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Cláudio Gontijo

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UNIVERSIDADE FEDERAL DE MINAS GERAIS
FACULDADE DE CIÊNCIAS ECONÔMICAS
CENTRO DE DESENVOLVIMENTO E PLANEJAMENTO REGIONAL

FACTOR PRICE EQUALIZATION IN A RICARDIAN FRAMEWORK

Cláudio Gontijo
Professor do Departamento de Ciências
Econômicas da UFMG

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SUMÁRIO

1. INTRODUCTION

One important piece of the modern neo-classical theory of international trade is the celebrated Factor Price Equalization Theorem, developed independently by Lerner (1952, though written in 1932) and Samuelson (1948, 1949, 1953). This Theorem states that under certain conditions free trade leads to complete equalization of production factor rewards independently of factor mobility. A similar result - a tendency towards equalization of the profit rate – was obtained by Mainwaring (1978) in a Ricardian-Sraffian framework, but under the assumption that all trading countries share the same technology. The objective of this article is to discuss this result assuming technological differences among trading countries.

This article is composed by four sections. Section 2 presents a simple model of long-run equilibrium prices for an economy with commodity-money. Section 3 discusses the relationship between balance of trade, economic growth and profitability. A numerical example is shown in section 4.

2. A SIMPLE RICARDIAN MODEL OF "NATURAL PRICES"

A simplified version of the Ricardian system of "natural prices" for an closed economy without joint production and fixed capital in which the turnover time of circulating capital is unitary and money is a commodity may be expressed as (see Morishima, 1990, pp. 126-146):[1]

$$\mathbf{p} = \mathbf{p}\,\mathbf{A} + w\,\mathbf{a_0} + r\,(\mathbf{p}\,\mathbf{A} + w\,\mathbf{a_0}) = (1 + r)\,\mathbf{p}\,(\mathbf{A} + \mathbf{d}\,\mathbf{a_0}) \qquad (2.1)$$

$$p_o = 1 \qquad (2.2)$$

where **p** is the row vector of long-run equilibrium prices; **A** the n by n matrix of technical coefficients; $\mathbf{a_0}$ the row vector of labor coefficients; w the wage rate, r the rate of profits; **d** is the column vector representing a (given) wage basket; and p_o is the price of an unity of the commodity-money - say, a gold coin.

Since the profit rate is determined by solving the following characteristic equation:

$$\det\,[\,\text{[illegible]}\;-\;(\mathbf{A} + \mathbf{d}\mathbf{a_0})] = 0 \qquad (2.3)$$

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e[1] Land is excluded because Ricardo considers ground rent always as differential rent so that it does not nter price determination (the "natural price" of agricultural products are determined in the land of higher costs).

where

$$\lambda_{max} = 1/(1+ r) \tag{2.4}$$

the Frobenius Theorem ensures that the price vector **p** as well as the profit rate r are positive and unique provided that the economy is productive (see Graham, 1987, pp. 112-68; Pasinetti, 1977, pp. 267-76).

If it is assumed free trade, the price system becomes

$$\mathbf{p} = (1 + r)\,(\mathbf{p}\,\mathbf{A}_D + \mathbf{p}_{*}\mathbf{A}_{*} + (\mathbf{p}\,\mathbf{d}_D + \mathbf{p}_{*}\mathbf{d}_{*})\,\mathbf{a}_0) \tag{2.5}$$

where subscript D indicates domestically produced goods and $*$ indicates imported goods. Note that the hypothesis that money is a commodity implies that the exchange rate is unitary.

Rearranging the terms of (2.5) yields:

$$\mathbf{p} = \mathbf{p}_{*}(\mathbf{A}_{*} + \mathbf{d}_{*}\mathbf{a}_0)\,[\lambda \; * - (\mathbf{A}_D + \mathbf{d}_D\,\mathbf{a}_0)]^{-1} \tag{2.6}$$

where

$$1/\lambda_{max}\{\mathbf{A}_D + \mathbf{d}_D\mathbf{a}_{0D}\} - 1 > r > 1/\lambda_{max}\{\mathbf{A} + \mathbf{d}\mathbf{a}_0\} - 1 \tag{2.7}$$

Condition (2.7) is necessary and sufficient for having non-negative prices.[2] The reasons are the following. First, if $r < 1/\lambda_{max}\{\mathbf{A}_D+\mathbf{d}_D\mathbf{a}_0\} - 1$, some prices are negative. Second, if $r = 1/\lambda_{max}\{\mathbf{A}_D+\mathbf{d}_D\mathbf{a}_0\} - 1$, matrix $[\lambda \; * - (\mathbf{A}_D+\mathbf{d}_D\mathbf{a}_0)]$ is singular and, therefore, has no inverse. But if $r < 1/\lambda_{max}\{\mathbf{A}_D+\mathbf{d}_D\mathbf{a}_{0D}\} - 1$, matrix $[\lambda * - (\mathbf{A}_D+\mathbf{d}_D\mathbf{a}_0)]$ becomes a Maia0kovsky matrix, which is non-singular and non-negative by definition (Graham, 1987, pp. 169-70). However, r can not be lower than $1/\lambda_{max}\{\mathbf{A}+\mathbf{d}\mathbf{a}_0\} - 1$, which is the profit rate that would prevail in a situation of autarchy because in this case the entrepreneurs would stop importing everything.[3] Note also that **p** is not positive because both $(\mathbf{A}_{*} + \mathbf{d}_{*}\,\mathbf{a}_0)$ and $(\mathbf{A}_D + \mathbf{d}_D\,\mathbf{a}_0)$ have null columns.

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[2] Note that $\lambda_{max}\{\mathbf{A}_D+\mathbf{d}_D\mathbf{a}_{0D}\} \leq \lambda_{max}\{\mathbf{A}+\mathbf{d}\mathbf{a}_0\}$ because λ is a monotonically increasing function of the elements of its corresponding matrix and $(\mathbf{A}+\mathbf{d}\mathbf{a}_0)$ includes some columns that are missing in $(\mathbf{A}_D+\mathbf{d}_D\mathbf{a}_{0D})$.

3. BALANCE OF TRADE, GROWTH, AND PROFIT RATE EQUALIZATION

Assuming that there is no technical progress, consumers' preferences are constant over time,[4] it prevails free trade, and that workers do not save, imports of a given country are determined by:

$$\mathbf{x}_M = (1 + g)\,(\mathbf{A}_M + \mathbf{d}_M\,\mathbf{a}_0)\,\mathbf{x}_D + \mathbf{c}_M \tag{3.1}$$

while its production vector, $\mathbf{x}_D$, is given by:

$$\mathbf{x}_D = (1 + g)\,(\mathbf{A}_D + \mathbf{d}_D\,\mathbf{a}_0)\,\mathbf{x}_D + \mathbf{c}_D + \mathbf{x}_E \tag{3.2}$$

where g is the rate of economic growth; $\mathbf{x}_E$ stands for the vector of exports; and **c** is the vector of capitalists' demand for consumption goods.

Assuming equilibrium in the balance of trade:

$$\mathbf{p}\,\mathbf{x}_E = \mathbf{p}\,\mathbf{x}_M \tag{3.3}$$

and considering that a country's exports are the rest of the world's imports, $\mathbf{x}_{MR}$:

$$\mathbf{p}\,\mathbf{x}_E = \mathbf{p}\,\mathbf{x}_{MR} \tag{3.4}$$

it is easy to show using (3.1) and (3.2) that the growth rates of the country and the rest of the world are identical:

$$g = g_R \tag{3.5}$$

which is equivalent to say that countries connected by a free trade agreement tend to grow at the same rhythm .[5]

From (3.1) and (3.2) it follows that total supply is given by

$$\mathbf{x} = \mathbf{x}_D + \mathbf{x}_M = (1 + g)\,[(\mathbf{A}_D + \mathbf{A}_M) + (\mathbf{d}_D + \mathbf{d}_M)\,\mathbf{a}_0]\,\mathbf{x}_D + \mathbf{c}_D + \mathbf{c}_M + \mathbf{x}_E =$$

$$= (1 + g)\,\mathbf{A}^+\,\mathbf{x}_D + \mathbf{c} + \mathbf{x}_E$$

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[3] Condition (2.7) is striking, because it shows that specialization results in a higher rate of profits, apparently confirming the comparative advantage principle (see Mainwaring, 1973). This issue, however, will not be discussed in this paper.

[4] Pasinetti (1977, p. 191) classifies a model that considers such hypothesis as "quasi-dynamic".

[5] No wonder, since it is a consequence of the fact that growth is dependent on exports and that the exports of an area are equal to the imports of the other areas of the world.

Multiplying by **p** gives:

$$\mathbf{p\,x} = (1 + g)\,\mathbf{p\,A^{+}\,x_D} + \mathbf{p\,c} + \mathbf{p\,x_E}$$

Using (2.5) and (3.3) it follows that the growth rate is equal to the profit rate minus the proportion between the value of capitalists' consumption and the value of the inputs used in domestic production:

$$g = r - \mathbf{p\,c} / \mathbf{p\,A^{+}\,x_D} \tag{3.6}$$

From (3.5) and (3.6) it can be seen that the profit rates diverge in so far as the proportion that the ratio between the value of capitalists' consumption and the value of the inputs used in the domestic production are different among countries.

Finally, if is assumed that all countries growth at maximum speed, it can be concluded that the profit rate is the same throughout the world economy:

$$r = r_R \tag{3.7}$$

4. A NUMERICAL EXAMPLE

To understand clearly the issues involved in this discussion, suppose two economies characterized in the following way:

United Kingdom:

$$\mathbf{A_U} = \begin{bmatrix} 0.30 & 0.50 \\ 0.40 & 0.40 \end{bmatrix} \quad \mathbf{a_{0U}} = \begin{bmatrix} 0.30 & 0.50 \end{bmatrix}$$

$$\mathbf{d_U} = \begin{bmatrix} 0.10 \\ 0.20 \end{bmatrix}$$

Rest of the world:

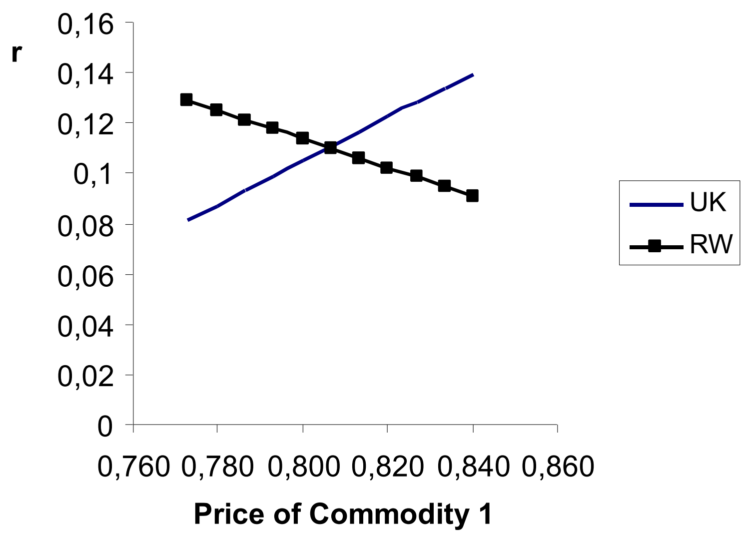
$$\mathbf{A_R} = \begin{bmatrix} 0.40 & 0.40 \\ 0.30 & 0.50 \end{bmatrix} \quad \mathbf{a_{0R}} = \begin{bmatrix} 0.50 & 0.30 \end{bmatrix}$$

$$\mathbf{d_R} = \begin{bmatrix} 0.20 \\ 0.10 \end{bmatrix}$$

Using (2.1) and (2.3) it can be seen that the United Kingdom's before trade profit rate, r_U, is 8.094%, while the Rest of the World's before trade profit rate, r_R, is 9.107% and that, if commodity 2 is taken as the *numeraire* of the price system ($p_{P2} = p_{R2}$ = $ 1), the price of commodity 1 is $ 0.77295 in the United Kingdom and $ 0.84028 in the Rest of the World. The nominal wage is $ 0.27730 in the United Kingdom and $ 0.26806 in the Rest of the World.

If free trade is established, United Kingdom will specialize in producing commodity 1 while the Rest of the World will produce only commodity 2. The profit rate in both economies will depend on the price of commodity 1, but if it is higher than $ 0.77295 and lower than $ 0.84028 it will be higher in both economies. Since commodity 1 is produced only in the United Kingdom and imported by the Rest of the World, the British profit rate will be proportional to its price while the Rest of the World's profit rate will be inversely proportional to it (see Figure 1).

Figure 1:Profit Rate and the Price of Commodity 1



The price vector that equalize the profit rate in both economies will be **p** = [0.80597 1.00000]. In this case, the world economy's profit rate will be 11.019%.

5. CONCLUSIONS

It can be concluded that capital mobility is not a necessary condition for profit rate equalization among trading countries. Free trade is sufficient to ensure equalization of the growth rate, which is equal to the profit rate minus the ratio between the value of capitalists' consumption and the value of the inputs used in domestic production. It follows that profit rates are equal when all trading countries growth at their maximum rate.

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