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TESTING FOR SEASONAL UNIT ROOTS IN A QUARTERLY SERIES OF BEEF CATTLE PRICES IN THE STATE OF SÃO PAULO (BRAZIL)

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ABSTRACT

The last decade has witnessed a great interest in integration and cointegration analysis of economic time series. This implies the detection of the so-called unit roots, which can be non-seasonal. The main objectives of this paper is to determine the order of seasonal integration and the nature of the seasonal process generating a particular set of data. Two different tests are carried out to achieve these objectives: the DHF and the HEGY tests. The results obtained indicate no seasonal unit roots but only a zero frequency unit root. This means that the series is $I_0(1)$ with a partly deterministic seasonal pattern and that after transformation by the) = (1-L) filter it can be modelled with seasonal dummy variables.

1. INTRODUCTION[1]

Since the 1980's applied economists and econometricians have paid more attention to the long-recognised fact that trended data can be potentially a major problem for empirical econometrics. The estimation of regression models containing variables with stochastic and/or deterministic trends may provide unreliable results. Such a procedure "at best ignores important information about the underlying (statistical and economic) processes generating the data, and at worst leads to nonsensical (or spurious) results" (Harris, 1995, page 1). For this reason it is now recognised that it is the applied researcher's responsibility to test for the presence of unit roots in the data and, if they are present, to choose appropriate modelling techniques. Furthermore, "the grim fact is that, in economics, most time series ... are subject to some type of trend." (Charemza and Deadman, 1992, page 143)

Applying first differences to trend variables to remove the non-stationary (stochastic) trend is not always a good solution to the problem, since this procedure eliminates any long-run information that the series may contain. The desire to model the short-run and long-run information contained in time series data while, at the same time, considering stationarity, has led to cointegration analysis and other related methods like general-to-specific modelling in econometrics.

The usual tests for integration and cointegration are applicable to non-seasonal data, i.e. variables measured annually or variables known to have no seasonal patterns. When the data are measured **s** times during the year (s = 4 for quarterly data, s = 12 for monthly, etc.) the relevant aspects to study are the **seasonal** integration properties of the series. These properties must be established before testing for non-seasonal integration.

Another recent development in economics is the recognition among econometricians and applied economists "that seasonal variation in many economic time series is often larger and less regular than often hitherto acknowledged..." (Hylleberg, 1994, page 153). Much more attention is being paid nowadays to modelling seasonal variations, and economic considerations are catching up with pure statistical theory in the analysis of this kind of phenomena. The recognition that economic agents may decide about the degree of smoothness of their consumption and production during the year in accordance with their preferences, expectations, costs and other constraints they may face, gave rise to the following definition**:** "Seasonality is the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are influenced by the endowments, the expectations and preferences of the agents, and the production techniques available in the economy." (Hylleberg, 1994, page 164)

Unfortunately, either because it is a difficult subject still under development or because there are few research works published, the seasonal properties of time series are ignored in many cases. One clear example of this can be found in the otherwise excellent paper by Athukorala and Menon (1994). The following rather lengthy citation helps to make this point**:** “We began the estimation

[1] This section leans heavily on the following books and journal articles: Charemza and Deadman (1992), Harris (1995), Hylleberg *et al.* (1990), Hylleberg (1994) and Franses (1996).

process by testing the time-series properties of the data. The results ... suggested that the two dependent variables and most of the explanatory variables are non-stationary processes of order 1. ...(We) tested for the existence of a stable steady-state relationship between the variables... These tests failed to produce any evidence concerning the existence of a cointegrating relationship in all cases. In theory, in the absence of cointegrating relationships between non-stationary series, the ideal choice for the time-series analyst is to model in (stationary) differences of the variables. ...(We) were reluctant to simply ignore the long-run relations embodied in level variables. Our preferred strategy was to employ the general to specific modelling procedure which minimises the possibility of estimating spurious relationships while retaining long-run information." (page 275) As can be seen, even though the methodological aspects received careful consideration from the authors, no mention is made about the fact that they are working with quarterly data that could present some seasonal pattern. Later on in the estimation process they even use seasonal dummy variables in their modelling, but no explanation is given about this particular 'property of the data'.

This paper is a case study which provides an example of the application of the techniques used to determine the seasonal properties of time series data. Two different tests are applied to determine the order of seasonal integration as well as the nature of the seasonal pattern present in a quarterly series of beef cattle prices in the State of São Paulo. The organisation of the article is as follows**:** Section 2 gives a brief methodological account of unit roots in general and of seasonal unit roots; Section 3 provides a review of the existing literature;[2] the data used to perform the tests and the results obtained are described in Section 4; Section 5 presents the conclusions.

2. UNIT ROOTS AND SEASONAL UNIT ROOTS

The problem of extracting the n^{th} roots of a complex number, say $z = r(\cos\theta + i\sin\theta)$, is equivalent to solving the equation

$$z_0^n = z \qquad (1)$$

for given $z_0 = r_0(\cos\theta_0 + i\sin\theta_0)$ and positive integer **n**. (Churchill, 1967) When $z \neq 0$, there are just **n** different solutions of equation (1), namely

$$z_0 = \sqrt[n]{r}\left(\cos\tfrac{\theta+2\pi k}{n} + i\sin\tfrac{\theta+2\pi k}{n}\right) \qquad (2)$$

where k = 0, 1, 2, ... , n-1. These are the **n** values of $z^{1/n}$.

In the particular case in which $z = 1$, since $(1 = \cos 0 + i\sin 0)$, the n^{th} roots of unity can be written

[2] To the best of our knowledge there is no other published research work on this subject matter in the Brazilian literature.

$$1^{1/n} = \cos\tfrac{2\pi k}{n} + i\sin\tfrac{2\pi k}{n} \tag{3}$$

(k = 0, 1, 2, ... , n-1).

"In the complex plane the n^{th} roots of unity are the vertices of a regular polygon of **n** sides inscribed in the circle $|z| = 1$, with one vertex at the point z = 1." (Churchill, 1967, page 15) Leaving aside the simple cases n =1 and n = 2 where the roots are real, in all the other cases most of the roots of unity are complex. However, all of them —real and complex— have modulus equal to one.

2.1. Seasonality

If a series has a seasonal pattern, then the differencing which removes seasonality should be of degree **s** rather than one, i.e. an operator $y_t - y_{t-s}$ should be applied rather than $y_t - y_{t-1}$. Often *s*–differencing also removes a trend —unless the trend is non-linear, in which case it may be necessary to take first differences of the *s*–differences in order to make the series stationary.

The well-known seasonal differencing operator $(1 - L^s)$ proposed by Box and Jenkins (1976) to model a seasonal process implies the following polynomial of order **s**,

$$(1 - L^s) = 0. \tag{4}$$

When s = 4 (quarterly data), the polynomial (4) has four roots of unity (or unit roots in the econometric parlance), namely 1, *i*, -1, -*i*, in the usual counter-clockwise direction on the unit circle. For monthly data, the 12 unit roots associated with the corresponding seasonal operator are**:** ± 1, ± *i*, 0.86 ± 0.5*i*, 0.5 ± 0.86*i*, -0.5 ± 0.86*i*, and -0.86 ± 0.5*i*.

When testing for seasonal unit roots in time series data, various kinds of seasonal processes can be studied. The following three cases are considered by Hylleberg *et al.* (1990)**:**

a) purely deterministic seasonal processes;

b) stationary seasonal processes;

c) integrated seasonal processes.

The deterministic case is a process generated by seasonal dummy variables. This process will never change its shape and any shocks affecting the system will die out with time. An example of a stationary seasonal process for s = 4 is the following**:**

$$y_t = \rho\, y_{t-4} + \varepsilon_t \qquad |\rho| < 1$$

whose spectrum has a peak at both seasonal periodicities $\pi/2$ and π, as well as at zero (long-run) frequency.[3] A series $\mathbf{y_t}$ is an integrated seasonal process if it has seasonal unit roots in its autoregressive representation. Using the Box–Jenkins seasonal differencing operator assumes the existence of a unit root at frequency zero and unit roots at the seasonal frequencies [the series is $I_0(1)$ and $SI_S(1)$ or $SI_S(1,1)$]. A series may be integrated of order higher than one and also of a different order at each frequency.[4]

If the seasonal differencing operator $(1 - L^4)$ (quarterly data) is factored as

$$\begin{aligned}(1 - L^4) &= (1 - L)(1 + L)(1 - i L)(1 + i L) \\ &= (1 - L)(1 + L + L^2 + L^3) \\ &= (1 - L^2)(1 + L^2)\end{aligned} \tag{5}$$

then it is possible to interpret it as being formed by different components which can be isolated if the appropriate operators are used.[5] In this way the polynomial $(1 - L)$ —the first difference operator— with root 1 corresponds to the trend. The $(1 + L)$ term with root 'minus 1' is an integrated quarterly process at 2 cycles per year and frequency π. Finally, since the roots i and $-i$ are indistinguishable, they are left together in the operator $(1 + L^2)$ which represents an integrated quarterly process at one cycle per year and frequency $\pi/2$.

Starting from this point, Hylleberg *et al.* (1990) transform the polynomial (5) to an expression that allows them to develop a testing strategy, which will be discussed in the next section.

3. REVIEW OF THE LITERATURE

All the work on integration with annual (or otherwise non-seasonal) data assumes a unit root which corresponds to a zero-frequency peak in the spectrum. It also assumes that there are no other unit roots in the system. Dickey, Hasza and Fuller (1984), following the methodology suggested by Dickey and Fuller (1979) for the zero-frequency unit-root case, propose a test of the hypothesis $\rho = 1$ against the alternative $\rho < 1$ in the model

$$y_t = \rho\, y_{t-s} + \varepsilon_t \ .$$

The DHF test for seasonal integration resembles a generalisation of the ADF (Augmented Dickey–Fuller) test for integration in annual data. For a series measured **s** times *per annum*, this test is based on the Studentised statistic for the OLS estimate of the parameter δ in the following auxiliary regression equation**:**

---

[3] See Appendix.

[4] For simplicity we will assume that the order of integration is the same at all seasonal frequencies.

[5] In general a difference filter $(1 - L^s) = (1 - L)(1 + L + ... + L^{s-1})$ can be decomposed in a part with one non-seasonal unit root and a part with $(s - 1)$ seasonal unit roots.

$$\Delta_s z_t = \delta\, z_{t-s} + \sum_{i=1}^{k} \delta_i\ \Delta_s\, y_{t-i} + \varepsilon_t \qquad (6)$$

where the variable $\mathbf{z_{t-s}}$ is constructed according to a procedure described in Dickey *et al*. (1984) and Charemza and Deadman (1992). Instead of the dependent variable being $\mathbf{\Delta_s z_t}$ in regression (6), it is possible to follow the practice adopted by Osborn *et al*. (1988) and use $\mathbf{\Delta_s y_t}$.

If the null hypothesis is not rejected, it is common to increase the order of non-seasonal differencing required to achieve stationarity, rather than to perform higher order seasonal differencing. Economic data series are generally believed to be $SI_s(0,0)$, $SI_s(0,1)$, or $SI_s(d,0)$, so that it is expected that using *s*-differences once will be enough to remove seasonal nonstationarity. Hence, if the $\delta = 0$ hypothesis is not rejected, the next step is to consider whether the variable is $SI_s(1,1)$ rather than $SI_s(0,1)$, with the former being the new null hypothesis and the latter the new alternative. Hence, the following auxiliary equation is constructed and estimated in the same way as in the case of the ADF test**:**

$$\Delta\Delta_s z_t = \delta\ \Delta_s y_{t-1} + \sum_i \delta_i \Delta\Delta_s y_{t-i} + \varepsilon_t\ . \qquad (7)$$

To perform the test, it is necessary, once again, to examine whether δ is significantly negative or not..

As mentioned above, the DHF paper describes regression estimators of coefficients in seasonal autoregressive models. “The percentiles of the distributions for time series that have unit roots at the seasonal lag are computed by Monte Carlo integration for finite samples and by analytic techniques and Monte Carlo integration for the limit case.” (Dickey *et al.*, 1984, page 355) Tables in the paper contain distributions that may be used to test the null hypotheses that a time series has seasonal unit roots.

“A major drawback of this (DHF) test is that it doesn’t allow for unit roots at some but not all of the seasonal frequencies and that the alternative has a very particular form, namely that all the roots have the same modulus.” (Hylleberg *et al.*, 1990, page 221)

Testing for seasonal integration using the DHF test is equivalent to testing for what is called *stochastic seasonality.* The simplest form of stochastic seasonality is the process expressed as

$$y_t = y_{t-s} + \varepsilon_t$$

where $\boldsymbol{\varepsilon_t}$ is a series of i.i.d. random variables. Such a process may exhibit a seasonal pattern which varies over time. In that case, $\mathbf{\Delta_s y_t}$, defined as

$$\Delta_s y_t = y_t - y_{t-s} = \varepsilon_t$$

is stationary. Quite a different (purely deterministic) seasonal process is the one represented by

$$y_t = \alpha_1 D1_t + \alpha_2 D2_t + \alpha_3 D3_t + \alpha_4 D4_t + \varepsilon_t$$

where the Di_t (i = 1, 2, 3, 4) variables are (quarterly) seasonal dummies.

The main difference between the two forms of seasonality described above is that in the deterministic seasonal model shocks die out in the long run, while they have a permanent effect in the alternative model. In the stochastic seasonal model, a positive shock at time **t** will increase the value of $\mathbf{y_t}$ as well as the value of $\mathbf{y_{t+s}}$, $\mathbf{y_{t+2s}}$, etc. Thus, it can be extremely important to be able to distinguish between the two types of seasonality. A test which was proposed to do this job is the HEGY test (Hylleberg *et al.*, 1990).

HEGY is a formal test for the existence of unit roots which are interpreted as an indication of a stochastic seasonal pattern, and stresses the possibility that a series may be transformed into stationarity by seasonal and/or other differencing. "The first goal of this paper is developing a testing procedure which will determine what class of seasonal process is responsible for the seasonality in a univariate process (page 216). ...(We) propose a test and a general framework for a test strategy that looks at unit roots at all the seasonal frequencies as well as the zero frequency." (Hylleberg *et al.*, 1990, page 221)

This test follows the well-known Dickey–Fuller test for a unit root at the zero frequency and the DHF test for testing for a unit root in models such as $y_t = \rho\, y_{t-s} + \varepsilon_t$ for s = 2, 4, 12. For quarterly data, the polynomial $(1 - L^4)$ can be factored as in (5). In order to test the hypothesis that the roots of the polynomial lie on the unit circle against the alternative that they lie outside the unit circle, the authors rewrite the autoregressive polynomial in a convenient form. The resulting testable model they propose is

$$\Delta_4 y_t = \mu_t + \pi_1 Y1_{t-1} + \pi_2 Y2_{t-1} + \pi_3 Y3_{t-1} + \pi_4 Y3_{t-2} + \sum_{i=1}^{k} \delta_i\, \Delta_4 y_{t-i} + \varepsilon_t \qquad (8)$$

which can be estimated by OLS and the statistics on the π's used for inference, and where**:**

μ_t may contain a constant, three seasonal dummies and/or a trend;

Y1 = $[(1 + L + L^2 + L^3)\, y_t]$ is the transformation retaining the unit root at the zero frequency (Figure II-D);[6]

Y2 = $[-(1 - L + L^2 - L^3)\, y_t]$ is the transformation that retains the unit root at the semi-annual frequency (Figure II-B);

Y3 = $[-(1 - L^2)\, y_t]$ is the transformation retaining the unit root at the annual frequency (Figure II-C).

The order of the lags (value of **k** in the summation) is determined using diagnostic checks such that the estimated error process is approximately white noise. The test is conducted by estimating the auxiliary regression in equation (8). Only if each of the π's are different from zero can

[6] Figures I and II are presented in the next section.

the series be said to have no unit roots at all and to be stationary. The test for the existence of a zero frequency unit root against a stationary alternative is a test of the null hypothesis $\pi_1 = 0$, against $\pi_1 < 0$, with the **t** statistics distributed as the usual Dickey–Fuller t-values. The test of a seasonal unit root at the semi-annual frequency π (or ½ of 2π) is a test of the null $\pi_2 = 0$, against $\pi_2 < 0$, with the same t-value distribution as in the zero frequency case. The test of an annual unit root is more complicated due to the presence of complex conjugate roots. There are two ways to conduct the test (Hylleberg *et al.*, 1990). One of them is a two-step procedure in which a test of the null hypothesis $\pi_3 = 0$ against $\pi_3 \neq 0$ with a two-tailed test is performed. The distribution of the t-value for this test is given in the HEGY paper. In case $\pi_3 = 0$ a test of $\pi_4 = 0$ against the alternative $\pi_4 < 0$ can be based upon the t-value of the π_4 estimate and critical values are supplied by the same authors.

4. SEASONALITY IN A QUARTERLY SERIES OF LIVE CATTLE PRICES: A CASE STUDY

4.1. The data

The series analysed in this paper is formed by quarterly prices received by producers of beef cattle in the State of São Paulo (Brazil) in the 1954-1996 period. It covers a total of 43 years or 172 quarters (Figure I–A). This series illustrates the complexity and variation of typical economic data. The original data, published by the Agricultural Economics Institute of the Agricultural Secretariat of the State of São Paulo, are monthly average (nominal) prices. The averages represent the whole State. Those prices were deflated using the General Price Index (IGP/DI) estimated by Fundação Getúlio Vargas (FGV). The monthly real prices were averaged into quarterly prices in order to perform this exercise. This procedure implies using a low-pass filter that filters away the high-frequency variations present in the data.

Panel A of Figure I shows that the seasonal pattern varies more in the later part of the period. This can also be seen in the first difference series of Panel B. In the next panel four series are presented**:** one is the 'first quarter series' containing all the first quarter observations of the period minus the calendar year average; another is the second quarter series, etc. This procedure is adopted to remove the trend and clarify the data message.[7] In the extreme case of completely regular and no changing seasonal pattern the four lines in Panel C of Figure I would be parallel straight lines, which obviously is not the case in our quarterly price series. It is apparent that a major change occurred around 1973 (observation number 20). The later period is also characterised by a larger price variance and coincides with the occurrence of high inflation rates in the country.

[7] This kind of graph is used by Hylleberg (1994).

TABLE I

Mean, standard deviation and coefficient of variation of price of beef cattle and of inflation rate in two periods(*)

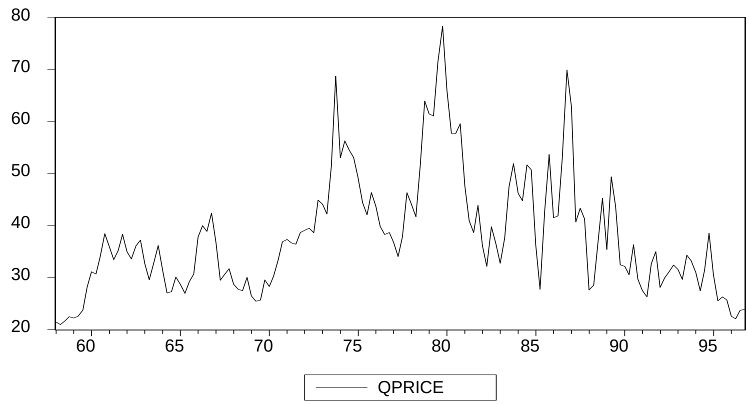
PERIOD	PRICE			INFLATION RATE		
	Mean	St.dev.	C. var.	Mean	St.dev.	C. var.
Jan/1954 – Jun/1973	30.66	6.27	20.45	1.66	0.79	47.59
Jul/1973 – Dez/1996	41.60	12.41	29.83	11.17	12.82	114.77

(*) Coefficient of variation in percentage.

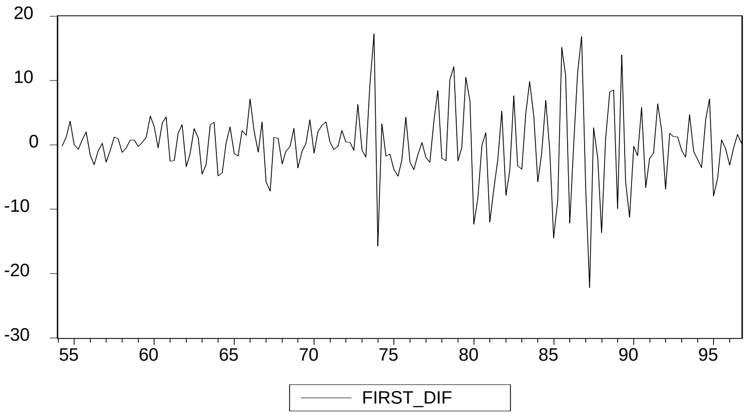
Table I presents the mean, standard deviation and coefficient of variation for the quarterly price and for monthly inflation rates (as measured by the General Price Index) in both periods. This table shows that the standard deviation of prices doubles in the second period. This coincides with the higher (and more volatile) inflation rates.

FIGURE I

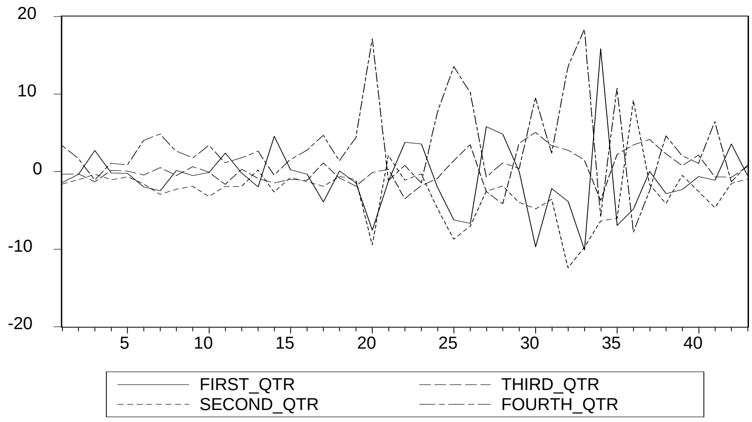
A **Quarterly Prices of Beef Cattle**



B **First Difference of Prices**



C The Four Quarters Less the Year Average

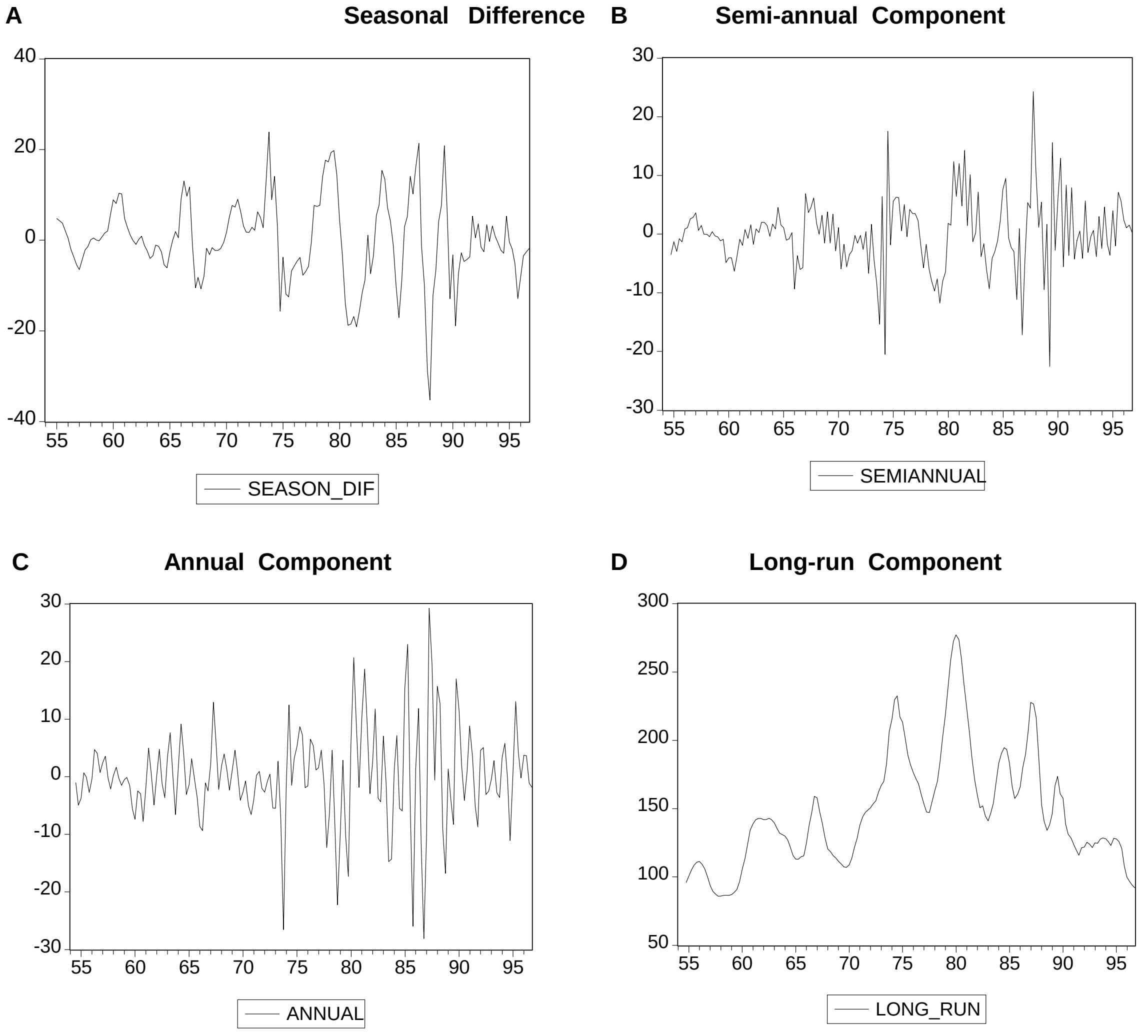


As it was explained in Section 2 the seasonal operator proposed by Box and Jenkins can be factored into four parts. This procedure allows us to describe some characteristics of the series which are associated with intra-annual cyclical components. If we apply the $(1 - L^4)$ seasonal operator to our series we obtain a series in which the seasonal pattern and the trends seem to have been removed (Figure II-A). The operator $(1 - L)$ produces the first difference series of Figure I-B. This series is called integrated at the long frequency zero.[8] The $[-(1 - L)(1 + L^2)\, y_t] = [-(1 - L + L^2 - L^3)\, y_t]$ operator is a transformation which preserves the frequency ½ corresponding to a two quarter period (Figure II-B). The transformation $[-(1 - L)(1 + L)\, y_t] = [-(1 - L^2)\, y_t]$ retains the frequency ¼, corresponding to a four-quarter period (Figure II-C). Finally, the transformation $[(1 + L + L^2 + L^3)\, y_t]$ removes the seasonal unit roots and preserves the long-run or zero frequency unit root. This is a seasonally adjusted version of our original series (Figure II-D).

[8] See Appendix.

FIGURE II

Transformations of quarterly beef cattle prices
1954(1) — 1996(4)



4.2. The results

The application of the DHF test to the quarterly series of beef cattle prices did not produce clear-cut results. The estimation of the auxiliary regression (6) produced a negative but non-significant estimate for δ which does not reject the null hypothesis that the process is $SI_4(0,1)$. Continuing with the test procedure, equation (7) was estimated. In this case, the appearance of a (nearly) singular matrix interrupted the estimation process of adding new lagged terms before white noise residuals were obtained.

Summing up, it was not possible to establish the order of integration of the seasonal process by means of the DHF test. No evidence of overdifferencing appeared though; if it had appeared, it would have meant that either the series is non-seasonal or that there is no differencing procedure to make it stationary. In these circumstances the use of the HEGY test is a useful alternative course of action to try to find out the nature of the seasonal pattern present in the data.

The execution of the HEGY test by means of estimation of the auxiliary regression (8) with a constant and three seasonal dummy variables produced the results shown in Table 2. The estimates from this table and the critical values provided by (Hylleberg *et al.*, 1990) are the following**:**

Parameters	Estimates	Crit. Values (5%)
π_1	-2.24	-2.91
π_2	-6.81	-2.89
π_3	-2.98	-1.96
π_4	-3.65	-3.38

As can be seen, since the $\pi_1 = 0$ hypothesis cannot be rejected, the presence of a non-seasonal unit root of value 1 cannot be rejected either. The situation is different in the other three cases since we reject the existence of seasonal unit roots. These results, together with the fact that two seasonal dummies have significant coefficients, seem to indicate that the seasonal pattern present in the data is a mixture of deterministic and stationary stochastic seasonality.. In this interpretation, the first and second quarters would correspond to seasonally low prices and the other two to prices above the yearly average.

TABLE 2

Results of the HEGY test(*)

LS // Dependent Variable is D(QPRICE,0,4)
Sample (adjusted): 1955:4 1996:4
Included observations: 165 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.4168	1.5924	3.4017	0.0009
Q1	-5.0669	1.2027	-4.2113	0.0000
Q2	-2.7947	1.3805	-2.0244	0.0447
Q3	-1.5546	1.2160	-1.2785	0.2030
Y1(-1)	-0.0208	0.0093	-2.2378	
Y2(-1)	-0.7769	0.1140	-6.8124	
Y3(-1)	-0.2302	0.0772	-2.9799	
Y3(-2)	-0.2773	0.0761	-3.6458	
D(QPRICE(-1),0,4)	0.0464	0.1141	0.4064	0.6850
D(QPRICE(-2),0,4)	0.2662	0.1123	2.3712	0.0190
D(QPRICE(-3),0,4)	-0.2367	0.0784	-3.0206	0.0030
R^2	0.768469		Mean dep var	-0.101310
(Adjusted) R^2	0.753434		S. D. dep var	9.189474
S. E. of regression	4.563068		Akaike info crit	3.100331
Sum squared residuals	3206.525		Schwarz criterion	3.307394
Log likelihood	-478.9022		F-statistic	51.11373
Durbin-Watson statistic	2.005245		Prob(F-statistic)	0.000000

(*)The probabilities corresponding to the t-values associated with the Yi variables are not reported because they do not belong to the usual t-Student distribution.

In order to investigate the adequacy of the estimated empirical model we used the Ljung-Box Q statistic to test for the joint significance of the first 60 autocorrelations in the estimated residuals. The null that all of the autocorrelations are zero was not rejected. The same result is obtained when the F-version of the Lagrange Multiplier (Breusch-Godfrey) serial correlation test is used to test for first, fourth and up to twelfth order residual autocorrelation. However, the ARCH test and White's heteroskedasticity test are significant indicating non-constant variances in the residuals. The $\chi^2(2)$ LM test for residual normality is also significant. The above evidence of heteroskedasticity in the disturbances invalidates the conventional standard error formulas and the associated inference procedures. As a consequence of this the results in Table II must be treated with some caution. It is evident that the changes shown in Figure I-C are not so much the result of changing seasonality but the effect of high inflation rates on price variability.

In spite of the doubts raised by the heteroskedastic residuals it is interesting to point out that our results are in line with many others obtained with very different data sets as reported in Osborn (1990), Otto and Wirjanto (1990), Hylleberg *et al*. (1993) and Mills and Mills (1992).[9] In the majority of those cases only the non-seasonal unit root is found. In our case, such a result implies that a first difference is enough to make the quarterly series of beef cattle prices stationary. The use of $\Delta\Delta_4$ or Δ_4 differencing filters would bring about overdifferencing. In addition to that, seasonal dummies can take care of the deterministic seasonal pattern present in the data. One obvious way to check these conclusions would be to run the HEGY test for the same series with monthly data, properly transformed to stabilise the variance, in order to see if these findings are confirmed.

5. SUMMARY AND CONCLUSIONS

Problems of spurious regression and correlation have been known since the early days of statistics and they may be extremely serious in econometrics since most economic series are trended. One possible procedure to induce a stationary mean is the use of differencing operators of various kinds. This has the drawback that important long-run information contained in the data is lost.

The tests for seasonal integration are a preliminary step if testing for seasonal cointegration and the estimation of cointegration vectors are attempted with potentially seasonal data. In addition to being more difficult than a similar task based on annual data, these procedures are still in a process of development. Currently, little published work in this field exists, specially in Brazil.

The Box-Jenkins approach assumes trends and seasonals to be stochastic and proposes a seasonal differencing operator to make a seasonal series stationary. The assumption of a certain differencing filter amounts to an assumption of the number of seasonal and non-seasonal unit roots in a time series.

The DHF test —an extension of the augmented Dickey-Fuller approach—applied to the quarterly series of beef cattle prices in the state of São Paulo did not produce clear-cut results in relation to the order of seasonal integration of the data. For this reason, the HEGY test becomes an important procedure to test for the presence of seasonal unit roots. This alternative method assumes the maximum number of unit roots to be four, i.e. one non-seasonal and three seasonal unit roots.

[9] Cited in Franses (1996).

"The HEGY test approach is in a sense a general-to-simple approach since it investigates the empirical adequacy of filters (1 - L) and (1 + L) against the 'more general' (1 - L^4) filter". (Franses, 1996, page 318)

The HEGY test shows that certain π_j are zero in cases where the corresponding unit roots are on the unit circle. Hence, testing for the significance of the π_j is equivalent to testing for seasonal and non-seasonal unit roots.

As $\pi_1 = 0$ the presence of a non-seasonal unit root of value 1 cannot be rejected. In the case of π_2 as it is statistically different from zero the seasonal unit root of value 'minus 1' can be rejected. The significance of the other two π estimates also results in the rejection of the corresponding seasonal unit roots.

In sum, the results of the tests indicate that the series under analysis has only one non-seasonal unit root and a partly deterministic seasonal pattern. For that reason, these seasonal fluctuations are probably caused by calendar and weather effects much more than by the behaviour of economic agents. In general, these kind of fluctuations are important because economic agents may take the seasonal variation in some variables into account when forming expectations and planning their future behaviour in relation to other variables.

As a consequence of the existence of heteroskedastic residuals in the model estimated to carry out the test, the whole testing procedure is under suspicion. This points the way for further research which could be performed with the original monthly data and the use of a larger number of seasonal unit roots.

APPENDIX

The characteristic equation $1\text{-}L^4 = 0$ associated with the seasonal differencing operator has four roots on the unit circle. The first root, in addition to having modulus one is also equal to 1. This root is associated to the so-called zero frequency related to the trend. The other three roots are called seasonal and are related to cycles inside the year. All four roots constitute a (multiplicative) group structure and their properties can be studied considering subgroups of that structure. However, it is possible to present an intuitive discussion of the seasonal frequencies based upon the following reasoning where we use s = 4 (quarterly data) and s = 12 (monthly) as concrete examples because these are the most usual data sampling periods used in economic research.

When s = 4 there are four different possible situations concerning intra-annual cycles**:**[10] no cycles at all, one, two or four cycles in the year.[11] The first possibility also implies no cycle per quarter, the second implies ¼ cycle per quarter, etc. So, taking 2π as corresponding to the whole year, we have**:**

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[10] In these two examples the degree of the polynomials are even numbers. This is important because in dealing with seasonality it does not make sense to consider degrees given by prime numbers. If the degree of the polynomial is a prime number we have the **primitive** n^{th} roots of unity.

[11] The case of three cycles per year is the conjugate partner of one cycle per year.

zero cycle per year $\mapsto$ zero cycle per quarter $\mapsto$ 0 x 2π = zero frequency
one cycle per year $\mapsto$ ¼ cycle per quarter $\mapsto$ ¼ x 2π = frequency $\pi/2$
two cycles per year $\mapsto$ ½ cycle per quarter $\mapsto$ ½ x 2π = frequency π.

The fourth possibility would give as result a 2π frequency which is equivalent to the first one.

Similar reasoning when s = 12 determines the following seasonal frequencies in addition to the (long-run) zero frequency**:** $\pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, $5\pi/6$, and π. In general, if we consider j cycles per year this corresponds to j/s cycles per sub-period and the seasonal frequencies are given by $2\pi j/s$ (j = 1, ..., s/2).

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