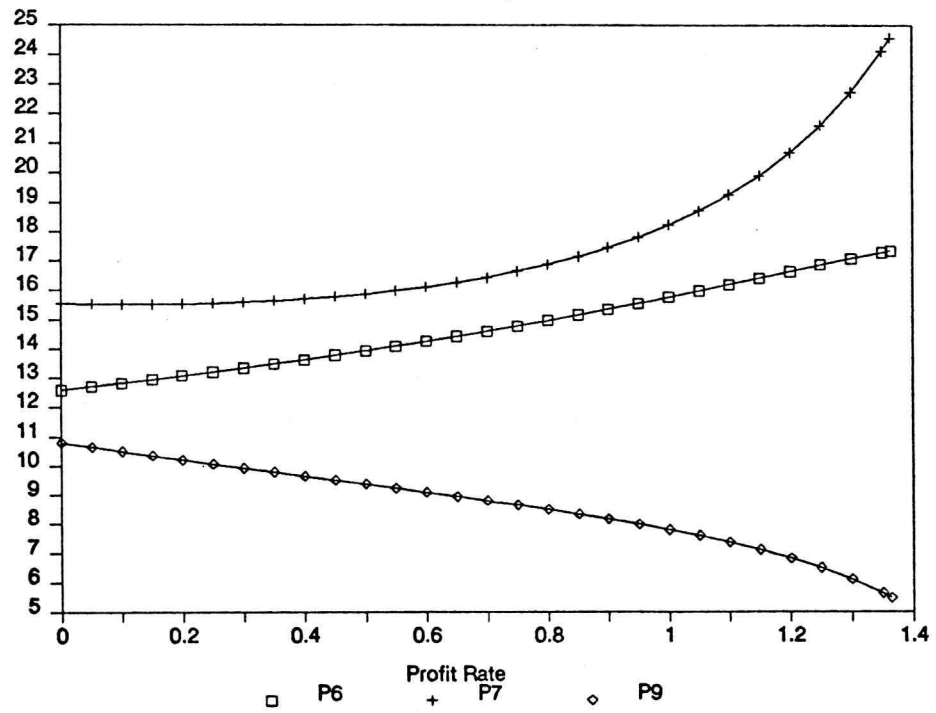

TABLE 1

BRAZILIAN INPUT/OUTPUT ACCOUNTS

NINE-SECTOR CLASSIFICATION

Sector	IBGE Activity code
1. Agriculture, fishing, and forestry	101 to 499
2. Mining and fuels	501 to 504
3. Metal industries	1101 to 1491
4. Chemicals	2001 to 2091
5. Agro-industry	2601 to 2899
6. Other manufacturing	1001 to 1091
	1501 to 1999
	2199 to 2502
	2901 to 3099
	5601
7. Construction	4201
8. Transport and distribution	5101 to 5204
9. Services	4001 to 4101
	5301 to 5504

Source: IBGE (1987, p. 49-51).

TABLE 2

BRAZIL, 1975

PRICES AS FUNCTION OF THE PROFIT MARGIN

r	w	p_1	p_2	p_3	p_4	p_5
0.00	1.000	44.21	7.93	9.13	6.69	25.18
0.05	0.919	43.15	7.90	9.31	6.90	25.90
0.10	0.845	42.09	7.87	9.49	7.11	26.59
0.15	0.778	41.02	7.85	9.70	7.32	27.24
0.20	0.717	39.95	7.83	9.91	7.52	27.85
0.25	0.661	38.87	7.81	10.14	7.72	28.43
0.30	0.609	37.79	7.80	10.39	7.91	28.97
0.35	0.561	36.71	7.80	10.65	8.10	29.46
0.40	0.517	35.61	7.80	10.94	8.28	29.90
0.45	0.476	34.51	7.80	11.26	8.45	30.30
0.50	0.437	33.39	7.82	11.60	8.62	30.65
0.55	0.401	32.26	7.84	11.97	8.78	30.93
0.60	0.368	31.11	7.87	12.38	8.94	31.16
0.65	0.336	29.94	7.91	12.82	9.09	31.32
0.70	0.306	28.75	7.95	13.32	9.23	31.41
0.75	0.278	27.53	8.01	13.86	9.35	31.42
0.80	0.252	26.28	8.08	14.47	9.47	31.34
0.85	0.226	24.99	8.17	15.15	9.58	31.15
0.90	0.202	23.66	8.27	15.91	9.68	30.86
0.95	0.179	22.27	8.39	16.78	9.76	30.43
1.00	0.156	20.81	8.54	17.77	9.82	29.85
1.05	0.135	19.28	8.71	18.90	9.86	29.09
1.10	0.114	17.64	8.91	20.22	9.88	28.12
1.15	0.093	15.88	9.16	21.76	9.87	26.88
1.20	0.072	13.95	9.45	23.60	9.82	25.33
1.25	0.051	11.82	9.81	25.80	9.73	23.37
1.30	0.029	9.41	10.25	28.51	9.57	20.88
1.35	0.007	6.62	10.81	31.90	9.34	17.68
1.36	0.000	5.74	11.00	33.01	9.25	16.62

TABLE 2

BRAZIL, 1975

PRICES AS FUNCTION OF THE PROFIT MARGIN

r	w	p_6	p_7	p_8	p_9
0.00	1.000	12.59	15.55	8.99	10.80
0.05	0.919	12.70	15.53	8.81	10.65
0.10	0.845	12.82	15.52	8.64	10.50
0.15	0.778	12.95	15.52	8.46	10.36
0.20	0.717	13.07	15.53	8.29	10.21
0.25	0.661	13.21	15.56	8.13	10.07
0.30	0.609	13.34	15.59	7.96	9.93
0.35	0.561	13.48	15.64	7.80	9.79
0.40	0.517	13.62	15.70	7.65	9.65
0.45	0.476	13.77	15.77	7.49	9.52
0.50	0.437	13.92	15.86	7.34	9.38
0.55	0.401	14.08	15.97	7.19	9.24
0.60	0.368	14.24	16.10	7.05	9.10
0.65	0.336	14.41	16.25	6.90	8.96
0.70	0.306	14.59	16.43	6.76	8.81
0.75	0.278	14.77	16.64	6.63	8.66
0.80	0.252	14.95	16.87	6.50	8.51
0.85	0.226	15.14	17.14	6.37	8.35
0.90	0.202	15.34	17.45	6.24	8.18
0.95	0.179	15.54	17.81	6.12	8.00
1.00	0.156	15.75	18.22	6.00	7.81
1.05	0.135	15.96	18.70	5.88	7.60
1.10	0.114	16.18	19.26	5.76	7.38
1.15	0.093	16.40	19.91	5.65	7.12
1.20	0.072	16.63	20.68	5.54	6.84
1.25	0.051	16.85	21.61	5.44	6.51
1.30	0.029	17.07	22.73	5.34	6.11
1.35	0.007	17.27	24.12	5.24	5.64
1.36	0.000	17.32	24.57	5.21	5.49

Data source: IBGE (1987).

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