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TESTING FOR SEASONAL UNIT ROOTS USING MONTHLY DATA

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SUMÁRIO

RESUMO

O objetivo deste artigo é estudar as propriedades de uma série temporal com observações mensais dos preços recebidas pelos produtores de boi gordo no Estado de São Paulo. Em particular, são aplicados os procedimentos de teste propostos por Beaulieu e Miron (1993) para determinar a presença de raízes unitárias na freqüência zero e/ou nas freqüências sazonais. Estes testes mostram a existência de uma raiz unitária na freqüência zero e indicam que não existem raízes unitárias nas freqüências sazonais. Estes resultados implicam que a primeira diferença da série é estacionária e pode ser modelada com variáveis *dummy* sazonais.

1. INTRODUCTION

Economic time series use to have very distinctive characteristics like heteroscedasticity, kurtosis, seasonality, serial correlation, trends, etc. Out of this list, the study of seasonal variations has a long history in the analysis of economic time series (Nerlove *et al.*, 1988; Hylleberg, 1992). Until not long ago, seasonal features of economic time series were viewed as a nuisance void of inherent economic interest. An illuminating citation which shows this point of view occurs in Hylleberg (1994), where William Stanley Jevons is quoted from an 1862 paper expressing this kind of opinion. Since those times until recently, the usual practice continued to be to focus on seasonally adjusted data, at least in the field of macroeconomics. Typically, this result is achieved by applying specific seasonal adjustment filters. This view dominated applied time series econometrics until its drawbacks, as well as the possible economic relevance of seasonality, have been fully recognized.

The model selection techniques popularized by Box and Jenkins (1976) recommend the use of the seasonal filter $(1-L^{12})$ to get rid of seasonal variations in the data. Such a filter is appropriate only in the case the series is seasonally integrated (which implies the existence of 12 unit roots). However, in the case only one unit root is present the use of this filter yields an overdifferenced series; in such a case, applying the $(1-L)$ filter would be sufficient to make the series trend stationary, while (deterministic plus stationary stochastic) seasonality can be handled by the inclusion of seasonal dummies. This overdifferencing may cause trouble in the construction of time series models because the (partial) autocorrelation pattern becomes hard to interpret. Furthermore, we may expect estimation problems because of the introduction of moving average polynomials with roots close to the unit circle. On the other hand, underdifferenced series may yield unit roots in their autoregressive parts, and classical arguments as those given by Granger and Newbold (1974), for time series containing neglected unit roots apply. So, it is important to test for (seasonal) unit roots.

The objective of this paper is to attempt to make a contribution by discussing the application of a testing procedure to determine the seasonal properties of monthly data. In this sense, it is a simple exercise that uses a rather sophisticated technique, similar to that reported in Aguirre (1997) for quarterly series. However, this kind of study necessarily precedes any other analysis about the seasonality of any monthly series and any cointegration analysis where this series is included. Out of the three main definitions of seasonality offered in the literature (deterministic, stationary stochastic, and nonstationary stochastic seasonality due to the presence of unit roots) it is the last one that raises the most troubling statistical issues.

The plan of the paper is as follows. Section 2 presents three typical seasonal models used in empirical work. Section 3 provides a brief reference to unit root tests starting from the so-called zero frequency unit root tests, the seasonal unit root tests for quarterly data, up to the corresponding tests for monthly data that are used in this paper. The data used and the results appear in Section 4 and the conclusions in the last section.

2. MODELLING SEASONAL PROCESSES

In the case of economic series with several observations per year, and unless the data have been seasonally adjusted by popular but frequently criticized routines such as Census X-11, they generally exhibit seasonal patterns which may be treated by including dummies in the system or modelled with additional unit roots at seasonal frequencies.

Several different time series models of seasonality are conceivable. The most common hypotheses for this kind of models are: 1) seasonal patterns can be represented by deterministic dummies, and 2) the series is seasonally integrated.[1] These assumptions are related to the following three classes of processes: purely deterministic seasonal processes; (covariance) stationary processes; and integrated seasonal processes. The first class includes those processes generated by purely deterministic components such as a constant term and seasonal dummy variables. In the following (simple) example variable y_t —observed s times each year— is generated solely by seasonal intercept dummies:**[2]**

$$y_t = \sum_{i=1}^{s} \alpha_i Di_t + \varepsilon_t \qquad (1)$$

where the Di_t ($i = 1, 2,..., s$) take value 1 when t lies on season i, and zero otherwise. This equation can be reformulated so as to avoid confounding the levels and the seasonals, in the following way:

$$y_t = \mu + \sum_{i=1}^{s-1} \alpha_i^* Di_t^* + \varepsilon_t \qquad (2)$$

where μ is the mean of the process and the coefficients α_i^* are constrained to sum zero. In order to make this constrain operative the Di_t^* dummies are defined to be 1 when t lies in season i, –1 when t lies in season s and zero otherwise. Finally, the above equation may also include deterministic trends with constant or variable coefficients across seasons, *i.e.*

$$y_t = \mu + \sum_{i=1}^{s-1} \alpha_i Di_t + \sum_{i=1}^{s} \beta_i [Di_t \times g(t)] + \varepsilon_t \qquad (3)$$

where $g(t)$ is a deterministic polynomial in t.[3]

[1] The concept of 'seasonal integration' may mean different things for different authors. See next section.

[2] A more general model may include an autoregressive and/or a moving average part.

[3] Note that all the above deterministic processes can be forecast and will never change their shape.

The second case —covariance stationary seasonal process— can be exemplified by the model expressed as

$$y_t = \rho\, y_{t-s} + \varepsilon_t \qquad (4)$$

where $|\rho| < 1$ and ε_t is a series of IID random variables.

If $\rho = 1$ in equation (4), we have a *seasonal random walk*, a process that exhibits a seasonal pattern which varies over time. This is the third class of seasonal processes listed above. In that case, $\Delta_s y_t$, defined as

$$\Delta_s y_t = y_t - y_{t-s} = \varepsilon \qquad (5)$$

is stationary. The main difference between these forms of seasonality is that in the deterministic and the stationary seasonal models shocks die out in the long run, while they have a permanent effect in the integrated model. That is to say, seasonally integrated processes have properties similar to those observed in the ordinary (zero frequency) integrated series. "...they have 'long memory' so that shocks last forever and may in fact change permanently the seasonal patterns. They have variances which increase linearly since the start of the series and are asymptotically uncorrelated with processes with other frequency unit roots" (Hylleberg *et al.*, 1990, p. 218).

Three different definitions of seasonal integration are those proposed by Osborn *et al*. (1988), by Engle *et al*. (1989) and a final one due to Hylleberg *et al.* (1990). According to the first one a variable is said to be integrated of order (d,D) —denoted I(d,D)— if the series becomes stationary after first-differencing d times and seasonal differencing D times, that is to say, X_t ~ I(d,D) if $(1-L)^d(1-L^s)^D X_t = \Delta^d \Delta_s^D X_t$ is stationary.[4] The second definition states that a time series is integrated of order d_0 and d_s, denoted SI(d_0,d_s), if $(1-L)^{d_0}[S(L)]^{d_s} X_t = \Delta^{d_0}[S(L)]^{d_s} X_t$ is stationary, where the polynomial expression *S(L)* is defined as $S(L) = 1 + L + L^2 + \ldots + L^{S-1}$.[5]

When variables do not present seasonal integration both definitions coincide, *i.e*., I(1,0) = SI(1,0), I(2,0) = SI(2,0), etc. On the contrary, whenever a series is seasonally integrated these definitions differ. This is so because $\Delta_s = (1-L^s)$ can be factored into $(1-L)S(L)$. In this way, the equivalent of I(0,1) is SI(1,1); I(1,1) = SI(2,1), and so on. In the same way, the SI(0,1) process —using Engle's definition— does not have an equivalent one if we use Osborn's concept. The SI definition will be the one used in this paper.

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[4] L is the usual lag operator.

[5] This polynomial is related to the decomposition of the $(1-L^s) = 0$ polynomial.

Finally, a third definition states that "a series x_t is an integrated seasonal process if it has a seasonal unit root in its autoregressive representation. More generally it is integrated of order d at frequency θ if the spectrum of x_t takes the form

$$f(\omega) = c(\omega - \theta)^{-2d}$$

for ω near θ. This is conveniently denoted by $x_t \sim I_\theta(d)$" (Hylleberg *et al.* 1990, p. 217).

The apparent variety of available models calls for specific statistical techniques to discriminate between various forms of seasonality. In the next section we present a brief summary of the methodology that will be used in this paper.

3. UNIT ROOT TESTS

The detection of unit roots started to be studied in annual data (the so-called zero frequency). The extension of the resulting methodologies to consider seasonal frequencies occurred in two stages: first, the researchers studied the application to quarterly data —with the appearance of three additional frequencies; second, monthly data were considered, which imply eleven seasonal frequencies in addition to the usual one. As soon as the new methods were known alternative procedures were proposed. In this way, not only parametric tests but also semiparametric, nonparametric and Bayesian techniques were put forward. For each one of them the three stage process was a natural development. Furthermore, in each case there were different proposals concerning the form of the null and alternative hypotheses, not to mention the large number of different data generating processes which were considered. The consideration of broken trend alternative hypotheses added even more material to this huge amount of literature.

The history of (non-seasonal) unit root tests starts with Dickey and Fuller (1979) and the well-known Augmented Dickey-Fuller (ADF) test with a non-stationary model as the null hypothesis.

The first test for seasonal integration resembles a generalization of the ADF test for integration in annual data. Dickey, Hasza and Fuller (1984) (DHF from now on), following the methodology suggested by Dickey and Fuller (1979) for the zero-frequency unit-root case, propose a test of the hypothesis ρ = 1 against the alternative ρ < 1 in the model $y_t = \rho y_{t-s} + \varepsilon_t$. The DHF test —as well as similar ones proposed in the following years— only allows for unit roots at all of the seasonal frequencies and has an alternative hypothesis which is considered rather restrictive, namely that all the roots have the same modulus. Trying to overcome these drawbacks Hylleberg *et al*. (1990) (from now on referred to as HEGY) propose a more general testing strategy that allows for unit roots at some (or even all) of the seasonal frequencies as well as the zero frequency. HEGY's methodology allows to test for unit roots at some seasonal frequencies without maintaining that unit roots are present at all seasonal frequencies.

3.1. Seasonal Unit Root Tests with Monthly Data

The HEGY (1990) procedure was extended for the case of monthly data in two different —though similar— directions. Franses (1991a, 1991b) discusses a method to distinguish empirically between models (2) and (5) presented above.[6] In his second paper this author shows that conventional autocorrelation checks cannot generally make this distinction because they are not discriminative. He also shows that considering a model like (5) or similar when (2) is more appropriate yields a deterioration of forecasting performance.

Beaulieu and Miron (1993) (B&M from now on) use —in a slightly different way— the approach developed by HEGY to derive the mechanics of another procedure to test for seasonal unit roots using monthly data. These authors derive the asymptotics of HEGY's procedure for monthly data and use Monte Carlo methods to compute the finite sample critical values of the associated test statistics. The main difference with Franses' (1991a, 1991b) methodology is that B&M use mutually orthogonal regressors, obtaining a different —somewhat more complicated— test equation.

Suppose that the series of interest (X_t) is generated by a general process like:

$$\varphi(L)\,X_t = \alpha_0 + \alpha_1 t + \sum_{k=2}^{12} \alpha_k D_{kt} + \varepsilon_t \qquad (6)$$

where ε_t is a white noise process and the deterministic terms include a constant, a linear trend and seasonal dummies. "We wish to know whether the polynomial in the backshift operator, $\varphi(L)$, has roots equal to one in absolute value at the zero or seasonal frequencies. In particular, the goal is to test hypotheses about a particular unit root without taking a stand on whether other seasonal or zero frequency unit roots are present" (Beaulieu and Miron, 1993, page 307).

The auxiliary regression model that allows to perform the test is given by the following equation:

$$\varphi(L)^{*}\,Y13_t = \alpha_0 + \alpha_1 t + \sum_{k=2}^{12} \alpha_k D_{kt} + \sum_{k=1}^{12} \pi_k Yk_{t-1} + \varepsilon_t \qquad (7)$$

where Yk_t $(k = 1, 2, \cdots, 13)$ are auxiliary variables obtained by appropriately filtering the variable under study (X_t).[7] The $\varphi(L)^{*}$ polynomial is a remainder with roots outside the unit circle which allows the augmentation necessary to whiten the errors in the estimation of the above equation. "In order to test

[6] Actually, Franses' models are more general, since they include autoregressive and moving average parts, but the distinction between deterministic and stochastic seasonality is the same.

[7] The definitions of these auxiliary variables are reproduced in Appendix I. For details see Hylleberg *et al.* (1990) and Beaulieu and Miron (1993).

hypotheses about various unit roots, one estimates [the test equation] by Ordinary Least Squares and then compares the OLS statistics to the appropriate finite sample distributions based on Monte Carlo results" (Beaulieu and Miron, 1993, page 309). The inclusion or not of a trend in the deterministic part of model (7) depends upon the hypothesized alternative to the null hypothesis of 12 unit roots.

So, there are twelve possible unit roots, one non-seasonal and eleven seasonal. Out of the eleven seasonal unit roots one is real and the other ten form five pairs of complex conjugates.[8] B&M provide the asymptotic distribution of the statistics necessary to perform the tests: t_1, t_2, t_k and t_{k+1}, where $k \in \{3, 5, 7, 9, 11\}$. They also prove that the asymptotic distribution of the five t_k statistics are the same as those of the five t_{k+1}.

For ease of notation B&M write that *k* is 'odd' if $k \neq 1$ and $k \in \{3, 5, 7, 9, 11\}$ and that *k* is 'even' if $k \neq 2$ and $k \in \{4, 6, 8, 10, 12\}$. They show that all the 'odd' statistics have the same distribution when different deterministic regressors are included in the regression. The same result is shown to be true in the case of the 'even' statistics. The distributions of $t_2, \cdots, t_{12}$ are independent of constant and trend terms. These terms only affect the distribution of t_1. Also, the distribution of t_2 when dummies are included in the regression is the same as that of t_1 when only a constant is included. The finite sample distributions obtained by Monte Carlo methods display all the characteristics of the asymptotic distributions mentioned in this paragraph.

4. AN APPLICATION

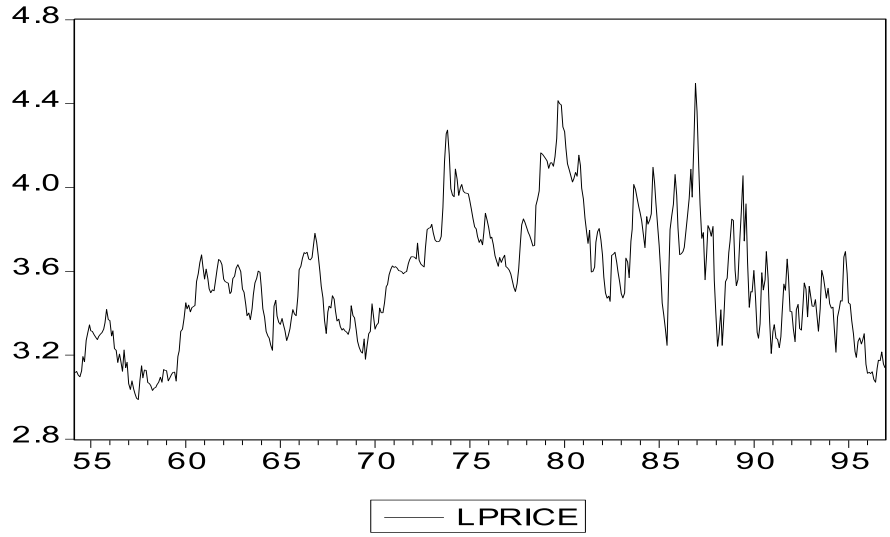
The series analyzed in this paper is formed by monthly prices received by producers of beef cattle in the State of São Paulo in the 1954-1996 period. The original data, published by the Agricultural Economics Institute of the Agricultural Secretariat of the State of São Paulo, are average (nominal) prices. The averages represent the whole State. Those prices were deflated using the General Price Index (IGP/DI) estimated by Fundação Getúlio Vargas (see Figure I).[9]

[8] See Appendix I.

[9] A careful frequency analysis of the price index series showed no evidence of seasonality.

FIGURE I

Logarithm of Monthly Beef Cattle Prices
State of São Paulo, 1954–1996



SOURCE: see text.

Explaining the general behavior of this series Mueller (1987) says that in the 1954–1979 period prices were subjected to demand pressures because, as demand increased as a result of general economic growth, supply did not follow. After 1980 several factors that influenced these prices were at play. On the supply side, there were important improvements in production technologies, the most important of which seems to have been the adoption of new varieties of pastures. On the demand side, there was a significant loss of purchasing power of consumers due to the economic recession of the 1981-1984 period. That loss was more significant for higher salaries,[10] affecting the high income-elasticity group of consumers with highest rates of consumption of beef in the country. The seasonal fluctuations observed in the price series of the State of São Paulo are due to the alternate occurrence of rain and drought seasons which affect the availability of grass and the supply of cattle (Margarido *et al.*, 1996).[11] This characteristic is similar to that observed in the beef market of the U.S. where cyclical annual variations in real prices are also attributed to bring about supply fluctuations.

Concerning price variance, it is evident that an increase occurred in the period 1972-1990. This phenomenon can be interpreted as increased uncertainty in these markets (Mueller, 1987) brought about by price controls at different levels of the marketing chain (and, sometimes, in all of them), erratic policies concerning official (subsidized) loans to the cattle production sector, etc.

[10] The rules dictated by the federal government to index public servant salaries to inflation implied real loses which were directly proportional to salary levels. During the recession years the private sector applied the same rules (Aguirre, 1984).

[11] The price series of fat cows, also sold by weight, presents these same intra-annual movements. The price series of other types of animals (calves, yearlings, 'unfinished' steers, etc.) sold on a 'per head' basis, do not show any seasonal variation.

We apply OLS to the auxiliary regression (7) in order to obtain the estimates of π_i and the corresponding standard errors. If all the estimated coefficients in this test regression are statistically different from zero the series presents a stationary seasonal pattern and the correct procedure to model the series would be using seasonal dummies. In case $\pi_i = 0$, for *i = 1,…,12*, the series is seasonally integrated and it is appropriate to use the seasonal difference filter $(1 - L^{12})$.

If $\pi_1 = 0$, then the presence of root +1 (zero frequency) cannot be rejected. There will be no seasonal unit roots if π_2 through π_{12} are significantly different from zero. When only some pairs of π's are equal to zero, one should consider using the corresponding implied operators. Abraham and Box (1978) show how this kind of operators may sometimes be enough.

TABLE I

Regression results to test for unit roots

Null Hypotheses	Estimated Statistics	Critical values[a]	
		5%	10%
$\pi_1 = 0$	– 1.82	– 2.81	– 2.53
$\pi_2 = 0$	– 4.08*	– 2.81	– 2.52
$\pi_3 = 0$	– 4.93*	– 3.29	– 1.90
$\pi_4 = 0$	– 6.28*	– 1.90	– 1.49
$\pi_5 = 0$	– 9.55*	– 3.29	– 1.90
$\pi_6 = 0$	4.59	– 1.90	– 1.49
$\pi_7 = 0$	– 2.51**	– 3.29	– 1.90
$\pi_8 = 0$	– 9.08*	– 1.90	– 1.49
$\pi_9 = 0$	– 8.67*	– 3.29	– 1.90
$\pi_{10} = 0$	1.70	– 1.90	– 1.49
$\pi_{11} = 0$	– 1.85	– 3.29	– 1.90
$\pi_{12} = 0$	– 6.26*	– 1.90	– 1.49
$\pi_3 = \pi_4 = 0$	32.25*	6.42	5.42
$\pi_5 = \pi_6 = 0$	59.64*	6.42	5.42
$\pi_7 = \pi_8 = 0$	43.28*	6.42	5.42
$\pi_9 = \pi_{10} = 0$	41.25*	6.42	5.42
$\pi_{11} = \pi_{12} = 0$	21.31*	6.42	5.42

[a] All critical values are from Beaulie and Miron (1993).
The test regression has a constant, eleven dummies and five lagged terms.
SOURCE: see text.

Table I presents a summary of the results obtained in performing the B&M tests in order to check for the integration of the series in its seasonal and nonseasonal parts, under the null hypotheses that the series is SI(1,1). The null about the presence of a unit root at the zero frequency is tested with the "t" statistic of the hypothesis $\mathrm{H}_0\colon \pi_1 = 0$ (called t_1 by B&M). The null hypotheses about the existence of seasonal unit roots are tested, in each frequency, by means of the "t" statistic associated with $\mathrm{H}_0\colon \pi_i = 0$, for *i = 2, 3, …,12*, and/or by means of the "F" statistics corresponding to the joint hypotheses $\mathrm{H}_0\colon \pi_i = \pi_{i+1} = 0$, for *i = {3, 5, 7, 9, 11}* which take into account all pairs of conjugate complex roots.[12] The significance tests for π_1 and π_2 are one-sided as well as those corresponding to π_i for 'even' *i*. On the contrary, those corresponding to 'odd' values of *i* should be two-sided.

In our case, the data reject the presence of unit roots at all seasonal frequencies. However, the existence of a unit root at the zero frequency cannot be rejected. These results imply that the seasonality present in this monthly series is partly deterministic and partly stationary stochastic, a conclusion that agrees with that of Aguirre (1997) for the quarterly averages of the same price series. As a consequence, the first difference of this series may be modeled with seasonal dummies to take seasonality into account. Prices are seasonally high in the July–November period.

In view of the above results we may assume that the model $(1-L)X_t = \sum_{i=1}^{12} \alpha_i Di_t + \varepsilon_t$ with 12 seasonal dummy variables will be a good description of the deterministic and stationary stochastic seasonal variation in the series. Based on this model we estimated recursively the coefficients corresponding to the twelve dummies obtaining the results shown in Figure II. As can be seen, neither of these recursive coefficients are exactly constant, though their variations are limited to a very narrow interval in each case. What the tests show is that this variation in the coefficients of the seasonal dummy variables is not statistically significant, and that it is correct to model the series with seasonal dummies with constant coefficients.

5. SUMMARY AND CONCLUSIONS

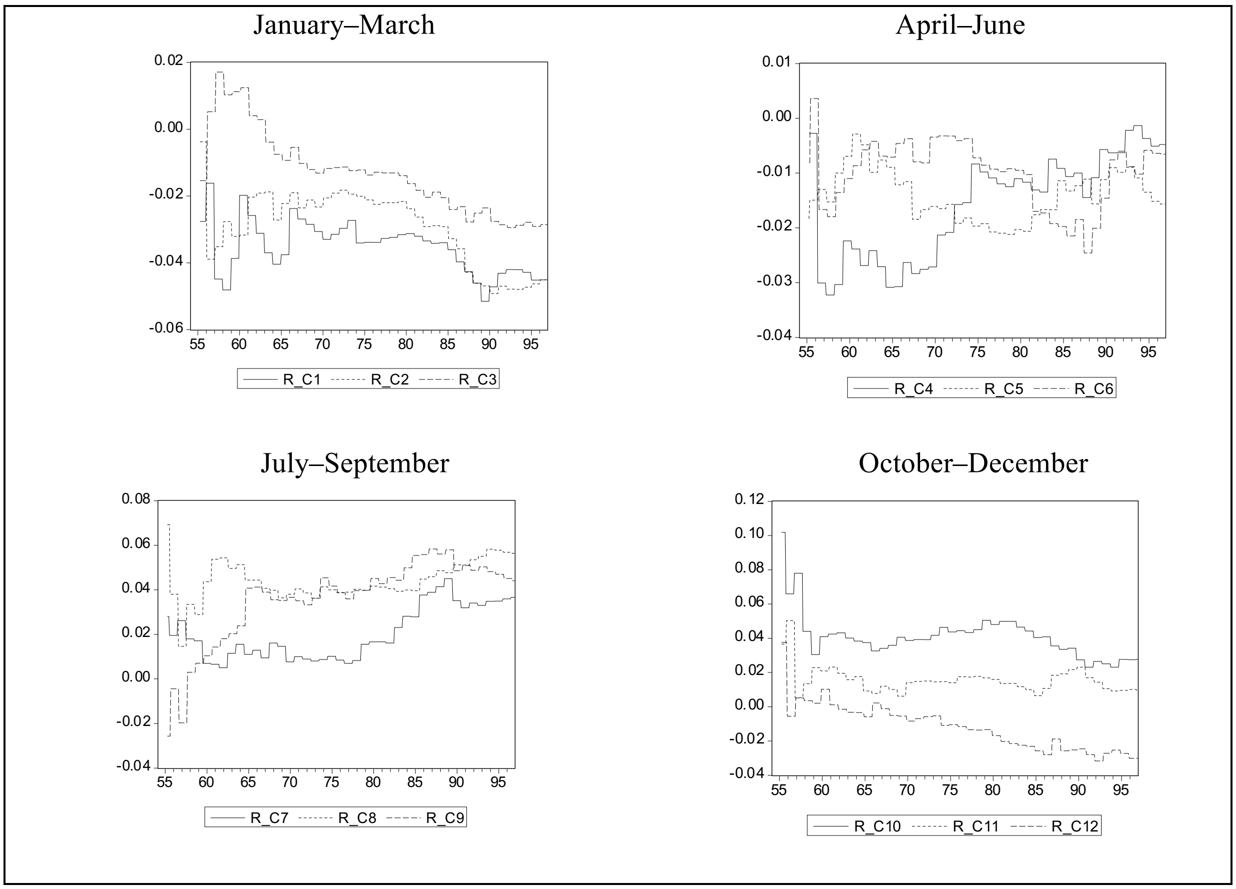
In a paper published in this journal Aguirre (1997) applies the so-called HEGY methodology to test for seasonal unit roots in processes that may also exhibit deterministic or stationary stochastic seasonality using quarterly data. In this paper we use Beaulieu and Miron's (1993) extension of that methodology for the case of monthly data. This procedure also allows to test for a unit root at the zero frequency when unit roots may be present at some or all of the seasonal frequencies.

[12] Franses (1991a) also obtains an F-statistic to test the joint hypothesis $\mathrm{H}_0\colon \pi_3 = \cdots = \pi_{12} = 0$, for the presence of unit roots in all the seasonal frequencies.

In other words, the objective of the present exercise was to look for evidence on the presence of seasonal unit roots in a microeconomic time series of monthly prices by applying a procedure specifically designed to test for the existence of nonseasonal as well as seasonal unit roots. Since the statistical properties of alternative seasonality models are also different, the imposition by assumption of one kind when another is present can result in serious biases and wrong inferences not to mention poor forecasting performance. It is therefore useful to establish empirically what kind of seasonality is present in the data.

FIGURE II

Coefficients of seasonal dummies obtained with recursive regressions



SOURCE: see text.

The analyzed data are the series of monthly average prices of steers in the State of São Paulo. The data reject the presence of unit roots at all seasonal frequencies but not at the so-called zero frequency. These results imply that the seasonality present in the series is partly deterministic and partly stationary stochastic, a fact that allows to model it just using seasonal dummy variables with constant coefficients.

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APPENDIX I

SEASONAL UNIT ROOTS

If a series has a seasonal pattern, then the differencing which removes seasonality should be of degree *s* rather than one, *i.e*. an operator $y_t - y_{t-s}$ should be applied rather than $y_t - y_{t-1}$. Often *s*–differencing also removes a trend —unless the trend is non-linear, in which case it may be necessary to take first differences of the *s*–differences in order to make the series stationary.

In the case of monthly data, the characteristic equation $(1 - L^{12}) = 0$ associated with the seasonal differencing operator has twelve roots on the unit circle, *i.e.*: $\pm 1; \pm i; -\dfrac{1 \pm i\sqrt{3}}{2}; \dfrac{1 \pm i\sqrt{3}}{2}; -\dfrac{\sqrt{3} \pm i}{2}$; and $\dfrac{\sqrt{3} \pm i}{2}$. The root $+1$ corresponds to the zero frequency, the only nonseasonal root. The seasonal roots are: -1 which corresponds to frequency π (6 cycles per year) and the five pairs of conjugate complex roots which correspond to {3,9}, {8,4}, {2,10}, {7,5}, {1,11} cycles per year with frequencies $\pm \frac{\pi}{2}$, $\mp \frac{2\pi}{3}$, $\pm \frac{\pi}{3}$, $\mp \frac{5\pi}{6}$, and $\pm \frac{\pi}{6}$ respectively (see Table AI).

TABLE AI

Unit roots and frequencies in the case of monthly data

Seasonal unit roots	Frequencies	
	In terms of π	Cycles/year
-1	π	6
$\pm i$	$\pm \frac{\pi}{2}$	3 , 9
$-\frac{1}{2}(1 \pm \sqrt{3}i)$	$\mp \frac{2\pi}{3}$	8 , 4
$\frac{1}{2}(1 \pm \sqrt{3}i)$	$\pm \frac{\pi}{3}$	2 , 10
$-\frac{1}{2}(\sqrt{3} \pm i)$	$\mp \frac{5\pi}{6}$	7 , 5
$\frac{1}{2}(\sqrt{3} \pm i)$	$\pm \frac{\pi}{6}$	1 , 11

SOURCE: see text.

Taking into account the above twelve unit roots the $(1-L^{12})$ polynomial can be written as the product of twelve factors, each factor involving one of the roots. Based on this decomposition, and using X to stand for the logarithm of price, the following auxiliary variables were defined and calculated in order to perform the tests reported in section 4:[13]

$$Y1_t = (1+L+L^2+L^3+L^4+\cdots+L^{11})X_t$$

$$Y2_t = -(1-L+L^2-L^3+\cdots-L^{11})X_t$$

$$Y3_t = -(L-L^3+L^5-L^7+L^9-L^{11})X_t$$

$$Y4_t = -(1-L^2+L^4-L^6+L^8-L^{10})X_t$$

$$Y5_t = -\tfrac{1}{2}(1+L-2L^2+L^3+L^4-2L^5+L^6+L^7-2L^8+L^9+L^{10}-2L^{11})X_t$$

$$Y6_t = \frac{\sqrt{3}}{2}(1-L+L^3-L^4+L^6-L^7+L^9-L^{10})X_t$$

$$Y7_t = \tfrac{1}{2}(1-L-2L^2-L^3+L^4+2L^5+L^6-L^7-2L^8-L^9+L^{10}+2L^{11})X_t$$

$$Y8_t = -\frac{\sqrt{3}}{2}(1+L-L^3-L^4+L^6+L^7-L^9-L^{10})X_t$$

$$Y9_t = -\tfrac{1}{2}(1-L-2L^2-L^3+L^4+2L^5+L^6-L^7-2L^8-L^9+L^{10}+2L^{11})X_t$$

$$Y10_t = \tfrac{1}{2}(1-\sqrt{3}L+2L^2-\sqrt{3}L^3+L^4-L^6+\sqrt{3}L^7-2L^8+\sqrt{3}L^9-L^{10})X_t$$

$$Y11_t = \tfrac{1}{2}(\sqrt{3}+L-L^3-\sqrt{3}L^4-2L^5-\sqrt{3}L^6-L^7+L^9-\sqrt{3}L^{10}+2L^{11})X_t$$

$$Y12_t = -\tfrac{1}{2}(1+\sqrt{3}L+2L^2+\sqrt{3}L^3+L^4-L^6-\sqrt{3}L^7-2L^8-\sqrt{3}L^9-L^{10})X_t$$

The last auxiliary variable, defined as $Y13_t = (1-L^{12})X_t$, is the dependent variable in equation (7) of section 3.1.

[13] For details see Beaulieu and Miron (1993). Franses' (1991a, 1991b) alternative methodology uses only seven auxiliary variables to perform this test.

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